1 Introduction

The development of credit risk management over the past few years has brought rating migration modeling to the forefront of attention. A chain of major sovereign defaults on foreign-currency denominated debts explains the impetus – the 1994-1995 Mexican-peso crisis, the 1997 Asian currency crisis, the 1998 Russian ruble devaluation and, more recently, the 2001 credit failure in Argentina. These events increased the financial institutions concerns about their risk exposure to emerging market borrowers. Sovereign credit ratings have since played a major role in modern risk management, valuation and international capital allocation. Financial institutions use ratings to feed VaR models (using, for instance, J.P. Morgan's Credit Metrics tool), to price risky loans anddetermine concentration limits.[1], [2] Furthermore, the New Basel Accord (Basel II, 2001) permits banks to use internal rating systems to determine the regulatory capital against their credit exposure. Finally, ratings are key inputs to prominent models for the term structure of credit-spreads (Jarrow et al., 1997) and the pricing of credit derivatives (Kijima and Komoribayashi, 1998).

All applications require a mapping of the rating history into transition probabilities which emphasises the quest for accurate estimation of the latter. The defaulted amount of sovereign debt and the subsequent scale of losses in the last decades exceed by far those of corporate defaults. Nonetheless, implementation of extant risk management approaches to sovereigns has received scant attention. Sovereign migration modeling is fraught with di culties, mainly because of data limitations – sovereign rating histories are relatively short and the cross-sectional dimension is also small. Accurate migration matrix estimation presumes not only enough observations but also enough transitions from each rating category. Thus, the problem becomes more pervasive for low credit-quality (emerging market) issuers because until very recently ratings were mainly produced for industrialized sovereigns.[3] These issues raise concerns about the reliability of sovereign transition matrix estimates. One remedy could be to use samples of corporate bond ratings which have a much longer history.[4] This presumes that the sovereign migration process resembles that of corporates whereas empirical evidence reveals significant discrepancies. Jackson and Perraudin (2000) find that, on average, sovereign credit spreads are substantially lower than those of equally rated corporates. Cantor and Packer (1996) find that sovereign ratings exhibit more discrepancies than corporate ratings across agencies. Nickell et al. (2000) show that the transition probabilities of US industrials di er significantly from those of equally rated sovereigns.

[1] For surveys of credit portfolio models see Crouchy et al. (2000), Gordy (2000) and Saunders and Allen (2002).

[2] The price of, say, a Baa bond has been found to increase by about 1-2% upon upgrade and to decrease by 30-50% upon default (Saunders and Allen, 2002, ch.6).

[3] In 1993 Moody's foreign currency bond ratings pertained to 24 sovereigns of which only 8 were non-industrialized countries. The number of rated sovereigns has increased substantially over the last decade (e.g. 72 in 2004) with a higher proportion of emerging economies (76% in 2004).

[4] Moody's corporate ratings database contains about 60,000 annual observations from 1970 to today.

The usual framework for modeling rating migration is a discrete time-homogeneous Markov process. Homogeneity means that the migration probabilities are constant over time. This assumption simplifies estimation but its plausibility is questionable. Time-heterogeneity is by now a stylized fact of corporate rating migrations. The latter has been attributed either to non-Markovian behaviour or to business cycle dependence. Several studies address the hypothesis of rating drift or momentum which implies that prior rating changes carry predictive power for the direction of future changes and hence the rating process is non-Markov. Downgrade and, to a small extent, upgrade momentum in corporate ratings is supported by Lucas and Lonski (1992), Altman and Kao (1992), Carty and Fons (1993), Altman (1998), Kavvathas (2001), Bangia et al. (2002) and Lando and Skodeberg (2002). The duration e ect, another non-Markov property, is the relationship between the time spent in a given rating and the transition probability. Lando and Skodeberg (2002) and Kavvathas (2001) both confirm earlier findings by Carty and Fons (1993) on the presence of duration e ects in corporate ratings but, the evidence on the direction of the relationship is conflicting. The strong impact of business cycles variation on transition probabilities, and especially on default probabilities of speculative bonds has been documented in Belkin et al. (1998), Nickell et al. (2000), Bangia et al. (2002) and Kavvathas (2001). On the other hand, discrete modeling entails loss of information regarding the exact timing of rating changes and the duration in each rating. The discrete multinomial approach typically adopted by leading rating agencies (Carty and Fons, 1993) and in most of the academic literature (Bangia et al., 2002) produces zero probability estimates for unobserved transitions by construction. For corporate ratings, Lando and Skodeberg (2002) first illustrate how continuous hazard rate methods can facilitate probability estimates of transitions (e.g. from Aa to default) that are not observed in the data. These probability estimates are of consequence because such transitions can actually occur given a large enough sample. This argument is implicitly supported by Basel II which establishes a minimum probability of 0.03% for such (unobserved) rare events. Further, a positive risk weight is assigned for sovereigns rated Aa to Baa which implies that Basel II recognizes the risk of default from these ratings. Against this background, a spur of research on the direct estimation of migration matrices is emerging that draws upon continuous time (intensity-based) methods and incorporates time-heterogeneity.

To our knowledge, only two studies focus on the estimation of sovereign transition matrices in the literature and both consider discrete time approaches. Wei (2003) proposes a multi-factor Markov chain model that accommodates time-heterogeneity but recognizes the limitations of the model to sovereign debt due to the small samples available. Hu et al. (2002) exploit bond ratings and sovereign default data in an homogeneous ordered probit framework to overcome the small-sample problem. Very little is known on the finite-sample properties of credit migration matrix estimators particularly in the context of sovereigns. There are two comparative studies available (Jafry and Schuermann, 2004; Christensen et al., 2004) but their focus is corporate debt. The extensive evidence that sovereign and corporate credit ratings behave differently prompts the thought that a given estimation method may have different properties in these two worlds. The present paper is thus motivated in two directions. First, the properties of the underlying Markov process for corporates and sovereigns may be different. For instance, there is consensus that corporate rating transition probabilities are time-varying and therefore hazard rate methods that allow for this heterogeneity are superior. However, this is unclear for sovereigns. Second, given that sovereign ratings are more stable than corporate ratings, continuous methods are expected to be even more fruitful relative to discrete ones.

The purpose of this paper is to compare different approaches to the estimation of rating transition probability matrices for sovereigns. The estimation methods we consider differ with respect to their assumptions – a discrete multinomial (or cohort) method and two continuous hazard rate (or duration) methods. The hazard methods differ in that one imposes the assumption of time-homogeneity whereas the other relaxes it. A matrix-norm statistic based on the spectral decomposition is used to gauge the dynamics or overall mobility implied by each transition matrix.[5] We assess the finite sample bias and variability of rival estimators. In order to conduct statistical tests we deploy a bootstrap method that facilitates the empirical distribution of the transition probabilities and of the mobility statistic. Two conjectures are explored. One is that continuous (versus discrete) estimation methods bring efficiency gains in the small samples of sovereign transitions. The other is that there is heterogeneity in the sovereign rating migration process.

The contribution of this study to the international credit risk literature is threefold. First, it investigates the value of continuous estimators that incorporate full information on the exact timing of rating transitions and on rating duration. Second, it assesses whether imposing the assumption of homogeneity results in transition matrix estimators that exhibit larger small-sample bias and variability than heterogeneous estimators. Third, it provides the first tests for the presence of rating momentum and duration effects for sovereigns which invalidate the Markov assumption and induce a particular type of time-heterogeneity. For this purpose, we employ spectral analysis and panel logit models. We find that the continuous (homogeneous) hazard rate estimator produces more reliable default probability estimates than the discrete multinomial approach. The efficiency of the former is further enhanced upon relaxing the homogeneity assumption. The homogenous estimator generally produces relatively high sovereign default probabilities. Furthermore, there are significant downgrade momentum and duration effects, consistent with the extant evidence on corporate ratings.

The remainder of the paper is organised as follows. Section 2 outlines the Markov chain framework for the rating process and discusses the three estimators under study. Section 3 introduces the dataset. Section 4 discusses the bootstrap simulation results. Section 5 elaborates on the findings for non-Markov effects while a final section concludes.

[5] Less dynamics means smaller probability mass at off-diagonal positions or equivalently, a smaller likelihood of transitions.

Methodology

This section discusses three distinct credit migration estimators. The mainstream literature on migration matrix estimation essentially rests on two assumptions. First, the future rating only depends on the current rating and is independent of the rating history (Markov property). Second, the transition probabilities are constant over time (homogeneity). We start by outlining the time-homogeneous continuous Markov chain model for credit ratings.

Let S denote the transition space and $i = 1, 2, ..., K; i \quad S$ are the available credit ratings. Moody's bond rating system has seven coarse states (Aaa, Aa, A, Baa, Ba, B, C), twenty three finer states (Aaa1, Aaa2, Aaa3, Aa1,..., Ca) plus the default (D) state. We adopt the broad scale so $i = 1$ is the highest (Aaa) rating and $i = K = 8$ denotes default. Let $P(s,t)$ denote the $K \times K$ transition probability matrix generated by a continuous Markov chain η so that

$$p_{ij}(s,t) = \Pr(\eta_t = j \,|\eta_s = i) \ , \ s < t \tag{1}$$

is the probability that a sovereign rated i at time s migrates to rating j at time t. If the Markov chain is homogeneous, then the transition matrix only depends on the transition horizon ($\Delta t = t - s$) but not explicitly on time and thus we have a family of transition matrices, $P_{\Delta t}$, indexed by Δt. The transition matrix for horizon $n\Delta t$ is simply $(P_{\Delta t})^n$.[6] Hereafter, P denotes the transition matrix for a homogeneous Markov chain over a one-period (i.e. $\Delta t = 1$) horizon

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1K} \\ p_{21} & p_{22} & p_{23} & \cdots & p_{2K} \\ \vdots & & \ddots & & \vdots \\ p_{K-1,1} & p_{K-1,2} & p_{K-1,3} & & p_{K-1,K} \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix} \tag{2}$$

where $p_{ij} \geq 0 \quad i, j$, $\sum_{j=1}^{K} p_{ij} = 1 \quad i$ and default is treated as an absorbing state.

Discrete Multinomial (DM) Estimator

The conventional migration matrix estimator is a discrete multinomial (DM) approach based on annual migration frequencies. Transitions away from a given state $i = 1, 2, ..., K$ over a one-year horizon are assumed to follow a multinomial distribution with $K - 1$ outcomes and associated probabilities p_{ij}, $j = 1, .., K$, $i \neq j$. Let $N_i(t)$ denote the number of sovereigns that start year t in state i and $N_{ij}(t, t+1)$ the number of those that migrate to j by the start of year $t+1$. The annual migration frequency over is $\frac{N_{ij}(t,t+1)}{N_i(t)}$.

[6] See Norris (1997) for a depth-in discussion of continuous Markov chains.

Let us assume homogeneous Markov chain dynamics for the rating process (time independence) and also independence between the multinomial experiments for di erent issuers (cross-section independence). Accordingly, the maximum likelihood estimator (MLE) of the one-year transition probability based on the pooled ratings is defined as

$$\hat{p}_{ij} = \sum_{t=1}^{T} w_i(t) \frac{N_{ij}(t,t+1)}{N_i(t)} = \frac{\sum_{t=1}^{T} N_{ij}(t,t+1)}{\sum_{t=1}^{T} N_i(t)} \tag{3}$$

where T is the number of sample years and $w_i(t) = \frac{N_i(t)}{\sum_{t=1}^{T} N_i(t)}$ is the weight for the year t migration frequency. Many studies use instead the sample average of the year-on-year migration frequencies $\tilde{p}_{ij} = \frac{1}{T}\sum_t \frac{N_{ij}(t,t+1)}{N_i(t)}$ as an estimator (Bangia et al., 2002; Hu et al., 2002). However, this coincides with (3) only when the number of sovereigns rated i remains constant over the sample period, $N_i(t) = N_i$. This implies that the annual rating inflow is equal to the outflow which is implausible.

Israel et al. (2001) argue that the transition probability matrices obtained from the discrete estimator (3) are typically not consistent with a continuous Markov chain process mostly because of the concentration of probability mass around the main diagonal. Moreover, the one-year discrete estimator (3) neglects information about within-year rating transitions and about rating duration. In the context of sovereign ratings on which data is scarce it is crucial to account for as much additional information as possible. This motivates the following hazard rate estimators.

Homogeneous Hazard Rate (HHR) Estimator

Let Λ_t be the heterogenous Markov chain generator or intensity matrix

$$\Lambda_t = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} & \dots & \lambda_{1K} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} & \dots & \lambda_{2K} \\ \vdots & \vdots & \ddots & & \vdots \\ \lambda_{K-1,1} & \lambda_{K-1,2} & \lambda_{K-1,3} & & \lambda_{K-1,K} \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$

where the o -diagonal transition intensity

$$\lambda_{ij}(t) \equiv \lim_{\Delta t \to 0^+} \frac{p_{ij}(t \le \tau_i \le t + \Delta t \mid \tau_i \ge t)}{\Delta t} \ge 0 \tag{4}$$

is defined as the instantaneous rate of transition from rating i to j at $\tau = t$ conditional upon being ('surviving') in state i up to time t. The diagonal element

$$\lambda_{ii}(t) \equiv \lambda_i(t) \equiv -\sum_{j=1, j\neq i}^{K} \lambda_{ij}(t), \; i \neq j \tag{5}$$

is the hazard rate function or the instantaneous rate of transition from i at time t conditional upon being in state i up to time t. The probability of leaving rating i over any time horizon Δt can be approximated by $\lambda_i(t)\Delta t$. The entry λ_{ij} in a transition intensity matrix Λ is driven by a random duration τ_i which is subject to censoring or discontinuation at both ends of the sample. For instance, one does not know for how long beyond the end of the sample each individual survived in a given rating; this is a case of right censoring.[7] Any rating withdrawals also result in right censored durations.

The maximum likelihood formulated to estimate Λ under the assumption of independent durations accounts for the contribution of each possible transition or, equivalently, duration (i.e. from i to j, $j = 1, ...K$, $j \neq i$) to the overall transition probability for rating i. The contributions of N_{ij} migrations (or durations) from state i to state j occurring at times t_m, $m = 1, ..., N_{ij}$, and of N_{ic} censored durations (at rating i) occurring at t_m, $m = 1, ..., N_{ic}$, formulate the likelihood function for the transition from state i as

$$L_i = \left[\prod_{j=1, j\neq i}^{K} \prod_{m=1}^{N_{ij}} \lambda_{ij}(t) \exp(- \int_0^{t_m} \left\{ \sum_{j=1, j\neq i}^{K} \lambda_{ij}(t) \right\} du) \right] \prod_{m=1}^{N_{ic}} \exp(- \int_0^{t_m} \left\{ \sum_{j=1, j\neq i}^{K} \lambda_{ij}(t) \right\} du)$$

If the intensities are homogenous, $\Lambda_t = \Lambda$, maximisation of the log-likelihood with respect to λ_{ij} gives a simple closed-form solution for the transition intensities

$$\hat{\lambda}_{ij} = \frac{N_{ij}(T)}{\sum_{m=1}^{N_i} t_m} \tag{6}$$

where $N_{ij}(T)$ is the number of transitions from i to j, $(i \neq j)$, within the window $[0, T]$ such that the total number of observations in i within this period is $N_i(T) = N_{ij}(T) + N_{ic}(T)$. The above MLE can also be written as $\hat{\lambda}_{ij} = \frac{N_{ij}(T)}{\int_0^T Y_i(u)du}$ where $Y_i(u)$ is the number of sovereigns rated i at time u and so the denominator of $\hat{\lambda}_{ij}$ gives the total time spent in state i by all the sovereigns in the sample. The transition probability matrix over a Δt horizon is estimated as

$$\hat{P} = \exp(\hat{\Lambda}\Delta t), \ \Delta t \geq 0 \tag{7}$$

where $\exp(\hat{\Lambda}\Delta t) = \sum_{k=0}^{\infty} \frac{(\hat{\Lambda}\Delta t)^k}{k!}$.[8]

The homogeneous hazard rate (HHR) estimator based on Eq.(7) is expected to be more e cient than the DM counterpart, Eq. (3), for several reasons. First, over a given time horizon of one year (Δt), if a country migrated from Aaa−→Aa−→ A, then the transition to the intermediate state Aa contributes to the estimation of the transition probability from Aaa to Aa and from Aa to A through (6). In the discrete framework, however, this information is ignored. Second, the continuous estimator accounts for the exact date when a sovereign receives a new rating and also for each rating duration. In the above example, the

[7] Left censoring refers to the corresponcing situation at the beginning of the sample window.

[8] It is only under homogeneity that a simple mapping from transition intensities to transition probabilities exists.A detailed exposition of failure time analysis can be found in Kalbfleisch and Prentice (2002).

time spent in the intermediate state Aa is accounted for in the estimation of the transition intensity $\hat{\lambda}_{ij}$ for $i = Aa$. Third, it readily accommodates right censoring by using information for the obligors up to the day of withdrawal and thus obligors ending the year as withdrawn ratings will not be discarded as in the DM estimator. Fourth, it generally yields non-zero probabilities of rare transitions between states, even if they are not observed in the sample, as long as an 'indirect' transition from one state to the other occurs. The DM estimates for these unobserved transitions would have been zero. For instance, suppose that no direct defaults from state Aa are observed, but we observe migrations from Aa to B and then to default. Then as long as the probability of migrating to B and the probability of default from B are both positive, the continuous estimate for the default probability from Aa is strictly positive. Finally, the estimates of the transition intensity matrix can be easily transformed into transition probabilities over any time horizon.

Non-homogeneous Hazard Rate (NHHR) Estimator

The nonparametric approach of Aalen and Johansen (1978) facilitates a generalization of the above continuous hazard rate estimator to allow for heterogeneity in the underlying Markov process. Let $P(s,t)$ be the rating transition matrix over the horizon $[s,t]$, as defined in (1). Assuming n transitions within the $[s,t]$ horizon, $P(s,t)$ can be consistently estimated by means of the Aalen-Johansen non-parametric product limit

$$\hat{P}(s,t) = \prod_{i=1}^{n} [I + \Delta \hat{A}(T_i)] \tag{8}$$

where T_i denotes a point in time over $[s,t]$ where a transition occurred, n is the total number of transitions (i.e. the number of days where at least one transition occurs) and

$$\Delta \hat{A}(T_i) = \begin{bmatrix} -\frac{\Delta N_{1.}(T_i)}{Y_1(T_i)} & \frac{\Delta N_{12}(T_i)}{Y_1(T_i)} & \frac{\Delta N_{13}(T_i)}{Y_1(T_i)} & \cdots & \frac{\Delta N_{1K}(T_i)}{Y_1(T_i)} \\ \frac{\Delta N_{21}(T_i)}{Y_2(T_i)} & -\frac{\Delta N_{2.}(T_i)}{Y_2(T_i)} & \frac{\Delta N_{23}(T_i)}{Y_2(T_i)} & \cdots & \frac{\Delta N_{2K}(T_i)}{Y_2(T_i)} \\ \vdots & \vdots & \ddots & & \vdots \\ \frac{\Delta N_{K-1,1}(T_i)}{Y_{K-1}(T_i)} & \frac{\Delta N_{K-1,2}(T_i)}{Y_{K-1}(T_i)} & \cdots & -\frac{\Delta N_{K-1.}(T_i)}{Y_{K-1}(T_i)} & \frac{\Delta N_{K-1,K}(T_i)}{Y_{K-1}(T_i)} \\ 0 & 0 & \cdots & \cdots & 0 \end{bmatrix} \tag{9}$$

where $\Delta N_{lj}(T_i)$ is the number of transitions from state l to j at time T_i.[9] The diagonal elements $\Delta N_{l.}(T_i)$ are the number of transitions away from state l at time T_i so that the sum of each row is zero and the rows of $I + \Delta \hat{A}(T_i)$ sum to 1. The number of sovereigns in state l just before time T_i is denoted $Y_l(T_i)$. The o -diagonal entries $\{\Delta \hat{A}(T_i)\}_{lj}$, $l \neq j$ thus represent the fraction of sovereigns at state l just before time T_i that migrated to state j at time T_i. The Aalen-Johansen method can be seen as the DM estimator (3) extended to infinitely short-time intervals (i.e. time points T_i). The average non-homogeneous hazard rate

[9] $N_{lj}(t,t+1)$ counts the total number of transitions observed from l to j from the starting date t until $t+1$ and $\Delta N_{lj}(T_i)$ is an increment of this process.

(NHHR) estimator of the transition matrix is

$$\hat{P} = w_0 \hat{P}(t_0, t_1) + w_1 \hat{P}(t_1, t_2) + ... + w_{T-1} \hat{P}(T-1, T) \tag{10}$$

where $\hat{P}(t_0, t_1)$, $\hat{P}(t_1, t_2)$ and so forth are particularizations of (8) for sequential, non-overlapping intervals and w_i is the proportion of rated issuers at t_i.

Measuring Overall Rating Mobility

The primal aspect in evaluating or comparing credit rating transition matrices is their mobility as opposed to stability – the latter refers to the diagonal probability mass whereas mobility (or dynamics) refers to the o -diagonal probability mass. The reason is that mispredicting the o -diagonal probabilities, and especially the default probabilities, implies greater economic costs. In the literature, this comparison is usually based on Euclidean distances (Israel et al., 2001; Bangia et al., 2000) and eigenvalue/eigenvector analysis. Schorrocks (1978) and Geweke et al. (1986) proposed indices of mobility for Markov matrices based on eigenvalues and determinants. A pitfall of all these metrics is that they typically cannot provide a clear signal because of the large concentration of probability mass along the diagonal which is typical of sovereign credit migration matrices. To circumvent this problem we employ Jafry and Schuermann's (2004) mobility measure based on singular value decomposition of the dynamic component of the transition matrix.

The dynamic part of the system or overall mobility matrix is given by

$$\tilde{P} = P - I$$

because the identity matrix I represents a fully stable (no migrations) system. The Jafry-Schuermann mobility estimator is defined as

$$m(\hat{P}) \equiv \frac{1}{K} \sum_{i=1}^{K} \left\{ \sqrt{e_i(\tilde{P}'\tilde{P})} \right\} \tag{11}$$

where $e_i(\tilde{P}'\tilde{P})$ denotes the i^{th} eigenvalue of $\tilde{P}'\tilde{P}$. Given the mobility matrices $\tilde{P}_A$ and $\tilde{P}_B$ the di erential measure

$$\Delta m(\hat{P}_A, \hat{P}_B) = m(\hat{P}_A) - m(\hat{P}_B) \tag{12}$$

has the nice property of largerly reflecting the concentration of the off-diagonal probability mass. Put di erently, in the case where $\tilde{P}_A$ and $\tilde{P}_B$ have close diagonal values, the typical di erential metrics are very likely to be zero whereas $\Delta m(\hat{P}_A, \hat{P}_B) \neq 0$. If the o -diagonal probability mass is diluted across a number of o -diagonal entries, then (11) is smaller than when it is concentrated in a few specific positions.[10]

[10] The metric does not distinguish, however, between mass concentrated closer or farther from the diagonal which would be a desirable property as distant transitions imply greater mobility and higher economic costs.

Bootstrapping the Rating Migration Process

Constructing asymptotic confidence intervals for the estimated transition probabilities is di cult in a continuous framework. This is because there is no simple closed-form expression for the asymptotic variance-covariance of the intensity matrix estimator. Moreover, in order to derive the variance-covariance of the probability matrix estimator, one will have to appeal to asymptotic theory to compute not only the variance-covariance of the intensity matrix estimator but also that of the matrix exponential. Besides, the sovereign rating samples available are relatively small (few transitions) and so asymptotic estimators may not behave well in this context. Regarding the mobility di erential estimator, Eq.(12), the asymptotic distribution is unknown. Given these shortcomings, we adopt Christensen et al.'s (2004) parametric bootstrap framework to compare alternative transition matrix estimators.

The DM estimator is compared with the HHR estimator by means of the following experiment. The continuous intensity matrix, Λ, and one-day transition matrix, $P = \exp(\frac{1}{365}\Lambda)$, are estimated from the observed rating histories. These estimates are used to construct artificial daily-rating data. For this purpose, the rating histories of the various obligors are conceptualized as independent realizations of a continuous, homogeneous Markov model. We thus construct R bootstrap samples $\{B_j\}_{j=1}^{R}$ with the same number of sovereign histories, N, as the observed sample. Each sovereign's lifetime h and initial rating X_0 are as in the observed sample.

Each sovereign daily rating history is constructed as follows. Suppose X_0=Baa for the sovereign at hand and that the probabilities of transition from rating Baa to Aaa, Aa, A, Baa, Ba, B, C and D are $\hat{p}_{41},$ $\hat{p}_{42},$ $\hat{p}_{43},$ $\hat{p}_{44},$ $\hat{p}_{45},$ $\hat{p}_{46},$ $\hat{p}_{47}$ and $\hat{p}_{48}$, respectively, with $\left(\sum_{j=1, j\neq 4}^{8} \hat{p}_{4j} = 1\right)$. We transform $\hat{P}$ into cumulative probability ranges for the initial rating, so that for initial rating Baa the first range is $(0,\, \hat{p}_{41}]$. Summing $\hat{p}_{41}$ and $\hat{p}_{42}$ gives the cumulative probability that the new rating is either Aaa or Aa and hence, the next probability range is $(\hat{p}_{41},\, \hat{p}_{41} + \hat{p}_{42}]$. The next range is $(\hat{p}_{41} + \hat{p}_{42},\, \hat{p}_{41} + \hat{p}_{42} + \hat{p}_{43}]$ and so forth until eight ranges are computed, one for each possible transition. The next daily rating, X_1, is obtained by randomly drawing from a uniform distribution, $r \quad i.i.d.U[0,1]$, and then matching the draw with one of the above cumulative probability ranges. For instance, if $r \quad (\hat{p}_{41} + \hat{p}_{42},\, \hat{p}_{41} + \hat{p}_{42} + \hat{p}_{43}]$ then $X_1 = A$. Another uniform random draw gives X_2 and so forth until the end of this sovereign's lifetime so that $\{X_1, ..., X_h\}$ is obtained. This simulation is conducted independently for the remaining sovereigns to construct the bootstrap sample B_j that contains N rating histories of varying length. After iterating this simulation R times we have the bootstrap data sets $\{B_j\}_{j=1}^{R}$. Then each B_j is transformed into N rating transition histories and rating durations. On the basis of the latter, we estimate R one-year transition matrices $\left\{\hat{P}_j\right\}_{j=1}^{R}$ using the DM and HHR approaches, Eqs. (3) and (7), respectively. Thus we obtain an $R \times 1$ vector of default probability estimates per rating (last column entries in $\hat{P}_j, j = 1, ..., R$) or equivalently, the bootstrap distribution of the one-year probability of default estimator (PD hereafter). We also obtain the empirical distribution of the mobility di erential $\left\{\widehat{\Delta m}_j\right\}_{j=1}^{R}$ (see Appendix A).

The bootstrap distribution of the PD for each rating facilitates confidence intervals to compare the e ciency of the estimators and to test hypotheses about di erences in the mean value. One can also assess the finite-sample bias by comparing the latter with the true (sample estimated) PD used in the bootstrap. Likewise, the bootstrap distribution of the mobility di erential statistic (12) facilitates hypothesis testing. We use $R = 1000$ following Efron and Tibshirani (1993) who argue that this generally su ces to obtain reliable bootstrap confidence intervals.[11]

Similar bootstrap simulations are conducted to compare the homogeneous (HHR) and non-homogeneous (NHHR) hazard rate estimators. In order to introduce heterogeneity in the DGP for ratings, we estimate one intensity matrix per year on the basis of the observed rating transitions. Using the matrices $\hat{P}_t = \exp(\frac{1}{365}\hat{\Lambda}_t), t = 1, 2, ..., T,$ a bootstrap daily-rating history is obtained for each sovereign as explained above, which allows for year-on-year heterogeneity. To make the comparison of the HHR and NHHR estimators more informative we consider 2-year transition matrices. Thus, Eqs. (7) and (8), are deployed so as to evaluate the probability of transitions over a 2-year horizon, the former imposing the latter relaxing time-homogeneity. In doing so the potential strength of the NHHR estimator can be revealed, given that in the simulated histories year-on-year heterogeneity may be present. To this end, both estimators are used to compute biannual transition matrices over the two year time spans 1982-1983, 1984-1985,..., 2002-2003. We then average over the entire sample using Eq.(10), where $t_1 - t_0, t_2 - t_1, etc.$ are 2-year horizons, to obtain the long-run average 2-year migration matrix. We were reluctant to introduce heterogeneity in the simulations across shorter periods than one year for two reasons. First, the generator matrix estimator, $\hat{\Lambda}_t$, over 6- or 4-month periods would be highly inaccurate because few sovereign transitions are generally observed in such short intervals. Second, the general stability of the sovereign ratings suggests that there is within-year homogeneity in the underlying Markov process.

The Dataset

The data is from the Moody's Default Risk Service database.[12] The assigned credit rating represents Moody's assessment on the likelihood of each issuer to honour any type of future debt payment. Structural features of the debt issues such as maturity, coupon structure, collateralization and seniority are all taken into account

[11] Nevertheless, in order to assess whether 1000 simulations guarantee convergence of the bootstrap, we experimented by performing two sets of 500 simulations. The estimated PD distributions using the two smaller (500) samples were virtually identical and comparable to those from the larger sample.

[12] S&P's and Moody's ratings are highly correlated. For instance, out of the 49 sovereigns rated by both Moody's and S&P's in September 1995, 28 had equal rating whereas for those with di erent ratings, the gap was only one notch, with 7 exceptions that were 2 notches apart (Cantor and Packer, 1996).

in the assessment. We consider only sovereigns which had foreign-currency bonds outstanding and rated by Moody's some time during March 5, 1981 through March 4, 2004. Thus the rating histories of the $N = 72$ sovereigns in the sample have di erent lengths $h_i, i = 1, 2, ..., N$.[13] For the DM approach, based on year-on-year ratings, the sovereigns are observed on the 5^{th} March. The analysis is based on the issuer's foreign-currency rating history rather than on the history of every single bond issue. Moody's occasionally assigns di erent ratings to bonds of the same issuer depending on characteristics.[14] In order to convert the sovereign bond ratings into a single issuer rating at any point in time we observe each issuer's lowest rating on senior unsecured bonds which have not yet matured or been repaid. The latter is the most meaningful indicator of a sovereign's likelihood of defaulting on any of its bonds (Moody's, 2003). The rating transition dates are thus recorded.

According to Moody's a sovereign default is defined to occur whenever a country defaults on any of its foreign-currency rated bonds. Moody's does not have a 'default' rating category as such but rather records default dates. While in default an issuer is still rated (e.g. as Caa), the rating representing the severity of default. For each country, we treat the date of the first default announcement as the default date – these are 17/08/98 (Russia), 30/11/98 (Pakistan), 25/08/99 (Ecuador) , 20/01/00 (Ukraine), 7/09/00 (Peru), 13/06/01 (Moldova), 30/11/01 (Argentina) and 15/05/03 (Uruguay). In order to identify the rating from which a transition to default took place we track the rating sequence up to the default date and throw out the transitions that occurred very close (within a month) to it. For instance, Ukraine was downgraded from Caa3 to Ca at day -15 in default event time and so the pre-default rating is Caa3 as the downgrade clearly reflected the pending default which was only delayed. Appendix B, shows the number of foreign-currency sovereign bond issuers rated by Moody's from 1981-2004 (Panel A) and provides a breakdown according to issuer's characteristics: geographical location, state of the economy and credit quality (Panel B). Most industrialised countries have ratings from the beginning of the sample period (dominating the sample until the mid 1990s) whereas many emerging economies were first rated later within the sample window. More specifically, 11 sovereigns were rated over 1981-1986 of which only 2 were emerging economies, while 17 industrial and 55 emerging markets were rated in 2003. Regarding non-investment grade ratings, none was observed before 1987, increasing to 18 in 1997 and up to around 25 during 1998-2004.

The initial date of the rating history or 'left' censoring varies across sovereigns. The same applies to

[13] We exclude countries that have been assigned ratings for domestic currency bond issues as well as for types of foreign currency debt issues other than bonds (i.e. foreign currency bank deposits, country ceilings). Excluded also are sub-sovereigns and municipals (local and regional) as well as bonds with convertible features. We have a total of 6,058 rating transitions experienced by 70 of the 72 sovereign issuers. The transitions per country range from just 1 for some small sovereigns (Bermuda, Egypt, El Salvador, Estonia, Latvia, Mauritius, Morocco, Oman) up to 961 for Sweden. Sovereigns that have never experienced a transition on any of their foreign bond issues are Chile and Guatemala, first rated in 1999 and 1997, respectively.

[14] For instance, Russia's 'MinFin' US dollar bonds have been generally rated lower than its Eurodollar bonds. During the 1999 Russian crisis, defaults occurred on the former but not on the latter.

the termination date or 'right' censoring. Termination can occur either at the end of the time window (i.e. March 4, 2004) or at an earlier point due to withdrawn rating or default. A sovereign receives a withdrawn rating (WR) status when bonds have matured, been repaid or called. Most often the latter reflects the issuer's temporary exit from the public bond market rather than having negative credit implications (Carty, 1997). Sovereign WRs are rather scarce because sovereigns, as opposed to corporates, rarely retire all their debt simultaneously – our sample contains only 5 countries that have, at any time, experienced a WR.[15] We follow the literature and exclude the WRs from the history of the specific sovereign (right censoring) and the sovereign is treated as a new independent issuer when the rating is resumed. Likewise, any rating assigned between the default date and the end of the default episode is discarded. The latter is defined as the date when the sovereign exceeds the B3 rating. The sovereign is treated as a new independent issuer when the default episode ends.[16] Half of the 8 defaulted countries (Pakistan, Peru, Russia and Ukraine) recover from default and reappear in our sample as new issuers whereas the rest (Argentina, Ecuador, Moldova and Uruguay) remain in default at the end of the sample window.

Finally, in order to focus on the broader rating scale (Aaa, Aa, A, Baa, Ba, B, C and default) used by Moody's prior to 1982, we label the numbered sub-categories according to the mother category, e.g. Baa1 and Baa2 are both treated as Baa. The lowest sub-categories, Caa1, Caa2, Caa3, Ca1, Ca2, Ca3, Ca, contain either very few or no observations at all and so are all merged into a C category. There are two reasons for restricting our analysis to the eight coarse ratings. First, this reduces the number of parameters to be estimated and increases the sample size of transitions per rating. Second, the reliability of finer transition matrix estimates in credit risk applications is doubtful and thus the coarse rating system has emerged as the industry standard.

The above considerations result in an e ective 1981-2004 sample of 81 sovereigns of which 4 are countries that re-emerged from default and 5 are countries that at some point received an WR, all of them terated as new issuers thenafter. Figure 1 displays the aggregate ratings distribution over all issuers and years in the sample. Investment grade sovereigns represent 70% of the sample on average. Appendix C provides summary statistics on the distribution of credit ratings per year. As a whole, we have a total of 759 country-year cases and 104 rating transitions. A discrete estimation framework (that ignores within-year rating transitions) captures only 80 of these transitions which means throwing out e ectively 23% of the observed migrations.[17],[18]

[15] Debt withdrawals are common for corporates, for instance, in the case of mergers or liquidation.

[16] This procedure is equivalent to treating the default state as absorbing.

[17] The only available study on sovereign transition matrix estimation, Hu et al. (2002), exploits only 26 S&P rating transitions during 1981-1998 in a discrete-time framework.

[18] The empirical analysis in this paper is conducted using Gauss 3.4 and LIMDEP 8.

Properties of Migration Matrix Estimates

In this section we compare the properties of rival transition matrix estimators. The statistical framework for the comparison is a parametric bootstrap. We first compare the 1-year probability matrices estimated from discrete and continuous type approaches. Second, we investigate the degree of discrepancy between the two estimators as the transition horizon increases from 1 to 3 years. Third, the added value of time-heterogeneous estimators is also assessed. For the latter purpose, we introduce year-on-year heterogeneity in the underlying Markov process and focus the comparison on a 2-year transition horizon. The discussion focuses primarily on the relative bias and e ciency of the PD estimates and on the overall mobility or dynamics implied by the transition matrix estimates. Finally, the presence of duration dependence and momentum in sovereign ratings is investigated.

Discrete versus Continuous Estimators

In the DM approach, Eq. (3), the 1-year transition matrix entry p_{ij} is estimated by cumulating the number of sovereigns rated i at the start of any year t in the sample $(t = 1, ..., T)$ and rated j at the start of year $t + 1$ and dividing the sum by the total number of sovereigns that started any year t in rating i. In the continuous HHR approach the generator matrix entry λ_{ij} is estimated using Eq. (6), that is, dividing the number of transitions from i to j observed during the T-length sample window over the total time spent (in years) at rating i by all sovereigns. The 1-year transition probability matrix is then obtained as the matrix exponential of the generator (horizon $\Delta t = 1$) for a 6^{th} order Taylor expansion $(k = 1)$. Table 1 reports the transition matrix estimates. Both transition matrices are diagonally dominant, indicating rating stability and the DM estimate is very similar to that reported by Moody's (2003). There is heavy concentration around the diagonal, that is, most observed transitions are towards a neigbouring rating. Higher migration activity and more distant transitions are associated with migration from lower credit qualities (Ba, B and C categories). Another common characteristic is that for each row (initial rating i) the transition probabilities decrease as one moves farther from the diagonal. This is referred to as row monotonicity and is a general feature of credit rating migration matrices (J.P.Morgan, 1997, p.73). A violation of monotonicity occurs for the B rating such that there is a higher probability of migrating to default than to C. A possible explanation could be the noisy nature of the data for the low B rating. The above e ect is unsurprisingly more prominent in the DM estimator.

The HHR probability estimates are positive for most transitions even if they have not been observed in the sample whereas for such cases, the DM estimates are zero. The HHR method spreads the o -diagonal probability mass over almost all ratings whereas the DM concentrates the probability mass around the diagonal. The HHR transition matrix estimate exhibits greater migration volatility for low ratings, that is, relatively less diagonal probability mass. One reason for these di erences is that continuous estimators better capture the rating dynamics. The PD of a C-rated sovereign suggested by the HHR method (28.9%) is relatively high as compared to the DM counterpart (16.7%). However, the duration of C is short, in most cases less than a year, as it is just a transitional status toward default. So, for instance, if a sovereign is at B at year beginning then is downgraded to C and ends the year in default, the DM will record a migration from B to D. On the other hand, the transition $B \to C \to D$ will be recorded by the continuous estimator. The short durations in C peak the default intensity $\hat{\lambda}_{C,D}$ leading to a higher estimate $\hat{p}_{C,D}$ from the continuous method.

We now deploy the parametric bootstrap technique described in Section 3 to compare the finite sample properties of the DM and HHR estimators. The bootstrap DGP is the homogeneous continuous Markov model characterized by the HHR transition matrix estimate (Table 1). To preserve space and without loss of generality we focus the comparison on the PDs.[19] Thus, the PD measures in Table 1 (bold) are referred to as true default probabilities. Figure 2 plots the kernel (Gaussian) density of the bootstrap 1-year PD estimates for the investment grade ratings. The bold vertical line signifies the true PD and the two dashed lines indicate the 95% confidence interval. For the three top ratings (Aaa, Aa, A) the DM estimates are not plotted because they are zero for all simulated paths. Table 2 shows, for each rating, summary statistics for the bootstrap PDs along with the true PDs. Clearly the shape of the empirical distribution di ers across ratings. The skewed nature of some of the densities inflicts the use of the median as a more informative measure of central tendency, thus the median is also reported in the table. We observe that the DM probability estimator has a smooth distribution only for the B category.For the three top ratings, the distribution of the HHR estimator is roughly exponential with upper 97.5% quantiles of 1.23×10^{-5}bp, 0.0009bp and 0.238bp, respectively and means quite close to the 'true' PD (small bias). Transitions to default from high ratings are not observed in the sample or in the simulated paths. However, the continuous method is able to provide an estimate of how rare such events are.

Regarding the Baa rating, the DM approach produces a zero PD estimate for most paths and a PD in the range $[60,\ 125]$ bp for a few paths (Figure 2) which leads to a rather high upper quantile of 75.2bp. In sharp contrast, the upper quantile of the HHR estimator's distribution is 9.72bp which means a notable increase in accuracy compared to the former estimator. The higher DM quantile is because about 2.5% of the simulated paths contain at least one transition from Baa to default $(0.0223\% \times 112 = 0.025$ where 0.0233% is the true PD from the Baa rating as in Table 1 and 112 is the number of Baa ratings observed at year-beginning, N_{Baa}, as shown in Appendix C). If one default occurs, then the DM probability estimate in simulation j is expected to be $1/N^j_{Baa}$where $E(N^j_{Baa}) = 112$ and $1/112$ is roughly 75.2. Our analysis can be used to assess the adequacy of the minimum probability at 3bp that has been established by the Basel

[19] Simulation results for all transition probabilities are available from the authors upon request.

Committee for unobserved events. This threshold probability clearly falls in the HHR confidence interval for Baa-rated sovereigns while it is well beyond the 97.5% quantile of the HHR distribution for the Aaa, Aa and A ratings. Hence, the Basel II threshold appears too conservative for these higher ratings.

The counterpart results for the lower ratings (Ba, B, C) are set out in Figure 3. This is not surprising given that direct default migrations from B are expected to be the most frequent – the total number of observations over the period are 132 for Ba, 85 for B and 6 for C so according to the true PDs the expected number of default migrations per simulation are 83.07bp$\times 132 = 0.010$, 487.2bp$\times 85 = 0.041$ and 2897bp$\times 6 = 0.017$, respectively. The confidence set of the DM estimator is similar to that of the HHR estimator for the B rating whereas the latter is more e cient (tighter confidence interval) for the Ba and C categories. The trimodal distribution of the HHR estimator for the Ba category is attributed to the fact that there are two main transitions from Ba in the true generator matrix, one is an upgrade from Ba to Baa , with 6.76% probability, and the other is a downgrade from Ba to C, with 8.55% probability. This creates a division of the simulations between those where the upgrade occurs and those where the downgarde occurs. When the up(down)grade occurs the PD de(in)creases significantly. In the case of no transition we obtain the middle peak, that is the PD directly from Ba which is 0.08% in the true transition matrix. In terms of bias, measured by the di erence between the true PD and the mean of the bootstrap distribution, the HHR estimator shows less bias than the DM estimator for the higher ratings whereas the opposite holds for the lower ratings. Moreover, as noted earlier, the HHR estimator captures the frequent short duration transitions from the low categories (on the way down to default) resulting in comparatively higher PD estimates (positive bias as opposed to negative bias of the DM estimator).[20]

We turn now to the issue of whether the di erence between the mean DM and HHR default probabilities is statistically significant. For this purpose, we conduct two-sided bootstrap tests for H_0:$PD_{HHR} - PD_{DM} = 0$. Table 3 (panel A) reports the summary statistics for the probability di erential measure, $(\widehat{PD}_{HHR} - \widehat{PD}_{DM})$, over replications. It turns out that all the 95% confidence intervals contain zero which suggests insignificant di erences, with the exception of the rating A for which the above null is rejected. In the latter case, the DM default probability estimate is zero because there are no default transitions from A observed in the sample whereas the more e cient HHR estimator gives a non-zero probability because it jointly exploits the information that there are transitions from A to Ba and Baa and from Ba to D. Therefore we infer that the DM estimator may underperform the HHR estimator in terms of bias for some mid ratings.

Finally, we address the question of whether there are significant di erences in the overall ratings mobility (or migration risk) implied by the DM and HHR methods. Table 3 (Panel B) provides the sample mobility di erential, $\widehat{\Delta m}$, as well as summary statistics of its bootstrap distribution. First, the DM matrix implies

[20] For the default probability from the Aaa, Aa and A ratings, the negative bias in the DM estimator is explained by the fact that it predicts 0 whenever no transitions are observed in the bootstrap sample, i.e. $N_{ij}(t, t+1) = 0$ for $i = Aaa, Aa, A$ and $j = D$.

higher overall mobility than the HHR matrix and the di erence is statistically significant as suggested by the 95% confidence interval not containing zero. This suggests that the concentration of probability mass at o -diagonal positions in the DM transition matrix is higher than that in the HHR counterpart. Upon closer inspection (Table 1) it is apparent that the DM matrix is relatively more sparse (large number of zero entries) and its transition probability mass is largely concentrated in those few ratings for which transitions have been observed in the sample (i.e. around the diagonal). These results are consistent with the discussion in Section 3, namely, that the presence of a few large o -diagonal terms inflates the $m(\tilde{P})$ metric considerably. Default probability mass is higher, in general, for the HHR. Moreover, the HHR matrix spreads the PD mass over more o -diagonal positions or equivalently, a larger number of ratings.

Increasing the Transition Horizon

The DM and HHR transition matrix estimators are expected to show more marked di erences when the horizon is beyond one year because the latter will allow for more rating activity. The probability matrix estimates for the 2- and 3-year transition horizons are presented in Table 4. It turns out that the DM estimator captures default risk only for one or two ratings, Baa and B, in contrast with the HHR estimator. One can notice that the DM estimate of the PD for Baa is about twenty-fold the HHR estimate – the 2-year default probabilities are 192bp and 9bp and the 3-year ones are 333bp, 22bp for the DM and HHR cases, respectively. The smaller HHR default probability for Baa is more plausible because sovereigns spend relatively long times in the mid Baa state (versus other ratings) on their way up(down) the rating scale – the latter is reflected in the denominator of Eq. (6) – which will thus pull down the default probability for Baa. The di erences are striking at the lowest end of the rating spectrum – for rating C the DM default probability estimate is zero throughout whereas the HHR estimates are 38.18bp and 35.49bp for the 2- and 3-year horizons, respectively. A non-zero probability of default from C over a relatively long horizon of 2 or 3 years is plausible given a large enough sample. These findings are just a reflection of a shortcoming of discrete versus continuous transition probability estimators, namely, biases in the former because it does not account for rating duration. The upshot is that the DM method underestimates the default probability for the highly risky sovereigns.[21] Regarding the overall mobility metric, $m(\tilde{P})$, it turns out that the DM estimator yields higher values than the HHR estimator for all horizons ranging from one to four years. Unsurprisingly, the di erential $\widehat{\Delta m}(\tilde{P}_{DM}, \tilde{P}_{HHR})$ increases with the time horizon.

[21] The DT estimates $\hat{p}_{C,B} = 1$ and $\hat{p}_{C,Ba} = 1$ (for the 2- and 3-year horizons, respectively) stem from one transition – Romania migrates from C to B after 2 years and to Ba after 3 years. All other C-rated issuers have durations shorter than 2 years and so their transitions are not captured by the DM method.

The Time-homogeneity Assumption

We now assess the role of the time-homogeneity assumption in the sovereign rating process by comparing the performance of the homogeneous (HHR) and the non-homogeneous (NHHR) approaches. It seems plausible that the longer the horizon, the more apparent the heterogeneity will be and so we focus the comparison primarily on the 2- and 3-year horizons. Table 5 presents the long-run average probability matrices from the HHR and NHHR estimators. In both cases biannual transition matrices are estimated sequentially over non-overlapping 2- or 3-year periods and the average transition matrix is obtained via an issuer weighted average as outlined in Eq.(10). The di erence between the two estimators stems from allowing or not for heterogeneous behaviour within the 2- or 3-year estimation windows. At first glance, the di erence between the above two hazard rate estimators is not as striking as that between the two homogeneous, one discrete (DM) and the other continuous (HHR), estimators. It is evident that all credit ratings exhibit greater migration in theNHHR matrix. Furthermore, there are discrepancies between the two estimators in the PDs with the NHHR estimator giving, in general, substantially higher default probabilities. It turns out that increasing the transition interval from 2 to 3 years exacerbates the di erences between the HHR and NHHR matrix estimates in terms of overall migration potential and, more specifically, transition to default. This is corroborated by the $\widehat{\Delta m}$ statistic which increases with the transition horizon. The overall mobility is larger for the NHHR method in accordance with our earlier observation of having greater o -diagonal probability mass. It turns out that the two estimators give di erent migration risk measures, which calls for caution in practical applications.

We now statistically compare the two continuous hazard rate estimators on the basis of the bootstrap replicates. To preserve space we focus on the PDs over a 2-year horizon that may be su cient for time variations in the rating process to become apparent. A di erent generator is used for each year (to introduce heterogeneity), as discussed in Section 3 and we take as the true PD over the sample window is the issuer-based weighted average of all the 2-year transition matrices (computed as the product of the annual transition matrices). The kernel density and summary statistics of the bootstrap PDs for investment grade issuers can be seen in Figure 4 and Table 6, respectively. For the Aaa and Aa ratings with zero true PD, the kernel densities are not plotted because these are zero for all simulated paths. The latter can be rationalized as follows. Direct transitions to default do not occur from high ratings. The two continuous estimators facilitate default intensities for such cases by capturing indirect transitions, but even these are non existant in such a short estimation window of 2 years. This explains why both approaches produce zero default intensities for the top ratings. For the A and Baa ratings, with non-zero true PD, both estimators produce smooth distributions (Figure 4). In the case of the A rating, the NHHR estimator yields zero probabilities for most simulated paths, and very few paths with probabilities in the range $(0, 2]$bp, giving a 97.5% confidence band of 0.45bp. Similarly, the corresponding upper quantile for the HHR estimator is 0.43bp, rendering the two comparable in terms of e ciency (Table 6).For the Baa category, the quantiles of the bootstrap distribution are again virtually the same. The comparison in terms of bias gives conflicting results depending on which measure (mean or median) is used, for both ratings.

Figure 5 plots the kernel density of the PD estimates for non-investment grade issuers. For the NHHR estimator the densities for the are smooth, both centered at the true default rate (c.f. true and mean PD in Table 6), except that for rating C. In contrast, for the HHR we obtain a high degree of biasedness, with the true default probability located at the right tail of the distribution. This vindicates that neglecting heterogeneity leads to underestimated default rates for the speculative ratings. However, the HHR estimator produces tighter confidence intervals than the NHHR estimator (more e cient) for both ratings. For the C rating the scarcity of transition data within the 2-year estination windows leads to uncentred distributions with wide confidence intervals for both etsimators. Table 7 reports the summary statistics for the bootstrap distribution of the 2-year PD di erential ($\widehat{PD}_{HHR} - \widehat{PD}_{NHHR}$) for all ratings. The only confidence interval not containing zero is that for rating B, thereby providing evidence that the HHR and NHHR estimators yield significantly di erent default probabilities for this rating. In essence, the HHR estimator significantly underestimates the PD from B whereas the mean estiamates are similar for all other ratings. However, in terms of overall mobility the two transition matrix estimates appear significantly di erent. More specifically, the mobility in the NHHR matrix is significantly larger than that in the HHR matrix which means more probability mass at o -diagonal positions. Put di erently, disregarding heterogeneities might result in transition matrix estimates that suggest less frequent rating migration (possibly between farther apart ratings) than that actually implied by the underlying time heterogeneous Markov DGP. A specific case of the latter, revealed in the above exercise, is that the PDs are all downward biased for the HHR.

Detecting Non-Markovian Behaviour

The DM, HHR and NHHR estimators build on the premise of a Markov migration process. To the best of our knowledge, the plausibility of the latter has not been assessed in the context of sovereign debt. In this section, we attempt to fill this gap in two ways. First, we test whether the transition matrix estimate conforms to the homogeneous Markov structure. Second, we investigate the presence of momentum and duration e ects.

Testing for Markov Structure of the Transition Matrix

Since each row of the transition matrix P sums to one, it follows that there exists a trivial unity eigenvalue which is the largest in magnitude ($q_1 = 1$) and thus the process reaches a steady state. The rate at which the process decays to the steady state is dictated by the second largest eigenvalue (q_2). It can also be shown that the eigenvector associated to q_2 provides the asymptotic distribution of survivors (sovereigns not ending in default) and thus provides insights on the rating towards which the survivors will converge asymptotically.[22] For a process to be homogeneous Markov, the eigenvalues of P associated with increasing horizons should decay exponentially (except for $q_1$which is unity for all horizons) and that the eigenvectors should remain constant. In other words, if q_{2t} is the 2nd largest eigenvalue of the migration matrix for transition horizon t, then $ln(q_{2t}) = -Ct, C > 0$. For a Markov process, this log-linear relationship has to hold for all subsequent eigenvalues, q_{3t}, q_{4t} etc The second to fifth largest eigenvalues of $\hat{P}$ for varying transition horizons from 1-4 years are plotted in Figure 6. The estimation of the transition matrix is based on the HHR method. The graph strongly supports the log-linear relationship above and consequently the Markov properties of the rating process.

Figure 7 shows that the 2nd largest eigenvector of $\hat{P}$ is very similar across horizons which provides additional evidence for Markovian behaviour. Moreover, the rating distribution exhibits a peak at Aaa. This suggests that the survivors in the long-term tend to settle at the highest rating. We carried out robustness checks for this analysis by using only the last 6 sample years (1998-2004) and the results did not change qualitatively.

Testing the Duration Effects Hypothesis

To test the e ect of duration on credit rating transitions we estimate panel logit models using monthly sovereign credit ratings over 23 years (March, 1981-March, 2004).[23] The duration measure, d_{it}, is obtained for each sovereign at the end of each month as the time elapsed since the last transition to the current state. To illustrate, consider a sovereign that experienced a rating transition to Ba in June 2002, then to B in September 2003 and has not moved since then. The duration in, say, June 2003 is 12 months and in March 2004 it is 6 months. We assume that the rating histories start at the beginning of our sample window in March 1981 (left-censored durations) which is not too restrictive because very few issuers had been rated before 1981. The e ect of duration on the rating transition probability is assessed separately for upgrades (UP) and downgrades (DW). To do so, we define the endogenous variables

$$UP_{it} = \begin{cases} 1 & \text{if sovereign } i \text{ was upgraded in month } t \\ 0 & \text{otherwise} \end{cases}$$

[22] The steady state solution is equal to the absorbing row of P, that is, for any transition matrix exhibiting at least one non-zero probability of default the state vector will settle at the default state. Put di erently, given enough time and a constant (homogeneous) migration matrix all sovereigns will end up in default. Since the rate of decay to the steady state of the sovereign migration process is very slow (long durations) the time homogeneity assumption is unlikely to hold over such long time periods. Hence, the economy and consequently the migration process often changes long before the default state is reached.

[23] The duration e ect is di erent from the 'ageing' e ect examined by Altman and Kao (1992) for corporations. The latter refers to the relation between the time since issuance of individual bonds and the upgrade (downgrade) probability.

$$DW_{it} = \begin{cases} 1 & \text{if sovereign } i \text{ was downgraded in month } t \\ 0 & \text{otherwise} \end{cases}$$

and estimate the following logit regression for each

$$y^*_{it} = \alpha + \boldsymbol{\beta} d_{it} + \gamma^{'} \mathbf{z}_t + \varepsilon_{it},\ \varepsilon_{it} \sim iid(0, \sigma^2_i), \quad i = 1, ..., N, \quad t = 1, ..., T \tag{13}$$

where y^*_{it} is a continuous latent variable such that $UP_{it} = 1$ for $y^*_{it} \geq 0$ and $UP_{it} = 0$ otherwise (likewise for DW_{it}) and $\mathbf{z}_t$ is a 7×1 year dummy vector (one per year) over 1998-2004 that controls for the fact that many emerging economies entered the sample after 1997. If durations are more stable over time for industrial countries, then the logit error variance should be smaller for them. To control for this groupwise heteroskedasticity, the error variance is allowed to di er between industrial and non-industrial countries according to $\sigma_i = [\exp(\psi + \xi r_{it})]$ where $\exp(\psi) = \frac{\pi}{\sqrt{3}}$, $r_{it} = 1$ if i is industrial, and ξ is a time constant parameter.

Appendix D summarises the observed rating durations. Two features emerge. First, average duration increases with rating quality, the only exception being the absorbing default rating. This can be explained in terms of the low transition probabilities for high credit-quality sovereigns – using survival theory, it can be shown that the expected duration in state i is negatively related to the probability of transition away from i. Second, the standard deviation of duration is also larger for the high credit-quality ratings but this is just a reflection of their relatively large durations.

We now address the question of whether duration influences the upgrade/downgrade probability. Table 8 reports the logit estimation (ML) results.[24] The duration coe cient is negative for both upgrades ($\hat{\beta} = -1.88$) and downgrades ($\hat{\beta} = -1.66$) which suggests that transition probabilities are negatively influenced by duration – the more time a sovereign spends in the current rating the less likely it is to be migrated. These findings are consistent with those in Lando and Skodeberg (2002) for corporates and can be explained in terms of the common practice by rating agencies of upgrading/downgrading gradually notch by notch. The latter results in short durations and high migration risk for the low end of the rating scale and vice versa. Finally, note that the estimated error variance is higher for industrial countries ($\hat{\xi} > 0$) albeit not significantly. The positive parameter is an artefact of the high durations that inflate the variance of higher ratings. The standardised durations are comparable across ratings, thereby justifying the insignificant result (see Appendix 4).

[24] Our primary focus is on testing e ects that are 'internal' to the rating process and would automatically violate the Markov assumption. Nevertheless, equation (13) was reestimated by substituting a business cycle dummy for d_{it}. We find that the downgrade probability is significantly higher in recessions, however, the result is marginal.

Testing the Rating Momentum Hypothesis

In order to test for momentum e ects in sovereign rating migrations we define

$$UM_{it} = \begin{cases} 1 & \text{if sovereign } i \text{ was upgraded to the current rating over } [t-1, t-23] \\ 0 & \text{otherwise} \end{cases}$$

$$DM_{it} = \begin{cases} 1 & \text{if sovereign } i \text{ was downgraded to the current rating over } [t-1, t-23] \\ 0 & \text{otherwise} \end{cases}$$

which are referred to as the upward and downward momentum indicators, respectively (and t denotes months).[25] First, we estimate the upgrade logit regression

$$y_{it}^{*} = \alpha + \boldsymbol{\beta} v_{it} + \gamma^{'} \mathbf{z}_t + \varepsilon_{it}, \ \varepsilon_{it} \sim iid(0, \sigma_i^2), \quad i = 1, ..., N, \quad t = 1, ..., T \tag{14}$$

where $UP_{it} = 1$ for $y_{it}^{*} \geq 0$ (as defined in the previous section) and $v_{it} \equiv UM_{it}$. A similar logit is estimated to test for downgrade momentum.

The estimation results are presented in Table 8. The downgrade logit estimates provide strong support for the momentum hypothesis with a highly significant coe cient at $\hat{\beta}$=1.20. This suggests that a downgrade in the previous two years significantly increases the current downgrade probability. These findings are in line with the extant evidence for corporate debt (Lando and Skodeberg, 2002; Nickell et al. 2000). Moreover, the residual variance is significantly lower for industrialized countries ($\hat{\xi} = -0.17$) which is in line with the more stable downgrade momentum history for these economies. In sharp contrast, the upgrade logit provides no evidence of momentum.

Conclusion

Sovereign credit ratings and the associated migration probabilities play a major role in modern credit risk management, valuation and international capital allocation. Di erent estimators for rating migration matrices have been proposed. However, the extant methodologies have been mainly applied to assess corporate credit risk. Very little is known on the finite-sample properties of these estimators in the context of sovereign ratings. This paper contributes to the literature by comparing the finite-sample properties of three di erent credit migration estimators – the discrete multinomial approach considered as the industry standard and two continuous, hazard rate approaches that di er in how they treat time-heterogeneity. The discrete multinomial approach is appealing because of its simplicity, while at the other end the time-heterogeneous hazard rate approach is computationally the most expensive. The comparison is conducted through a parametric

[25] Let R_{it} denote the rating for country i at period t. We initially set $UM_{it} = 0$ and compare $R_{i,t-1}$ and $R_{i,t-j}$ for $j = 2, ..., 23$ sequentially. For instance, if $R_{i,t-j} < R_{i,t-1}$ for $j = 2$ then an upgrade occurred at $t-1$ and $UM_{it} = 1$ and the comparison stops, otherwise $j = 3$ and so forth. Thus, we construct UM and DM.

bootstrap method that facilitates empirical confidence intervals and bias measures. The three estimators have in common that they build on the Markov property for the rating evolution which implies that the future rating is independent of the rating history. In a panel logit framework, tests are conducted for the presence of two non-Markov effects in the sovereign rating process known as rating momentum and rating duration.

Significant differences are found between discrete and continuous type estimators, on the one hand, and between homogeneous and heterogeneous estimators, on the other. The continuous estimators yield more accurate default probabilities than the standard discrete approach and are, in general, significantly less biased. The transition probability matrices also differ significantly in the mobility or dynamics that they imply. The discrete estimator provides matrices with a larger concentration around the main diagonal. As the transition horizon increases, time heterogeneities become apparent and the difference between the two hazard rate estimators is more marked. Larger migration mobility is implied by the heterogenous estimator, while the default probabilities from the homogeneous estimator are generally downward biased. The heterogenous estimator emerges as less biased for non-investment grade ratings, though not less efficient in order to inarguably overweigh its computational costs. From a regulatory point of view, it turns out that the lower bound of 3bp recently established by the Basel Committee as the minimum transition probability for rare events is relatively conservative for the high credit-quality ratings. Another important implication, in the light of the New Basel Accord, is that the choice of either a discrete or a continuous framework for the estimation of sovereign default probabilities may result in substantially different levels of capital requirements. Therefore, further research is required to address the important question of whether the statistical differences unveiled have any economic impact.

There is evidence of non-Markov effects in the sovereign ratings which implies a specific type of heterogeneity in the rating process and might well advocate the use of continuous time-heterogeneous estimators. Logit regression estimates suggest negative duration dependence for both downgrade and upgrade transition probabilities. Rating momentum effects are significant for downgrades (but not for upgrades) which is consistent with the rating agencies' practice of sequentially reducing a sovereign's credit-quality grade. These findings have important implications for risk management. For instance, in terms of pricing credit sensitive instruments it turns out that the rating momentum of a sovereign and its duration in the current rating may entail information about the future value of its debt obligations. In the case of multisovereign portfolios, upgrade and downgrade duration dependence for different assets is likely to cancel out on average so the effect might be less pronounced. This might not be the case for the momentum dependence because such effect is mostly present in downgrade migrations. In the light of this, a question that warrants further investigation is whether downgrade momentum is priced in sovereign debt or whether it essentially leaves room for arbitrage.

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Appendix A: Bootstrapping the Ratings Migration Process

The bootstrap simulation method in steps is as follows:

1. Estimate the generator matrix Λ from the observed sample of sovereign ratings (N sovereigns)

2. Calculate the probability transition matrix as $P(\Delta t) = \exp(\Lambda \Delta t)$. Choose the transition horizon Δt for the analysis. Transform the transition matrix into cumulative probability ranges for each rating.

3. The initial rating X_{0i} and and lifetime h_i for each sovereign $(i = 1, ..., N)$ are those in the observed sample. Obtain the next period state, X_{1i}, by generating a random draw $r \quad i.i.d.U[0,1]$ and matching it with the cumulative probability ranges corresponding to X_{0i}. For instance, if r $(p_{11} + p_{12}, p_{11} + p_{12} + p_{13}]$, then the rating associated with the 3^{rd} column of P represents X_{1i}.

4. Repeat step 3 with starting state X_{1i} to simulate X_{2i} and so forth. The artificial rating history for sovereign i is $\{X_{0i}, X_{1i}, ..., X_{h_i}\}$

5. Repeat steps 3 and 4 for all sovereigns $i = 1, ..., N$. The output is the bootstrap dataset B_j that contains the N rating histories.

6. Transform B_j into sequences of rating transition and durations. Use the latter to compute the transition probability matrix $\hat{P}(\Delta t)$ for the horizon of interest using the DTM and HHR estimation methods. Retain the default probability vector $P\hat{D}_j$ and compute the mobility differential $\Delta \hat{m}_j$.

7. Repeat steps 3 to 6 a large number, R, of times. The parameters of interest are the R default probability vectors $\left\{P\hat{D}_j\right\}_{j=1}^{R}$ and R mobility differential scalars $\{\Delta \hat{m}_j\}_{j=1}^{R}$ from the DM and HHR estimation methods.

Appendix B: Moody's Foreign-Currency Sovereign Bond Issuers

A. List of Moody's-rated foreign currency sovereign bond issuers

Year	New Ratings	Countries
1981-1986	11	Australia, Austria, Canada, Denmark, Finland, New Zealand, Norway, Panama, Sweden, United Kingdom, Venezuela
1987	5	Argentina, Brazil, Italy, Malaysia, Portugal
1988	3	Belgium, Ireland, Spain
1989	2	China, France
1990	2	Iceland, Thailand
1991	1	Mexico
1992	0	None
1993	2	Trinidad and Tobago, Turkey
1994	5	Colombia, Indonesia, Japan, Philippines, Uruguay
1995	5	Barbados, Bermuda, Greece, Pakistan, South Africa
1996	4	Israel, Jordan, Mauritius, Poland
1997	9	Bulgaria, Croatia,Kazakhstan, Lebanon, Lithuania, Moldova, Oman, Russia, Slovenia
1998	8	Bahamas, Costa Rica, Cyprus, Ecuador, Guatemala, Jamaica, Romania, Ukraine
1999	12	Belize, Bolivia, Czech Republic, Dominican Republic, El Salvador, Egypt, Estonia, Hungary, Korea, Latvia, Peru, Slovakia
2000	3	Chile, Morocco, Qatar
2001	0	None
2002	0	None
2003	0	None
Total	72	

Ratings are observed on 5th March every year.

A. Characteristics of Moody's-rated foreign currency sovereign bond issuers

						Level		Region				
Year	Rated sovereigns	IG	NIG	WR	Default	Industrial	Emerging	Asia	Latin Amer.	East. Europe	Africa	Mid. East
1981	11	11	0	0	0	9	2	0	2	0	0	0
1982	11	11	0	0	0	9	2	0	2	0	0	0
1983	11	9	0	2	0	9	2	0	2	0	0	0
1984	11	9	0	2	0	9	2	0	2	0	0	0
1985	11	9	0	2	0	9	2	0	2	0	0	0
1986	11	8	0	3	0	9	2	0	2	0	0	0
1987	16	12	2	2	0	11	5	1	4	0	0	0
1988	19	15	3	1	0	14	5	1	4	0	0	0
1989	21	17	3	1	0	15	6	2	4	0	0	0
1990	23	19	3	1	0	16	7	3	4	0	0	0
1991	24	19	3	2	0	16	8	3	5	0	0	0
1992	24	19	5	1	0	16	8	3	5	0	0	0
1993	26	19	5	2	0	16	10	3	6	0	0	1
1994	31	21	9	1	0	17	14	5	8	0	0	1
1995	36	24	11	1	0	17	19	6	10	1	1	1
1996	40	28	11	1	0	17	23	6	10	2	2	3
1997	49	31	18	0	0	17	32	7	10	8	2	5
1998	57	32	25	0	0	17	40	6	15	12	2	5
1999	69	40	27	0	2 (2)	17	52	7	20	17	3	5
2000	72	42	26	0	4 (2)	17	55	7	21	17	4	6
2001	72	44	24	0	4 (1)	17	55	7	21	17	4	6
2002	72	44	25	0	3 (2)	17	55	7	21	17	4	6
2003	72	45	24	0	3 (1)	17	55	7	21	17	4	6

Ratings are observed on 5th March every year. The rated sovereigns per year are categorised by credit quality and state of the economy. IG stands for Investment Grade issuers, NIG for Non-Investment Grade issuers and WR for Withdrawn Ratings. The numbers in parentheses are default entries for the given year. The rated emerging markets are further categorised by regional location.

Appendix C: Moody's Ratings Distribution

A. Number of rated sovereigns. Groups by rating and year

	Rating										
Year	Aaa	Aa	A	Baa	Ba	B	C	Default	Total	IG	NIG
1981	8	3	0	0	0	0	0	0	11	11	0
1982	8	3	0	0	0	0	0	0	11	11	0
1983	6	3	0	0	0	0	0	0	9	9	0
1984	6	3	0	0	0	0	0	0	9	9	0
1985	5	4	0	0	0	0	0	0	9	9	0
1986	6	2	0	0	0	0	0	0	8	8	0
1987	8	2	1	1	2	0	0	0	14	12	2
1988	7	6	1	1	1	2	0	0	18	15	3
1989	8	6	2	1	1	2	0	0	20	17	3
1990	7	7	4	1	1	2	0	0	22	19	3
1991	5	9	4	1	2	1	0	0	22	19	3
1992	4	10	4	1	2	2	0	0	23	19	4
1993	4	10	4	1	3	2	0	0	24	19	5
1994	5	9	6	1	7	2	0	0	30	21	9
1995	4	11	6	3	9	2	0	0	35	24	11
1996	4	11	7	6	8	3	0	0	39	28	11
1997	4	13	6	8	12	6	0	0	49	31	18
1998	5	13	6	8	17	8	0	0	57	32	25
1999	6	12	9	13	14	10	3	2	69	40	29
2000	7	11	7	17	14	11	1	2	70	42	28
2001	7	11	8	18	12	11	1	1	69	44	25
2002	9	9	8	18	14	11	0	2	71	44	27
2003	13	5	14	13	13	10	1	1	70	45	25
Total	146	173	97	112	132	85	6	8	759	528	231
%	19	23	13	15	17	11	1	1	100	70	30

Ratings are observed on 5th March. Sovereigns with withdrawn ratings are eliminated from the year of withdrawal to the year of new rating. Default sovereigns are discarded from the year following default to the year of recovery. IG and NIG stand for Investment Grade and Non-Investment Grade issuers, respectively.

B. Percentage of rated sovereigns. Groups by rating and year

	Rating										
Year	Aaa	Aa	A	Baa	Ba	B	C	Default	Total	IG	NIG
1981	73	27	0	0	0	0	0	0	100	100	0
1982	73	27	0	0	0	0	0	0	100	100	0
1983	67	33	0	0	0	0	0	0	100	100	0
1984	67	33	0	0	0	0	0	0	100	100	0
1985	56	44	0	0	0	0	0	0	100	100	0
1986	75	25	0	0	0	0	0	0	100	100	0
1987	57	14	7	7	14	0	0	0	100	86	14
1988	39	33	6	6	6	11	0	0	100	83	17
1989	40	30	10	5	5	10	0	0	100	85	15
1990	32	32	18	5	5	9	0	0	100	86	14
1991	23	41	18	5	9	5	0	0	100	86	14
1992	17	43	17	4	9	9	0	0	100	83	17
1993	17	42	17	4	13	8	0	0	100	79	21
1994	17	30	20	3	23	7	0	0	100	70	30
1995	11	31	17	9	26	6	0	0	100	69	31
1996	10	28	18	15	21	8	0	0	100	92	28
1997	8	27	12	16	24	12	0	0	100	63	37
1998	9	23	11	14	30	14	0	0	100	56	44
1999	9	17	13	19	20	14	4	3	100	58	42
2000	10	16	10	24	20	16	1	3	100	60	40
2001	10	16	12	26	17	16	1	1	100	64	36
2002	13	13	11	25	20	15	0	3	100	62	38
2003	19	7	20	19	19	14	1	1	100	64	36

See note in Table A.

Appendix D: Summary Statistics for Sovereign Rating Durations

Rating	Mean	StDev	Max	Min	StDev/Mean
Aaa	78.94	78.65	280.03	12.10	0.996
Aa	50.22	49.09	194.07	1.63	0.978
A	33.74	32.30	124.57	2.23	0.957
Baa	32.17	21.70	87.13	0.80	0.674
Ba	31.46	26.66	103.10	0.63	0.847
B	18.58	19.62	84.97	0.13	1.06
C	11.48	11.74	34.37	1.27	1.02
D	28.70	17.44	55.13	0.40	0.61

Duration is the number of months spent in each rating.

Data are from March 1981 to March 2004.

Table 1

One-year Homogeneous Rating Transition Probability Estimates

	Aaa	Aa	A	Baa	Ba	B	C	D
i) DM estimator: transition probabilities								
Aaa	0.94444444	0.05555556	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
Aa	0.06395349	0.92441860	0.01162791	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
A	0.00000000	0.04123711	0.88659794	0.06185567	0.01030928	0.00000000	0.00000000	0.00000000
Baa	0.00000000	0.00000000	0.10000000	0.85454545	0.02727273	0.01818182	0.00000000	0.03333333
Ba	0.00000000	0.00000000	0.00000000	0.07575758	0.83333333	0.06818182	0.01515152	0.00757576
B	0.00000000	0.00000000	0.00000000	0.00000000	0.07142857	0.84523810	0.02380952	0.05952381
C	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.33333333	0.50000000	0.16666667
D	0	0	0	0	0	0	0	1
ii.1) HHR estimator: transition intensities								
Aaa	-0.06031064	0.06031064	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
Aa	0.06345418	-0.07499131	0.01153712	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
A	0.00000000	0.03898427	-0.11695282	0.07796855	0.00000000	0.00000000	0.00000000	0.00000000
Baa	0.00000000	0.00000000	0.10085659	-0.15968960	0.05883301	0.00000000	0.00000000	0.00000000
Ba	0.00000000	0.00000000	0.00000000	0.08026789	-0.18972411	0.10215914	0.00000000	0.00729708
B	0.00000000	0.00000000	0.00000000	0.00000000	0.08042178	-0.19531004	0.06893296	0.04595530
C	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.30290456	-0.75726141	0.45435685
D	0	0	0	0	0	0	0	0
ii.2) HHR estimator: transition probabilities								
Aaa	0.94326537	0.05640615	0.00032019	0.00000816	0.00000012	0.00000000	0.00000000	1.89×10^{-10}
Aa	0.05934618	0.92974209	0.01050315	0.00040064	0.00000771	0.00000020	0.00000000	**0.00000002**
A	0.00113833	0.03549046	0.89328254	0.06804843	0.00196704	0.00006660	0.00000103	**0.00000556**
Baa	0.00003750	0.00175118	0.08802438	0.85780798	0.04957648	0.00252941	0.00005040	**0.00022258**
Ba	0.00000074	0.00004600	0.00347151	0.06763890	0.83258448	0.08550168	0.00244961	**0.00830707**
B	0.00000001	0.00000092	0.00009220	0.00270316	0.06680802	0.83799106	0.04368674	**0.04871789**
C	0.00000000	0.00000005	0.00000665	0.00025650	0.00935562	0.22501918	0.47564963	**0.28971236**
D	0	0	0	0	0	0	0	1

The bootstrap DGP parameters (transition probabilities in bold) are estimated from the observed 1981-2004 ratings. DM stands for discrete multinomial and HHR for homogenous hazard rate.

Table 2
Summary Statistics for Bootstrap 1-year Default Probability Estimates

Panel A: DM estimator

Rating	True PD	Mean($\widehat{PD}$)	StDev($\widehat{PD}$)	Median($\widehat{PD}$)	95% Conf. Int.	Mean Bias	Median Bias
Aaa	1.9×10^{-10}	0	0	0	$[0,0]$	-1.89×10^{-10}	-1.89×10^{-10}
Aa	1.6×10^{-8}	0	0	0	$[0,0]$	-1.58×10^{-8}	-1.58×10^{-8}
A	5.6×10^{-6}	0	0	0	$[0,0]$	-5.56×10^{-6}	-5.56×10^{-6}
Baa	0.0002226	0.0002696	0.001502	0	$[0,0.00752]$	0.0000470	−0.0002226
Ba	0.008307	0.008905	0.008086	0.007519	$[0,0.02801]$	0.0005977	−0.0007883
B	0.04872	0.04661	0.02596	0.04412	$[0,0.1067]$	−0.002109	−0.004600
C	0.2897	0.2761	0.2591	0.2500	$[0,1]$	−0.01363	−0.03971

Panel B: HHR estimator

Rating	True PD	Mean($\widehat{PD}$)	StDev($\widehat{PD}$)	Median($\widehat{PD}$)	95% Conf. Int.	Mean Bias	Median Bias
Aaa	1.9×10^{-10}	2.3×10^{-10}	3.5×10^{-10}		$[0,1.2\times10^{-9}]$	3.5×10^{-11}	-1.01×10^{-11}
Aa	1.6×10^{-8}	1.8×10^{-8}	2.7×10^{-8}		$[0,9\times10^{-8}]$	2.6×10^{-9}	-7.99×10^{-9}
A	5.6×10^{-6}	6.6×10^{-6}	6.8×10^{-6}		$[1.8\times10^{-7},2.4\times10^{-5}]$	9.9×10^{-7}	-4.91×10^{-7}
Baa	0.0002226	0.0002701	0.0002625		$[1.1\times10^{-5},0.0009718]$	0.00004748	-7.59×10^{-6}
Ba	0.008307	0.01003	0.007823		$[0.0007534,0.02963]$	0.001727	0.0008696
B	0.04872	0.05861	0.04568		$[0.01333,0.1204]$	0.009890	0.004923
C	0.2897	0.3220	0.1882		$[0,0.7488]$	0.03231	0.01808

See footnote to Table 5.1. True PD are the 1-year default probability parameters in the bootstrap DGP for the ratings.
The bias is calculated as $Mean(\widehat{PD})-True(PD)$ or as $Median(\widehat{PD})-True(PD)$.

Table 3

Bootstrap Tests for Di erences between Discrete and Continuous Estimator

	Panel A: 1-year default probability di erential			
Rating	Mean$\Delta(\widehat{PD})$	StDev$\Delta(\widehat{PD})$	95% Conf. Int.	Null $\Delta(PD) = 0$
Aaa	2.3×10^{-10}	3.5×10^{-10}	$[0, 1.2 \times 10^{-9}]$	Not reject
Aa	1.8×10^{-8}	2.7×10^{-8}	$[0, 9 \times 10^{-8}]$	Not reject
A	6.6×10^{-6}	6.8×10^{-6}	$[1.8 \times 10^{-7}, 2.4 \times 10^{-5}]$	Reject
Baa	4.8×10^{-7}	0.001486	$[-0.006857, 0.000946]$	Not reject
Ba	0.001129	0.004858	$[-0.01078, 0.01024]$	Not reject
B	0.01200	0.03948	$[-0.01468, 0.04364]$	Not reject
C	0.04557	0.2171	$[-0.3796, 0.5269]$	Not reject

Panel B: Matrix mobility di erential

$\hat{m}$	$\Delta\hat{m}$	Mean($\Delta\hat{m}$)	StDev($\Delta\hat{m}$)	95% Conf. Int.	$\Delta m = 0$
0.1921(HHR) 0.1954(DM)	-0.003257	-0.09424	0.04160	$[-0.1801, -0.03485]$	Reject

See note in Table 5.1. The default probability di erential, $\Delta(PD)$, and the mobility di erential, $\Delta\hat{m}$, are computed as the HHR minus the DM estimator.

Table 4

Longer Horizon Homogeneous Transition Probability Matrices

	Aaa	Aa	A	Baa	Ba	B	C	D
Panel A: 2-year time-homogeneous transition matrices								
i) DM estimator: transition probabilities								
Aaa	0.89393939	0.10606061	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
Aa	0.13253012	0.84337349	0.02409639	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
A	0.00000000	0.09302326	0.83720930	0.04651163	0.02325581	0.00000000	0.00000000	0.00000000
Baa	0.00000000	0.00000000	0.19230769	0.73076923	0.03846154	0.01923077	0.00000000	0.01923077
Ba	0.00000000	0.00000000	0.01587302	0.12698413	0.73015873	0.11111111	0.01587302	0.00000000
B	0.00000000	0.00000000	0.00000000	0.02564103	0.12820513	0.71794872	0.02564103	0.10256410
C	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	1	0.00000000	0.00000000
D	0	0	0	0	0	0	0	1
ii) HHR estimator: transition probabilities								
Aaa	0.89309742	0.10566051	0.00118122	0.00005910	0.00000169	0.00000006	0.000000002	0.00000001
Aa	0.11116780	0.86814133	0.01920177	0.00143194	0.00005410	0.00000274	0.00000005	0.00000025
A	0.00419941	0.06488333	0.80432402	0.11930575	0.00677385	0.00045614	0.00001340	0.00004410
Baa	0.00027174	0.00625894	0.15432853	0.74518679	0.08414610	0.00858763	0.00029323	0.00092705
Ba	0.00001060	0.00032267	0.01195477	0.11480340	0.70235638	0.14498126	0.00706058	0.01851035
B	0.00000032	0.00001280	0.00062900	0.00913344	0.11247287	0.72636960	0.05735495	0.09402703
C	0.00000004	0.00000125	0.00008910	0.00162146	0.02943731	0.34594667	0.24106727	0.38183694
D	0	0	0	0	0	0	0	1
Panel B: 3-year time-homogeneous transition matrices								
i) DM estimator: transition probabilities								
Aaa	0.825	0.175	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
Aa	0.2	0.76363636	0.03636364	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
A	0.00000000	0.15384615	0.73076923	0.07692308	0.03846154	0.00000000	0.00000000	0.00000000
Baa	0.00000000	0.00000000	0.23333333	0.63333333	0.06666667	0.033333	0.00000000	0.03333333
Ba	0.00000000	0.00000000	0.05	0.15000000	0.6	0.175	0.025	0.00000000
B	0.00000000	0.00000000	0.00000000	0.05263158	0.10526316	0.78947368	0.05263158	0.00000000
C	0.00000000	0.00000000	0.00000000	0.00000000	1	0.00000000	0.00000000	0.00000000
D	0	0	0	0	0	0	0	1
ii) HHR estimator: transition probabilities								
Aaa	0.84869989	0.14865507	0.00245627	0.00018061	0.00000769	0.0000004	0.00000002	0.00000004
Aa	0.15640335	0.81410236	0.02643214	0.00288804	0.00016023	0.00001240	0.00000020	0.00000130
A	0.00873241	0.08931496	0.72970394	0.15754915	0.01317806	0.00131647	0.00006220	0.00014276
Baa	0.00083059	0.01262352	0.20379847	0.65546943	0.10786059	0.01656732	0.00065266	0.00219742
Ba	0.00004820	0.00095553	0.02325718	0.14715790	0.60046186	0.18628034	0.01267560	0.02916336
B	0.00000196	0.00005770	0.00180806	0.01754222	0.14351949	0.64937460	0.05295081	0.13474512
C	0.00000049	0.00000641	0.00040229	0.00419626	0.05450719	0.39994490	0.18600557	0.35493687
D	0	0	0	0	0	0	0	1

(.cont)

Panel C: Matrix mobility metric for varying horizon			
Transition horizon (years)	$\hat{m}$(HHR)	$\hat{m}$(DM)	$\Delta\hat{m}$
1	0.1921	0.1954	-0.0033
2	0.3155	0.3916	-0.0761
3	0.3802	0.4661	-0.0859
4	0.4007	0.5197	-0.1190

DM stands for discrete multinomial and HHR for homogenous hazard rate.
The mobility di erential, $\Delta\hat{m}$, is computed as the HHR minus the DM estimator.

Table 5

Heterogeneous versus Homogeneous Transition Probability Matrices

	Aaa	Aa	A	Baa	Ba	B	C	D
Panel A: 2-year horizon								
i) NHHR estimator: transition probabilities								
Aaa	0.92016251	0.07983749	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
Aa	0.17143835	0.80310470	0.02545695	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
A	0.00336537	0.06106579	0.78600696	0.13813270	0.01075966	0.00066952	0.00000000	0.00000000
Baa	0.00000000	0.00187119	0.15640571	0.71752941	0.11918620	0.00488992	0.00005879	0.00005879
Ba	0.00000000	0.00006791	0.01368107	0.10337922	0.72815887	0.13087669	0.00475398	0.01908226
B	0.00000000	0.00000000	0.00015967	0.00525031	0.11132463	0.77381518	0.02885281	0.08059740
C	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.16111111	0.67063492	0.16825397
D	0	0	0	0	0	0	0	1
ii) HHR estimator: transition probabilities								
Aaa	0.95925929	0.04074071	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
Aa	0.09644019	0.88791886	0.01564095	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
A	0.00188575	0.02971148	0.88767981	0.07820094	0.00239812	0.00011247	0.00000428	0.00000715
Baa	0.00012088	0.00078962	0.08153693	0.85864221	0.05674503	0.00195668	0.00006480	0.00014386
Ba	0.00000352	0.00003103	0.00337684	0.06105312	0.85362392	0.07054356	0.00244397	0.00892405
B	0.00000125	0.00000005	0.00015837	0.00338075	0.05672224	0.86721028	0.03246649	0.04006059
C	0.00000000	0.00000000	0.00000346	0.00023973	0.00426602	0.07854813	0.78923259	0.12771007
D	0	0	0	0	0	0	0	1
Panel B: 3-year horizon								
i) NHHR estimator: transition probabilities								
Aaa	0.89704219	0.10295781	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
Aa	0.24791551	0.71512515	0.03695934	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
A	0.00477332	0.08545516	0.75038362	0.14407716	0.01308519	0.00162502	0.00002053	0.00058000
Baa	0.00000000	0.00271339	0.22693730	0.58257029	0.17611914	0.00949798	0.00016421	0.00199768
Ba	0.00000000	0.00028635	0.02743720	0.12277370	0.62134554	0.19443597	0.00740273	0.02631851
B	0.00000000	0.00005206	0.00412252	0.01114124	0.13311615	0.70687773	0.03902732	0.10566297
C	0.00000000	0.00000000	0.00017013	0.00221167	0.04689865	0.19957862	0.50550336	0.24563758
D	0	0	0	0	0	0	0	0
ii) HHR estimator: transition probabilities								
Aaa	0.96124149	0.03875851	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
Aa	0.11331804	0.86813047	0.01817890	0.00036531	0.00000720	0.00000007	0.00000000	0.00000000
A	0.00239263	0.03718017	0.87399071	0.08338458	0.00288064	0.00015946	0.00000692	0.00000488
Baa	0.00015714	0.00128565	0.08588658	0.84221222	0.06832629	0.00195097	0.00010080	0.00008034
Ba	0.00000442	0.00004905	0.00303206	0.04954478	0.85712529	0.08371660	0.00341873	0.00310906
B	0.00000012	0.00000179	0.00013095	0.00189599	0.03459065	0.86930118	0.04151851	0.05256081
C	0.00000002	0.00000019	0.00002111	0.00036164	0.00919448	0.14159845	0.62098425	0.22783986
D	0	0	0	0	0	0	0	1

(.cont)

Panel C: Matrix mobility

Transition horizon (years)	$\hat{m}(\text{HHR})$	$\hat{m}(\text{NHHR})$	$\Delta\hat{m}$
1	0.1219	0.1303	0.0618
2	0.1352	0.2399	0.0755
3	0.1718	0.3146	0.0786
4	0.1192	0.4210	-0.0204

HHR stands for homogenous hazard rate and NHHR stands for non-homogeneous hazard rate estimator. The mobility di erential, $\Delta\hat{m}$, is computed as the HHR minus the NHHR estimator.

Table 6

Summary Statistics for Bootstrap 2-year Default Probability Estimates

Panel A: HHR estimator							
Rating	True PD	Mean($\widehat{PD}$)	StDev($\widehat{PD}$)	Median($\widehat{PD}$)	95% Conf. Int.	Mean Bias	Median Bias
Aaa	0	0	0	0	0	0	0
Aa	0	0	0	0	0	0	0
A	4.8×10^{-6}	8.5×10^{-6}	1.3×10^{-5}	4.3×10^{-6}	$[0, 4.3 \times 10^{-5}]$	3.7×10^{-6}	-4.9×10^{-7}
Baa	1.2×10^{-4}	0.0001625	0.0001979	0.00009753	$[0, 0.0006900]$	4.2×10^{-5}	-2.3×10^{-5}
Ba	0.02519	0.009670	0.007105	0.008813	$[0.0006235, 0.02601]$	-0.01552	-0.01638
B	0.09319	0.04327	0.01940	0.04068	$[0.01313, 0.08646]$	-0.04992	-0.05251
C	0.2232	0.1095	0.06122	0.1079	$[0, 0.2437]$	-0.1137	-0.11535

Panel B: NHHR estimator							
Rating	True PD	Mean($\widehat{PD}$)	StDev($\widehat{PD}$)	Median($\widehat{PD}$)	95% Conf. Int.	Mean Bias	Median Bias
Aaa	0	0	0	0	$[0, 0]$	0	0
Aa	0	0	0	0	$[0, 0]$	0	0
A	4.8×10^{-6}	3.7×10^{-6}	1.7×10^{-5}	0	$[0, 4.5 \times 10^{-5}]$	-1.1×10^{-6}	-4.8×10^{-6}
Baa	1.2×10^{-4}	0.0001218	0.0002324	0	$[0, 0.0007751]$	1.7×10^{-6}	-0.0001202
Ba	0.02519	0.02390	0.01384	0.02305	$[0.00265, 0.05420]$	-0.001292	-0.002139
B	0.09319	0.09017	0.03512	0.08712	$[0.02992, 0.1674]$	-0.003022	-0.006063
C	0.2232	0.1690	0.09032	0.1673	$[0, 0.3522]$	-0.05425	-0.05589

The bootstrap DGP parameters (true transition probabilities) are estimated from the observed 1981-2004 ratings. The bias is calculated as $Mean(\widehat{PD}) - True(PD)$ or as $Median(\widehat{PD}) - True(PD)$.HHR stands for homogeneous hazard rate and NHHR for non-homogeneous hazard rate estimator.

Table 7
Bootstrap Tests for Di erences between Homogeneous and Heterogeneous Estimator

	Panel A: 2-year default probability di erential			
Rating	Mean$\Delta(\widehat{PD})$	StDev$\Delta(\widehat{PD})$	95% Conf. Int.	Null $\Delta(PD) = 0$
Aaa	0	0	0	–
Aa	0	0	0	–
A	-4.8×10^{-6}	2×10^{-5}	$[-4 \times 10^{-5}, 3.5 \times 10^{-5}]$	Not reject
Baa	-4.1×10^{-5}	0.0002902	$[-0.0006211, 0.0006058]$	Not reject
Ba	0.01423	0.009264	$[-0.0002048, 0.03332]$	Not reject
B	0.04689	0.02326	$[0.007110, 0.09616]$	Reject
C	0.05947	0.06409	$[-0.04547, 0.2122]$	Not reject

	Matrix mobility metric di erentials				
$\hat{m}$	$\Delta\hat{m}$	Mean($\Delta\hat{m}$)	StDev($\Delta\hat{m}$)	95% Conf. Int.	$\Delta m = 0$
0.1352(HHR) 0.2399(NHHR)	0.1047	0.08104	0.01623	$[0.05331, 0.1201]$	Reject

See note in Table 5.6. The di erential statistics, $\Delta(\widehat{PD})$ and $\Delta\hat{m}$, are computed as the HHR minus the NHHR value.

Table 8
Logit Test Results for Non-Markov E ects

Panel A: Duration e ect

	Upgrade		Downgrade	
Coe cient	estimate	t ratio	estimate	t ratio
β	-1.88	-5.33	-1.65	-5.74
γ_{1998}	1.99	2.71	-0.31	-0.37
γ_{1999}	0.48	0.39	1.68	3.12
γ_{2000}	1.36	1.69	1.49	2.62
γ_{2001}	3.81	4.88	0.37	0.43
γ_{2002}	2.39	2.89	1.94	2.70
γ_{2003}	3.78	5.67	-0.08	-0.12
γ_{2004}	-0.44	-0.67	-0.90	-1.66
α	-1.57	-1.50	-0.41	-1.15
ξ	0.27	1.04	0.11	0.44

Panel B: Momentum e ect

	Upgrade		Downgrade	
Coe cient	estimate	t ratio	estimate	t ratio
β	-1.24	-1.02	1.20	2.94
γ_{1998}	1.65	2.75	-0.37	-0.46
γ_{1999}	0.60	0.71	1.21	2.67
γ_{2000}	0.84	1.19	0.71	1.47
γ_{2001}	1.83	3.23	-0.79	-0.99
γ_{2002}	0.84	1.16	0.35	0.63
γ_{2003}	2.61	5.47	-0.28	-0.45
γ_{2004}	0.90	1.08	-0.08	-0.12
α	-6.37	-14.46	-5.18	-14.06
ξ	-0.027	-0.35	-0.17	-2.10

The logit estimates are based on monthly ratings March, 1981-March 2004. The logits in Panel A and B model the probability of upgrade/downgrade as a function of duration and rating drift (β), respectively. γ_j are dummies, α is an intercept and ξ is a parameter in the disturbance variance equation to capture groupwise heteroskedasticity (industrial/non-industrial sovereigns).

Figure 2: Discrete versus Continuous Bootstrap Default Probability Estimates
(high credit-quality ratings)

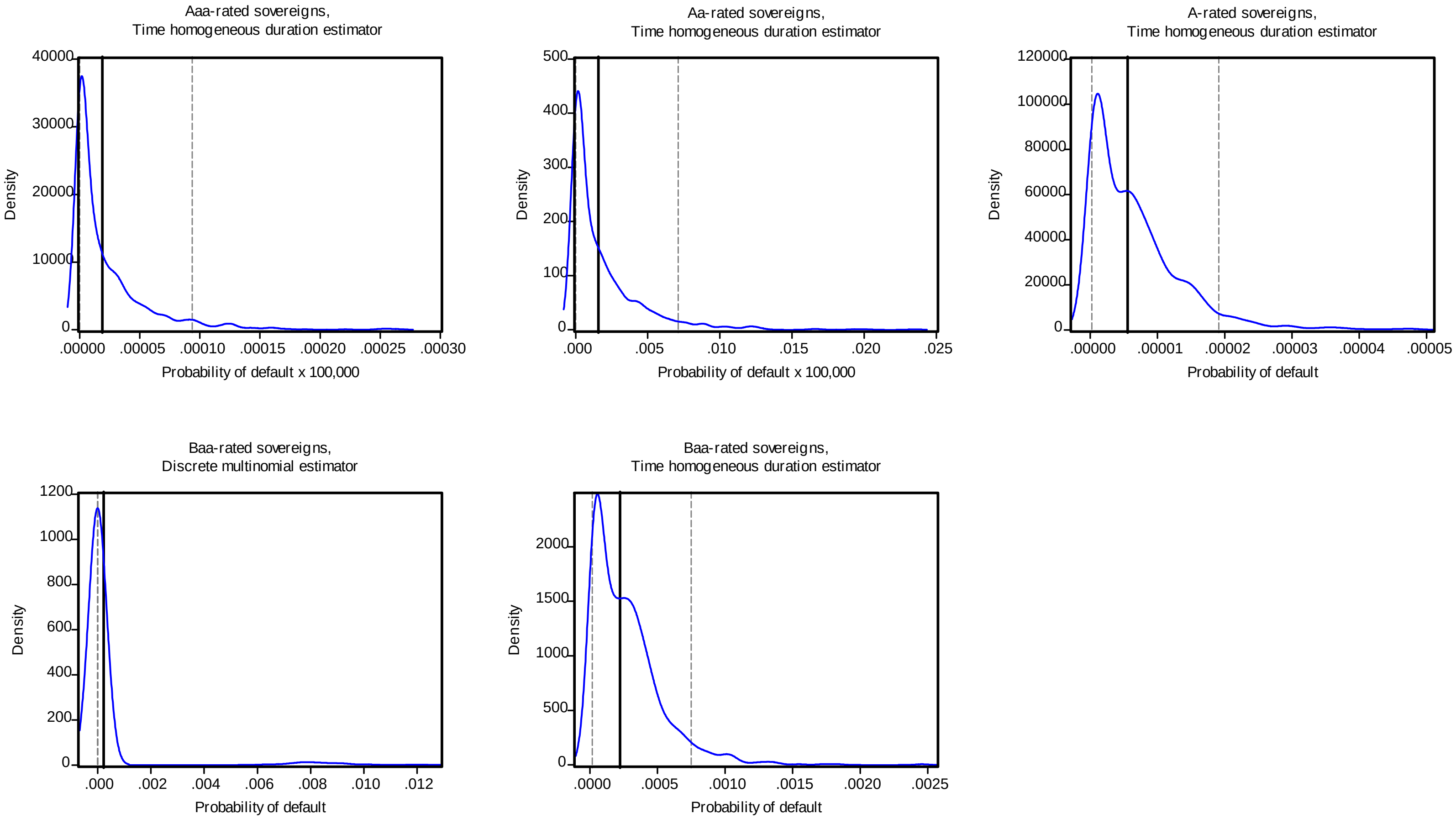

Figure 3: Discrete versus Continuous Bootstrap Default Probability Estimates
(low credit-quality ratings)

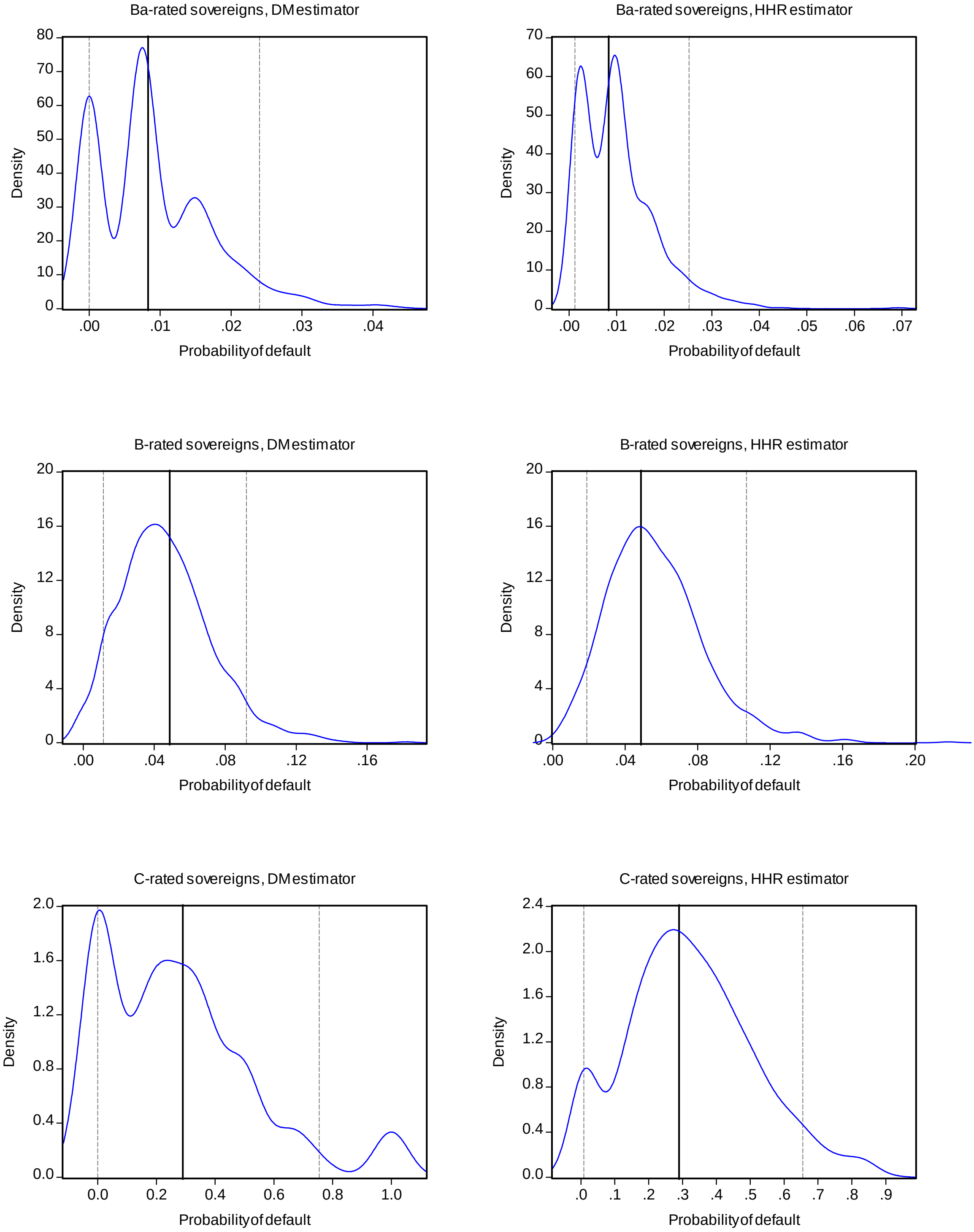

Figure 4: Homogeneous versus Heterogeneous Bootstrap Default Probability Estimates
(high credit-quality ratings)

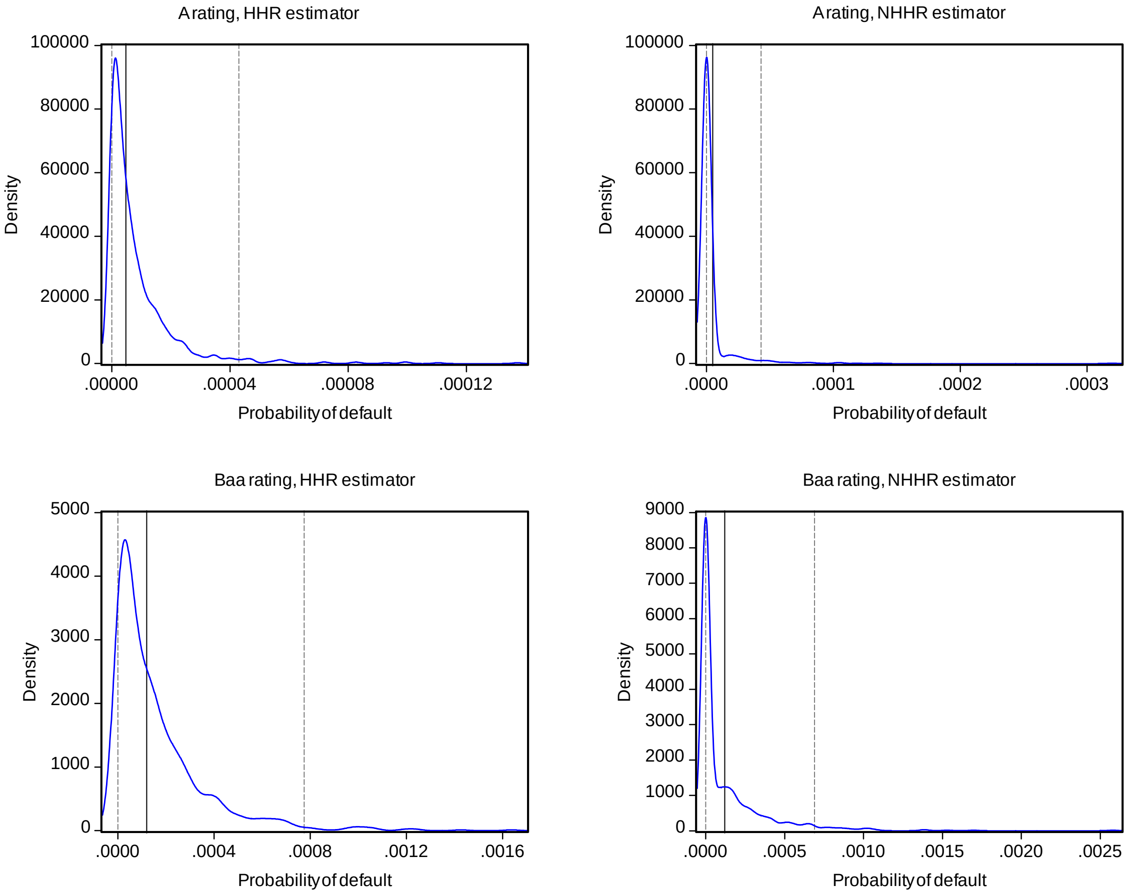

Figure 5: Homogeneous versus Heterogeneous Bootstrap Default Probability Estimates
(low credit-quality ratings)