Fundamental Capital Valuation for Growing Firms: A Real Option Approach

Chung Baek*
Brice Dupoyet**
Arun J Prakash**

Address for Correspondence

Arun J Prakash
Department of Finance
CBA, Florida International University
Miami, Florida 33199
(305) 348-2680
prakasha@fiu.edu

*University of Nebraska, Lincoln
**Florida International University, Miami

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Abstract

We propose a new approach to the pricing of young growing firms' fundamental equity values by combining the corporate debt valuation of Merton (1974) and the rational pricing of internet companies of Schwartz and Moon (2000). The Schwartz and Moon (2000) methodology is made more realistic by allowing for stochastic volatility in revenue growth and used to compute an estimate of the firm's fundamental value. We recover the firm's fundamental equity value by subtracting the fundamental debt value – obtained using Merton (1974) debt valuation technique – from the fundamental firm value. We test the methodology on six firms from the Information Technology (IT) sector and show how, contrary to the popular belief that most IT companies are overvalued we find that it is only the case for four of them while the other two are actually undervalued. The approach described in the paper is expected to provide guidance to academics and practitioners in need of evaluating the share price of young firms having yet to reach stability of growth.

I. Introduction

The groundbreaking work of Black and Scholes (1973) in the field of option pricing has opened the doors to a myriad of valuable extensions in a wide variety of areas. One of these extensions is the application of option theory to real assets in an attempt to value a firm's capital. The pricing of these real options isn't without difficulties, however, as pointed out by Edward (2003). Real options can be mathematically challenging, may lack simplicity, are a relatively new instrument, may use non-tradable underlying assets, and finally may not accurately reflect the complex reality faced by managers. Moreover, the specific traits of the underlying real asset may be difficult to estimate before being used as inputs to the model. Nevertheless, despite all of these issues, real options do come in very handy when traditional pricing tools fail to provide us with a clear answer to a valuation problem.

Real options are often associated with the pricing of enterprises considered having significant growth opportunities, and as such, tend to be involved with the valuation of high-tech firms such as electronics and pharmaceutical companies. Among others, Lint and Pennings (2001) demonstrate how real options are able to capture the value of the flexibility present at each stage of a new product development, Frigo (2003) addresses the usefulness of real option valuation and Banerjee (2003) shows how valuation by real options of the R&D component of a pharmaceutical company can help account for a high market price otherwise unexplained by more traditional valuation tools. Our study focuses on how the equity of a growing firm can be evaluated using a real option approach. For the task at hand, we select six Information Technology - IT from here on - firms likely to benefit from the technique described in the next paragraph.

Merton (1974) shows how one can price risky discount bonds using an argument following that of Black and Scholes (1973). In a recent article, Schwartz and Moon (2000) propose a methodology to estimate the fundamental total value of a firm in cases where traditional valuation methods are difficult to implement, such as in the case of internet companies. We approach equity valuation under a new angle by combining these two techniques to compute an estimate of the fundamental equity value of firms in the IT sector as the difference between their fundamental total value and their debt value. Since equity market value is easily observable, it is then straightforward to compare whether a firm's stock is priced accurately. Despite the common perception that IT firms are often not worth their share price, our results show that not all firms turn out to be overvalued: the ratio of undervalued to overvalued firms is two to four.

We also study the debt characteristics of these firms, but since observing the current market value of the debt is a challenge[1], we are not able to estimate whether the debt itself is overvalued or undervalued. Instead, we focus on the risk premiums associated with the bonds. Under the assumption that the firm's debt can be expressed as a zero-coupon bond, we find that two of the firms have debt with fundamental risk premiums close to 0% while the rest have fundamental risk premiums ranging from 0.1% to 1.87%. In other words, the first two are close to the riskless rate while the other four appear slightly riskier.

The paper is organized as follows. Section 2 reviews how options relate to the value of a firm's debt and equity. Section 3 describes data collection. The techniques used to estimate the fundamental value of the firms' debt and equity are covered in detail in section 4. Empirical results are described in section 5 and section 6 concludes.

II. Real Option Theory with Debt and Equity

This section begins with a brief review of how options relate to the valuation of a firm. Suppose that a company with total value X has outstanding issues of debt and equity with respective values D and E. Debt holders' claims having priority over those of the equity holders, the equity holders are called residual claimants. If X is greater than D, the firm pays the debt holders, and the equity holders collect the difference X-D. If X is less than D, the debt holders claim the assets of the firm and the equity holders receive nothing. The payoff to the equity holders is thus identical to the payoff associated with a call option where the underlying asset is the value X of the firm and the exercise price is the amount of debt D. Note that the bondholders face a different payoff structure, albeit again relating to options. As long as X is larger than D, the debt holders are guaranteed to receive D. But if X falls below D, debt holders lose the amount D-X. The payoff to the bondholders is thus related to the payoff associated with being short on a put option where the underlying asset is the value X of the firm and the exercise price is the amount of debt D.

Now suppose that we are able to obtain the "true" total value of the firm. This number should be distinguished from the sum of the current debt and equity market values, as the true total firm value is not necessarily the same as the total market value. This fundamental value of the firm, once estimated, could then be compared against observed market values of the company – provided that both are observable – in an attempt to measure the levels of over(under) valuation. Alternatively, the total fundamental firm value could be estimated along with fundamental debt value, fundamental equity value could be inferred from the difference, and comparisons between theoretical and market share values could be made. This is the approach followed in the paper.

III. The Data

The firms are selected from a list of top 20 IT companies ranked by revenue level in the Washington Post. Eight out of 20 companies are immediately eliminated for lack of accounting data. In addition to these, companies with more than 10 years of business history are also taken out of the sample as we are targeting firms that haven't reached their steady growth stage yet. Empirically we observe that it takes an average of about 13 years for a firm in the IT sector to do so, leading us to remove six more firms. The six remaining companies are: Bearing Point Inc, EPlus Inc, Costar Group Inc, Online

Resources Corp, Steel Cloud Inc, and Inteli Data Technologies Corp – for our valuation model.

All data needed were obtained from COMPUSTAT and 10-Q quarterly reports. Table 1 shows the range of data available from COMPUSTAT for each firm. Data for Bearing Point Inc were available from 1998 to 2003, data for EPlus Inc from 1995 to 2003, data for CoStar Group Inc from 1996 to 2003, data for Online Resources Corp from 1997 to 2003, data for Steel Cloud Inc from 1996 to 2003, and data for Inteli Data Technologies Corp from 1994 to 2003. We conduct our valuation on the basis of quarterly data and a future long-term horizon of 25 years[2]. As mentioned earlier, we face some limitations when estimating the fundamental value of the debt. The debt characteristics not always being described in detail in the 10-Q quarterly reports, we assume that all cash flows are the face values of risky bonds[3].

IV. The Model

The estimation of the value of equity in this paper relies on combining two separate capital valuation techniques: the corporate debt valuation of Merton (1974) and the rational pricing technique of internet companies of Schwartz and Moon (2000). The approach used by Merton (1974) is used to infer an estimate of the fundamental value of the debt, and the valuation methodology of Schwartz and Moon (2000) is used to estimate the fundamental value of the firm. The technique described in Schwartz and Moon (2000) is modified somewhat in order to allow for stochastic volatility in revenue growth and to reduce the number of parameters needed for the simulation. The technique from Merton (1974) is adapted to our problem, and assuming that the firm's various obligations can be lumped into a representative single bond we construct a zero-return portfolio and obtain a closed-form solution estimate of the fundamental debt value by solving the resulting partial differential equation.

4.1 Fundamental firm value

Since the firm as a whole is not a financially traded asset itself, we use the methodology proposed by Schwartz and Moon (2000) originally introduced to estimate the true value of the firm under a stochastic environment. We also convert their continuous-time model into a discrete-time version using quarterly accounting data. One drawback of the methodology, however, is the large number of parameters that must be estimated prior to the implementation of the Monte Carlo simulation. For tractability purposes, we thus make a few simplifying assumptions enabling us to reduce the number of parameters.

The key aspect of this valuation method is to use a risk-neutral discounted cash flow analysis to derive the value of the firm. The present value of the firm is the sum of the after-tax expected cash flows under the risk-neutral measure, discounted back to the present at the risk-free rate. The value today is therefore:

$$X_0 = E^*(TCF_T e^{-rT}) \tag{1}$$

where X_0 is the present value of the firm at time 0, TCF_T is the total after-tax expected cash flow accumulated up to time T, r is the risk-free rate and E^* is the risk-adjusted expectation operator. Following Schwartz and Moon (2000), we assume that all cash flows generated remain as retained earnings, earn a rate of return equal to the risk-free rate and are distributed to the shareholders at a long-term horizon T. The liquidation value of the firm TCF_T is calculated as the total expected cash flows accumulated up to time T plus a multiple of EBITDA at time T. This implies that when the firm is liquidated at time T, the terminal value of the firm is assumed to be a multiple of EBITDA. We select the same multiple (10 times) as Schwartz and Moon (2000)[4]. Earnings are driven by revenues, and revenues are generated according to the following stochastic differential equations specified here under the risk-neutral measure[5]:

$$\frac{dR_t}{R_t} = (g_t - \lambda\sqrt{V_t})dt + \sqrt{V_t}\, dW_{1t} \tag{2}$$

$$d\log V_t = \kappa\,(\log V^M - \log V_t)dt + \varphi_1 dW_{2t} \tag{3}$$

$$dg_t = \kappa\,(g^M - g_t)dt + \varphi_2 dW_{3t} \tag{4}$$

$$d\varphi_2 = -\kappa\varphi_2 dt \tag{5}$$

where R_t is the revenue, the drift g_t is the expected growth rate on revenues, λ is the price of risk, V_t is a volatility of the percentage change on revenue, and W_t is a standard wiener process. V_t and g_t follow a mean-reverting process. Unlike Schwartz and Moon (2000), V_t in our model is allowed to be stochastic. Figure 1 indicates that revenue growth displays time-varying volatility characteristics for all firms in the sample. And since V_t is a direct component of the revenue process, one can presume that the stochastic volatility assumption should impact valuation results. V^M and g^M are respectively the long-term average volatility and growth rate, while κ is the speed of mean-reversion. The volatility φ_1 of the change in V_t is assumed to be constant and the volatility φ_2 of the change in the growth rate is assumed to decrease along with a deterministic path because in the long-run, the growth rate is expected to be stable. Following Schwartz and Moon (2000), we assume that all processes other than the revenue are uncorrelated with aggregate wealth, implying that the market price of risk associated with these processes is null and that all Brownian shocks are independent of one another.

The net after-tax income N_t can be written as follows:

$$N_t = \text{Pr}\,etax\ Income_t \quad if\ \ LC_{t-1} > \text{Pr}\,etax\ Income_t \tag{6}$$

$$N_t = \text{Pr}\,etax\ Income_t(1-Tc) + Tc \bullet LC_{t-1} \quad if\ \ \text{Pr}\,etax_t Income_t > LC_{t-1}$$

where LC_t is the loss carry-forward, and Tc represents the corporate tax rate. The dynamics of the loss carry-forward, LC_t, are described by

$$dLC_t = Max(-M_t dt, 0) \qquad \text{if } LC_t = 0 \tag{7}$$

$$dLC_t = -M_t dt \qquad \text{if } LC_t > 0$$

where M_t = Pretax $Income_t$. As in Schwartz and Moon (2000), we assume that the total cost is proportional to the revenue. However, we estimate the total firm cost differently, describing the total cost-to-revenue ratio by the following stochastic differential equation:

$$d\alpha_t = \kappa\,(\alpha^M - \alpha_t)dt + \varphi_3 dW_{4t} \tag{8}$$

where α_t is the ratio process, α^M is the long-run average ratio, φ_3 is the volatility of change in the ratio, and κ is the speed adjustment. The mean-reverting process for the ratio is reasonable because in the long run, the earning margin associated with this ratio is expected to be stable. The most important advantage of using this stochastic ratio process is our ability to overcome a critical problem faced by Schwartz and Moon (2000). Schwartz and Moon (2000) assume a constant ratio, implying that when the ratio estimated from past data is found to be larger than one the firm is expected to always produce negative earnings[6]. In our stochastic setting, however, the ratio would ultimately be able to revert to a reasonably low long-run level even with a starting value greater than one. To conduct the simulation we discretize equations (2) through (8), yielding:

$$R_{t+\Delta t} = R_t e^{(g_t - \lambda\sqrt{V_t} - 0.5V_t) + \sqrt{V_t}\varepsilon_1} \tag{9}$$

$$\log V_{t+\Delta t} = \log V^M (1 - e^{-\kappa_1}) + e^{-\kappa_1} \log V_t + \varphi_1 \sqrt{\frac{1 - e^{-2\kappa_1}}{2\kappa_1}}\, \varepsilon_2 \tag{10}$$

$$g_{t+\Delta t} = g^M (1 - e^{-\kappa_2}) + e^{-\kappa_2} g_t + \varphi_2 \sqrt{\frac{1 - e^{-2\kappa_2}}{2\kappa_2}}\, \varepsilon_3 \tag{11}$$

$$\varphi_{2,t+\Delta t} = \varphi_0 e^{-\kappa t} \tag{12}$$

$$\alpha_{t+\Delta t} = \alpha^M (1 - e^{-\kappa_3}) + e^{-\kappa_3} \alpha_t + \varphi_3 \sqrt{\frac{1 - e^{-2\kappa_3}}{2\kappa_3}}\, \varepsilon_4 \tag{13}$$

where ε is the standard normal distribution. Note that we need to estimate or have data for a total of 17 parameters in order to perform the Monte Carlo simulation. Although the number of parameters can seem large, we are able to estimate those parameters under reasonable assumptions. Finally, the true value of the firm is estimated by equation (1) using a Monte Carlo simulation and the fact that the total expected cash flows accumulated up to the future long-term horizon, T are:

$$TCF_T = e^{r\Delta t} TCF_{T-\Delta t} + N_T + Dp + 10EBITDA_T \tag{14}$$

where Dp represents the Depreciation expense.

4.2 Fundamental debt and equity value (A closed-form solution)

We derive the fundamental value of the firm's debt using the methodology proposed by Merton (1974). A zero-return portfolio is constructed and the resulting partial differential equation is solved in order to obtain a closed-form solution for the fundamental debt value. And since the fundamental firm value X is the sum of the fundamental equity value E and fundamental debt value D, the fundamental equity value can be recovered by computing the difference X-D. Assuming perfect market conditions and a known term structure, the continuous-time dynamics for the value of the firm are expressed by the following stochastic differential equation:

$$dX_t = (\mu X_t - \theta_S - \theta_d)dt + \sigma X dW_t \tag{15}$$

where X is the total value of the firm, μ is the instantaneous expected rate of return on the value of the firm per unit of time, θ_s and θ_d are the payouts per unit of time by the firm to equity-holders and debt-holders respectively, σ is the volatility of the value of the firm per unit of time, and dW_t is a standard wiener process. Note that to keep the model tractable the volatility σ is assumed constant over time.

We now let H=B(X, t) where B(X, t) is the market value of the debt and a function of the value of the firm and time. Since H is assumed to be affected by a single factor, X, the dynamics of H can be given by

$$dH_t = (\mu_H H_t - \theta_d)dt + \sigma_H H dW_{tH} \tag{16}$$

where the drift μ_H is the instantaneous expected rate of return on H, σ_H is the instantaneous standard deviation of H, and dW_{tH} is a standard Wiener process.

Applying Ito's lemma to equation (10) yields

$$dH = [\frac{1}{2} B_{XX}\sigma^2 X^2 + B_X(\mu X - \theta_S - \theta_d) + B_t]dt + B_X \sigma X dW \tag{17}$$

where B_{XX} is the second-order partial derivative of B with respect to X, B_X is the first-order partial derivative of B with respect to X, and B_t is the first-order partial derivative of B with respect to time t. Since equation (16) should be identical to equation (17), we obtain

$$\mu_H H = \frac{1}{2} B_{XX}\sigma^2 X^2 + B_X(\mu X - \theta_S - \theta_d) + B_t + \theta_d \tag{18}$$

and

$$B_X \sigma X dW = \sigma_H H dW_H \tag{19}$$

In order to obtain a partial differential equation for the market value of the debt function, we form a portfolio consisting of the firm, the debt, and the risk-free asset so that the net investment in this portfolio is zero. In other words, the sum of w_1, w_2, and w_3 is equal to zero where w_1 is the dollar amount invested in the firm, w_2 is the dollar amount invested in the debt, and w_3 is the dollar amount invested in the risk-free asset.

The instantaneous return of the portfolio dr_p therefore is

$$dr_P = w_1 \frac{dX + (\theta_S + \theta_d)\, dt}{X} + w_2 \frac{dH + \theta_d dt}{X} + w_3 r\, dt \tag{20}$$

$$= [w_1(\mu - r) + w_2(\mu_H - r)]\, dt + w_1 \sigma dW_t + w_2 \sigma_H dW_{tH}$$

In the absence of arbitrage, the expected return on the portfolio should be zero because the net investment is null. From this condition and equation (19), the existence of a non-trivial solution implies the following singular matrix condition:

$$(\mu - r)\frac{B_X X}{H} = (\mu_H - r) \tag{21}$$

Combining equations (18) and (21) yields

$$\frac{1}{2} B_{XX}\sigma^2 X^2 + B_X(rX - \theta_s - \theta_d) + B_t + \theta_d - r\, B = 0 \tag{22}$$

This partial differential equation must be satisfied by the market value function of the debt B(X, t). Now, suppose that F is the promised payment to the debt-holders at the maturity. As long as the market value of the firm, X, is larger than F, the firm will pay F to the debt-holders at maturity. If X is smaller than F, however, the firm will go bankrupt and the debt-holders will receive X only. This can be summarized by:

$$If\ \ X > F, \qquad B = F$$

$$If\ \ X < F, \qquad B = X$$

From these conditions, the boundary condition B(X, T) = Min[X, F] can be set for the maturity date T. The other boundary condition is immediately obtained from the non-negativity of the debt. At any time t, B(0, t) = 0. By using these two conditions and under the assumption that θ_s and θ_d are sufficiently small relative to X, if we let $\Delta = (\theta_s + \theta_d)/X$ we obtain the following closed form with details shown in the Appendix 1:

$$B(X,t) = Fe^{-r(T-t)} - [Fe^{-r(T-t)}\Phi(d_2) - Xe^{-\Delta(T-t)}\Phi(d_1)] + (1 - e^{-r(T-t)})\frac{\theta_d}{r} \tag{23}$$

$$d_1 = \frac{-\ln(\frac{X}{F}) - (r - \Delta + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 + \sigma\sqrt{T-t}$$

where Φ(·) is the standard cumulative normal distribution function. Since the firm value is closely related to the stock value, we use the standard deviation of stock returns as an estimate for σ. In equation (23), the fundamental debt value consists of three components. On the right hand side, the first term is the present value of a zero-coupon bond, the next bracket term is the value of a put option, and the last term is associated with the present value of a coupon stream related to the remaining maturity. In other words, if the value of the firm X goes below the face value of the debt, stockholders will turn over all of the assets to the debt-holders. The last term on the right hand side collapses to the present value of a perpetuity when coupons are made indefinitely. As, however, maturity approaches zero, the last term converges to zero. Since we assumed in section 4.1 that all cash flows remain as retained earnings until the firm is liquidated, the payout to the stockholders θ_s is zero. The fundamental debt value therefore depends on θ_d: for example, if θ_d is equal to zero, B(X, t) will be the fundamental debt value for a zero-coupon bond and if θ_d is not equal to zero, B(X, t) will be the fundamental value for a coupon-bond.

We can now obtain the fundamental value of the equity by subtracting the fundamental debt value from equation (23) from the fundamental firm value. We get:

$$E(X,t) = X - B(X,t) \tag{24}$$

where E(X,t) is the fundamental value of equity at time t given the fundamental firm value X.

4.3 Risk premium on risky debt

Our focus in this section is on the risk premium of the risky debt. Since in prior sections we assumed that the firm does not distribute its earnings but keeps them as retained earnings until liquidation occurs, θ_s must be zero. We are thus left with θ_d as the only form of payout by the firm. With continuous compounding, if we denote the yield-to-maturity on the risky bond by R and a discount function of the coupons by K(R), the value of the debt can be expressed as:

$$B(X,t) = F\, e^{-R(T-t)} + \theta_d K(R) \tag{25}$$

Rearranging equation (25) in terms of the risk premium yields

$$(R-r) = -\frac{1}{T-t} \ln\left[\frac{Be^{r(T-t)} - \theta_d e^{r(T-t)} K(R)}{F}\right] \tag{26}$$

Note that in the case of a zero-coupon bond, θ_d is equal to zero and the risk premium can be directly backed out from equation (25). When coupons are present, however, solving for the risk premium entails defining a function

$$G(R) = (R-r) + \frac{1}{T-t} \ln\left[\frac{Be^{r(T-t)} - \theta_d e^{r(T-t)} K(R)}{F}\right] \tag{27}$$

and solving for the yield-to-maturity R by numerically finding its roots.

V. Empirical Results

5.1 Simulation of fundamental firm value

Simulating the fundamental firm value requires parameters and starting values for the variables. Under the assumption that the past is a reasonable predictor of the future, we use statistical measures such as mean, variance, and covariance from past data. We also use the long-run variables' estimates obtained from the data of a stable firm identified from the IT sector. Figure 2 shows that Computer Sciences Corp. has the longest history in this sector. We thus select this firm as the representative stable firm. Figure 2 also shows that it takes about 13 years to reach the stable stage. In other words, the number of quarters taken to get the steady growth is about 52. Table 2 reports estimated parameters and starting values for the six firms. R_0, CF_0, and LC_0 are directly observable from the 2003 10-Q quarterly reports. The starting growth rate, g_0 is estimated as an average of changes in revenue over recent four quarters and g^M is estimated as an average of changes in revenue for the stable firm[7]. We calculate $\log V_0$ as the log variance of changes in revenue over the last four quarters and $\log V^M$ as the log variance of changes in revenue for the stable firm. Dp, the interest expense[8], and the risk-free rate are estimated as averages from past data. The tax rate is obtained from the tax table. Past historical pretax incomes of our sample firms reveal that a 35% tax rate is reasonable. The speed adjustment κ is estimated from the half life in reaching the stable growth[9]. The volatility φ_1 of the volatility process is estimated as the standard deviation of the log changes in revenue growth for the stable firm, and φ_0 as the variance of changes in revenue for the stable firm. The long-run average ratio, α^M, is obtained from regressing the stable firm's revenue on the stable firm's total expenses. Each firm's α_0 is estimated from the regression using the last four quarters' total expenses and revenue. The volatility φ_3 of the ratio process is estimated as the standard deviation of the changes in the quarterly ratio of the stable firm. Lastly, λ is estimated as the product of the correlation and the market risk premium divided by the standard deviation of the market portfolio (See Appendix 2).

With estimated values and starting values displayed in table 2, we simulate 10,000 paths and use equations (9) through (13) to estimate the fundamental values of the six firms. Table 3 reports the results. Since all annual data were obtained as of the fiscal year of each company, the fundamental value approximates the true value of companies as of the most recent fiscal year.

5.1.1 Sensitivity tests

To estimate how sensitive the results are to parameter values, we compute the change in the firm value given a 10% increase in the value of all the estimated parameters. Since, however, the same percentage increase in α^M makes the long-run average ratio larger than one, we let α^M and α_0 vary by 1% only. Table 4 shows that overall the fundamental firm values estimated by our model are not particularly sensitive to changes in the value of parameters. The directions of the change in firm value are similar across all sample firms. To sump up, V^M, φ_1, α^M, Dp and r appear to have more of an impact on the value of the firm than the rest. As predicted in Section 4.1, since V^M and φ_1 are part of the stochastic volatility process, the stochastic volatility assumption does turn out to be play an important role in the valuation.

5.2 Fundamental debt value

We now estimate the fundamental debt value for each company. This can be done based on sections 4.2 and 4.3. However, although the 10-Q quarterly reports provide the face value of the various obligations to be paid at different maturities, they do not provide a detailed description of the debt characteristics. Moreover, equation (23) is on the basis of risky bonds with one single maturity date. We thus assume that all obligations can be expressed in terms of risky bonds and calculate a face value-weighted duration in order to obtain an average maturity for all different debts. Then, based on the face value-weighted duration and the total face value, we estimate the fundamental debt value for a given coupon amount. Table 5 reports results for seven cases with coupon rates ranging from 0 to 6 percent.

Table 5 reports fundamental debt values for all seven cases and a risk premium estimate for the zero-coupon case. Note that the risk premium is theoretically independent of the coupon rate and thus for computational ease we use the zero-coupon case to infer the premium. Table 5 also shows that for a given face value, a higher coupon rate implies a higher fundamental debt value. While Online Resources Corp reveals premium bonds as the coupon rate increases, the rest of the firms prove to have discount bonds across all seven cases. If various market debt values were observable directly, comparisons with their respective fundamental values could be made. However, since debt values are not directly observable, the main conclusion that we can draw is that with the assumption of a zero-coupon, being judged by the fundamental risk premium, EPlus and InteliData Technologies seem to have the riskiest debts whereas Online Resources and Steel Cloud have obligations whose characteristics are close to that of riskless bonds.

5.3 Fundamental equity value

Since the equity value is the difference between firm value and debt value, we are now able to estimate the fundamental equity value of each company. Table 6 reports fundamental equity value and corresponding share prices across all levels of coupon rates.

Note that the fundamental stock price decreases with an increase in the coupon rate. The market price of a share of Bearing Point, Inc. at the end of the fourth quarter of 2003 was $10.09 and the number of its outstanding shares was about 191 million. For all cases, its fundamental stock price falls between $12 and $14, which implies that its share price was undervalued by about 25%. The stock of EPlus Inc. was traded for $15.60 and about 9 million shares were outstanding. Since for all cases the fundamental stock prices are between $21 and $23, the market price as of third quarter of 2003 was undervalued by about 30%. A possible explanation for this is that many IT stocks have been steadily decreasing during the past few years and that as a result some might have actually gone below fair levels. Inteli Data Technologies Corp., Online Resources Corp. and Steel Cloud Inc have fundamental stock prices consistent across all seven cases and their market prices consistently overvalued. Costar Group Inc displays the largest difference between the observed market price and the fundamental price. Its market price seems to be overvalued by about 480%. In the end, four out of six IT firms have overestimated market prices, indicating that their persistently high share price may be overestimating the value of the firm's equity. But it is important to note that the other two firms actually appear undervalued, somewhat possibly creating a dent in the saying that most IT firms are not worth their market price.

VI. Conclusion

This article shows how the fundamental equity value of a growing firm can be estimated by combining two capital valuation techniques. Six firms from the IT sector are selected and evaluated empirically. The corporate debt valuation of Merton (1974) is used in connection with the rational pricing technique of Schwartz and Moon (2000) as a way to determine the fairness of the share prices. The methodology by Schwartz and Moon (2000) on how to value internet companies is used to estimate the total value of the six IT firms selected. We adapt the corporate debt valuation method proposed by Merton (1974) to the valuation of the debt of various IT companies and derive a closed-form solution as a reasonable approximation to the debt value of these firms. We then recover fundamental equity values for the six firms by computing the difference fundamental and debt values, and show how the share price is actually undervalued relative to the fundamental share price in some of the cases. In summary, our equity valuation approach is expected to provide a useful tool to academics and practitioners involved in the estimation of share prices of young growing firms yet having to reach stability of growth.

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TABLE 1

Sample Firms	Range of Accounting Data
BearingPoint Inc	1st Qtr/1998 – 4th Qtr/2003
EPlus Inc	1st Qtr/1995 – 3rd Qtr/2003
Costar Group Inc	1st Qtr/1996 – 4th Qtr/2003
Online Resources Corp	1st Qtr/1997 – 4th Qtr/2003
SteelCloud Inc	1st Qtr/1996 – 4th Qtr/2003
InteliData Technologies Corp	1st Qtr/1994 – 4th Qtr/2003

TABLE 2

Estimated Value of Parameters and Starting Values for Simulation

This table provides the estimated values of various parameters as well as the starting value for simulation

Parameter	Bearing Point Inc	EPlus Inc	Costar Group Inc	Online Resources Corp	Steel Cloud Inc	InteliData Technologies Corp
R_0 (million)	780.135	79.801	25.27	9.722	9.031	4.362
CF_0 (million)	105.198	26.846	35.643	7.65	8.099	7.603
LC_0 (million)	188.9	0	0	88	32.6	0
r	0.01	0.01	0.01	0.01	0.01	0.01
Dp (million)	18.064	1.583	0.898	0.52	0.265	0.635
Int (million)	4.445	1.74	0	0.223	0.042	0.016
Tc	0.35	0.35	0.35	0.35	0.35	0.35
κ	0.049	0.082	0.069	0.058	0.069	0.116
λ	0.161	0.079	0.118	0.109	0.07	0.051
φ_0	0.002	0.002	0.002	0.002	0.002	0.002
φ_1	2.326	2.326	2.326	2.326	2.326	2.326
φ_3	0.024	0.024	0.024	0.024	0.024	0.024
α^M	0.95	0.95	0.95	0.95	0.95	0.95
α_0	0.948	0.946	0.999	0.91	0.989	1.087
$\log V^M$	-5.244	-5.244	-5.244	-5.244	-5.244	-5.244
$\log V_0$	-5.222	-4.968	-9.527	-3.811	-1.888	-4.219
g^M	0.019	0.019	0.019	0.019	0.019	0.019
g_0	0.043	0.024	0.039	-0.02	-0.012	-0.1

TABLE 3

Fundamental Firm Value

IT Company	Fundamental Firm Value
Bearing Point Inc (as of 4th Qtr/2003)	3145.20 million
EPlus Inc (as of 3rd Qtr/2003)	294.49 million
Costar Group Inc (as of 4th Qtr/2003)	151.67 million
Online Resources Corp (as of 4th Qtr/2003)	57.94 million
Steel Cloud Inc (as of 4th Qtr/2003)	36.55 million
InteliData Technologies Corp (as of 4th Qtr/2003)	51.64 million

Table 4

We tested the sensitivity of the firm value over 10% increase in all the parameters except for α^M and α_0. Since the same change in α^M and α_0 does not make sense, we changed the value of these two parameters by only 1%. The positive sign implies the increase in the firm value in Table 3 while the negative sign does the opposite.

Parameter	Percentage Change in Firm Value					
	Bearing Point	EPlus	Costar Group	Online Resources	Steel Cloud	InteliData Technologies
r	-4.14%	-3.96%	-3.58%	-3.45%	-3.23%	-3.87%
Dp	+3.63%	+3.45%	+3.67%	+5.92%	+4.71%	+8.17%
Int	+0.06%	+0.21%	0.00%	+0.14%	+0.03%	+0.02%
Tc	-2.52%	-2.44%	-1.72%	-0.16%	-0.74%	-0.17%
κ	-3.24%	+1.62%	-1.74%	+0.16%	+0.74%	+1.37%
λ	-2.24%	-1.79%	-1.25%	-0.67%	-1.12%	-0.17%
φ_0	+0.07%	+0.01%	+0.02%	0.00%	-0.03%	+0.02%
φ_1	-4.27%	-8.03%	-3.99%	-1.16%	+2.85%	-1.49%
φ_3	+2.01%	+1.50%	+1.42%	+0.26%	+0.90%	+0.08%
α^M	-5.08%	-5.77%	-4.85%	-2.05%	-3.39%	-1.03%
α_0	-2.13%	-1.54%	-1.29%	-1.19%	-1.48%	-0.33%
$\log V^M$	-4.34%	-7.62%	-4.13%	-1.07%	-0.03%	-1.36%
$\log V_0$	-3.35%	-3.04%	-1.33%	-0.88%	+0.77%	-0.29%
g^M	+3.23%	+3.37%	+2.85%	+0.83%	+1.37%	+0.45%
g_0	+4.02%	+1.37%	+2.06%	+0.54%	+0.38%	+0.41%

TABLE 5

Fundamental Debt Value and Risk Premium

	Company	Bearing Point	EPlus	CoStar Group	Online Resources	Steel Cloud	InteliData Technologies
	Face Value (million)	878.10	119.88	32.17	1.86	0.37	7.55
Case 1 (0% Coupon)	Fundamental Debt Value (million)	546.75	112.15	25.60	1.80	0.36	7.19
	Fundamental Risk Premium (%)	1.87%	0.27%	0.83%	0.00%	0.00%	0.10%
Case 2 (1% Coupon)	Fundamental Debt Value (million)	580.18	113.68	26.54	1.82	0.36	7.27
Case 3 (2% Coupon)	Fundamental Debt Value (million)	613.61	115.21	27.48	1.83	0.36	7.35
Case 4 (3% Coupon)	Fundamental Debt Value (million)	647.03	116.74	28.43	1.85	0.36	7.44
Case 5 (4% Coupon)	Fundamental Debt Value (million)	680.46	118.27	29.37	1.86	0.37	7.52
Case 6 (5% Coupon)	Fundamental Debt Value (million)	713.89	119.80	30.31	1.88	0.37	7.60
Case 7 (6% Coupon)	Fundamental Debt Value (million)	747.32	121.33	31.25	1.90	0.37	7.68

TABLE 6

Fundamental Equity Value and Stock Price

This table provides the fundamental equity value and fundamental stock price. These values are computed using the following equations:
Fundamental equity value = fundamental firm value – fundamental debt value.
Fundamental stock price = fundamental equity value / the number of shares.

	Company	Bearing Point (as of 4qtr/2003)	EPlus (as of 3qtr/2003)	CoStar Group (as of 4qtr/2004)	Online Resources (as of 4qtr/2003)	Steel Cloud (as of 4qtr/2003)	InteliData Technologies (as of 4qtr/2003)
Current Market	Stock Price (as of the end of qtr)	10.09	15.60	41.70	6.56	4.30	1.56
	Number of Outstanding Shares (million)	191.663	9.265	17.877	17.812	12.609	51.231
Case 1 (0% Coupon)	Fundamental Equity Value (million)	2598.45	182.34	126.07	56.14	36.19	44.45
	Fundamental Stock Price ($)	13.56	19.68	7.05	3.15	2.87	0.87
Case 2 (1% Coupon)	Fundamental Equity Value (million)	2565.02	180.81	125.13	56.12	36.19	44.37
	Fundamental Stock Price ($)	13.38	19.52	7.00	3.15	2.87	0.87
Case 3 (2% Coupon)	Fundamental Equity Value (million)	2531.59	179.28	124.19	56.11	36.19	44.29
	Fundamental Stock Price ($)	13.21	19.35	6.95	3.15	2.87	0.86
Case 4 (3% Coupon)	Fundamental Equity Value (million)	2498.17	177.75	123.24	56.09	36.19	44.20
	Fundamental Stock Price ($)	13.03	19.18	6.89	3.15	2.87	0.86
Case 5 (4% Coupon)	Fundamental Equity Value (million)	2464.74	176.22	122.30	56.08	36.18	44.12
	Fundamental Stock Price ($)	12.86	19.02	6.84	3.15	2.87	0.86
Case 6 (5% Coupon)	Fundamental Equity Value (million)	2431.31	174.69	121.36	56.06	36.18	44.04
	Fundamental Stock Price ($)	12.69	18.85	6.79	3.15	2.87	0.86
Case 7 (6% Coupon)	Fundamental Equity Value (million)	2397.88	173.16	120.42	56.04	36.18	43.96
	Fundamental Stock Price ($)	12.51	18.69	6.74	3.15	2.87	0.86

FIGURE 1

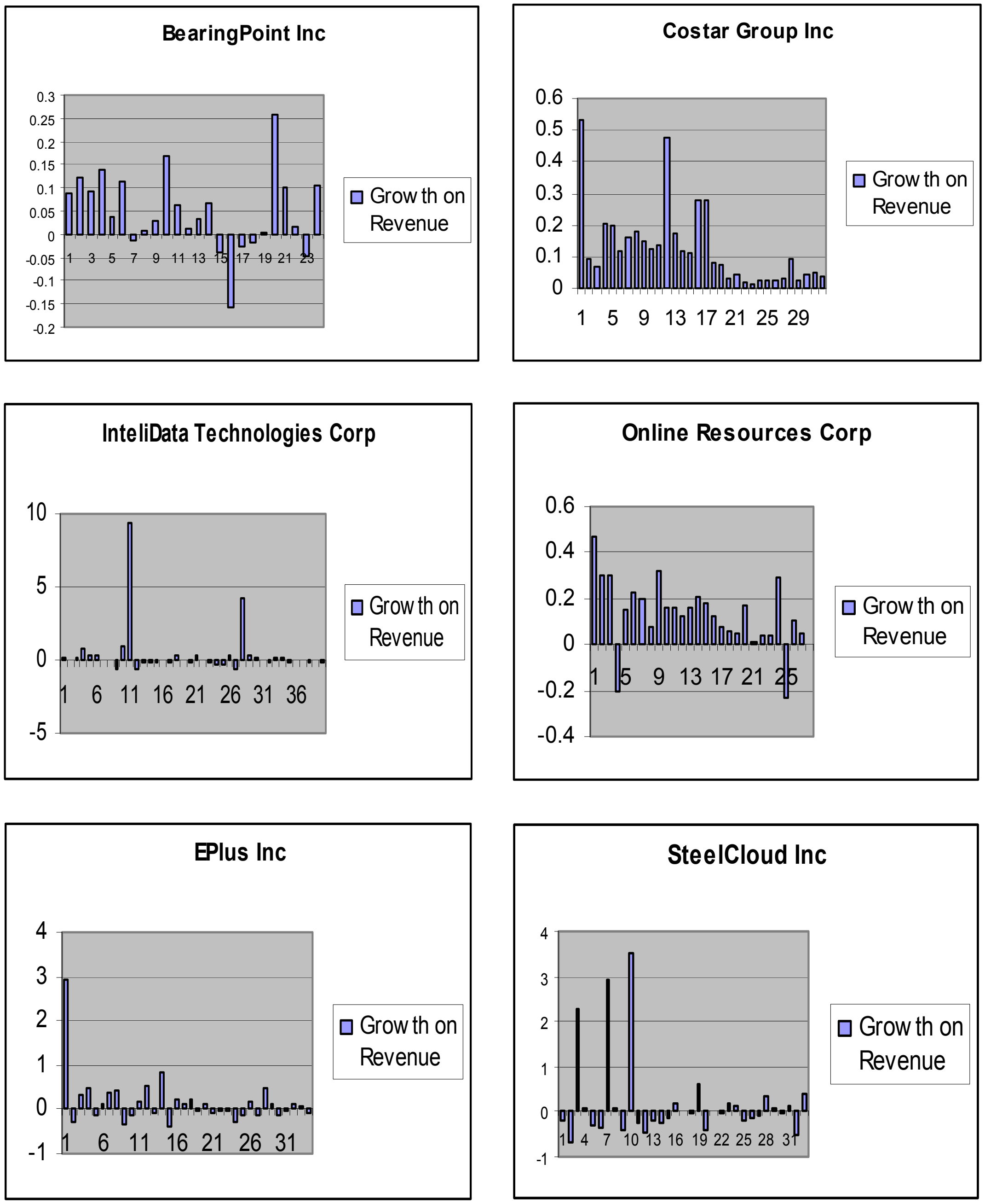
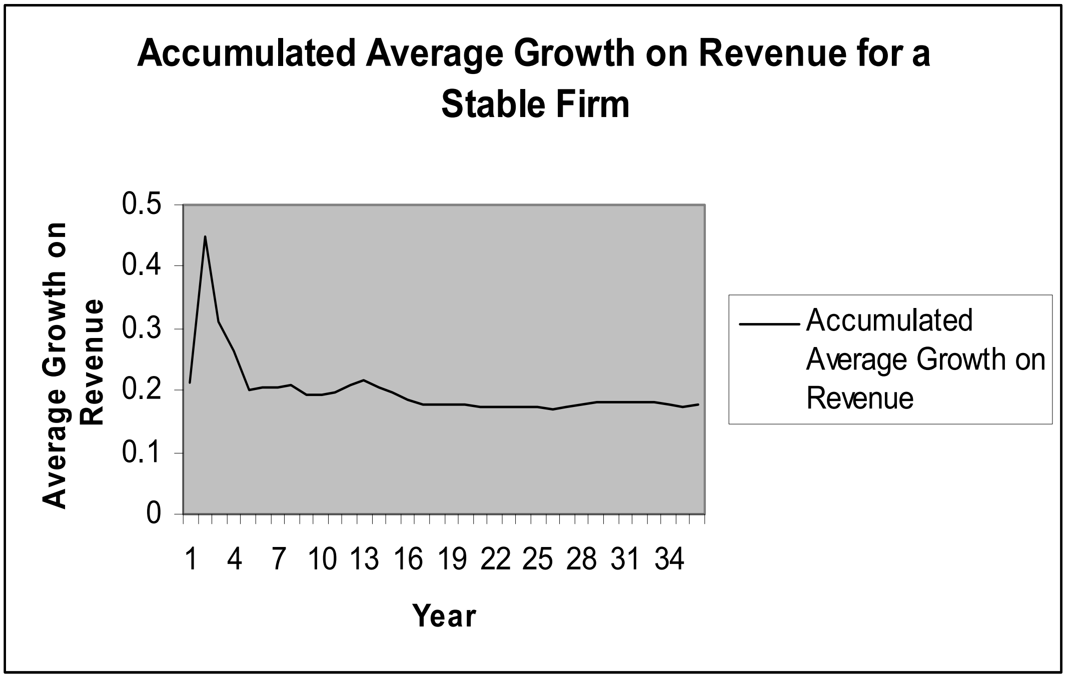

BearingPoint Inc
Grow th on Revenue
Costar Group Inc
Grow th on Revenue
InteliData Technologies Corp
Grow th on Revenue
Online Resources Corp
Grow th on Revenue
EPlus Inc
Grow th on Revenue
SteelCloud Inc
Grow th on Revenue

FIGURE 2


Accumulated Average Growth on Revenue for a Stable Firm
0.5
0.4
0.3
0.2
0.1
0
Average Growth on Revenue
1 4 7 10 13 16 19 22 25 28 31 34
Year
Accumulated Average Growth on Revenue

FOOTNOTE

1. Because there are several different liabilities in contractual obligations, some are tradable in the market but the others are not tradable (e.g., operating leases, purchasing obligation, etc).

2. We adopt the same horizon as Schwartz and Moon (2000).

3. According to this assumption, seven cases with different coupons are suggested.

4. This approach is often used by practitioners. (See Schwartz and Moon (2000))

5. The original process that is not risk-neutralized can be described as follows;

$$\frac{dR_t}{R_t} = g_t dt + \sqrt{V_t}\, dZ$$

where g_t is at the mercy of firm's risk preference.

6. Schwartz and Moon (2000) used analysts' projection due to this problem.

7. We restrict change in revenue between +10% and -10% per quarter to avoid statistical bias due to isolated large changes.

8. In our model, the interest expense is used to calculate EBITDA at the long-term horizon, T.

9. The speed adjustment is calculated as log 2 divided by the half life. For example, since the time period the stable firm took to reach steady growth is 52 quarters (13 years) and as of 4th Qtr/2004, Steel Cloud Inc's business history is 32 quarters (8 years), on average, it still seems to have about 20 more quarters to go to get the stable stage in its business. Therefore, the estimate of the speed adjustment for this firm is log 2/10 = 0.0693.

APPENDIX 1

The partial differential equation to solve is:

$$\frac{1}{2}B_{XX}\sigma^2X^2+B_X(rX-\theta_s-\theta_d)+B_t+\theta_d-rB=0$$

with boundary conditions B(0,t) = 0 and B(X,T) = Min(X,F)

With the assumption that θ_s and θ_d are sufficiently small relative to X, let's rewrite the PDE as follows:

$$\frac{1}{2}B_{XX}\sigma^2X^2+B_XX(r-\Delta)+B_t+\theta_d-rB=0$$

where $\Delta = (\theta_s + \theta_d) / X$.

Let $t = T - 2\tau/\sigma^2$, $X = Fe^x$, and $B(X,t) = Fb(x, \tau)$.

Since $B(X, T) = F\text{Min}(e^x, 1)$ and $B(X, T) = Fb(x, \tau)$, we have $b(x, \tau) = \text{Min}(e^x, 1)$

Rewriting the above PDE with respect to b, x, and τ yields

$$\frac{\partial b}{\partial \tau}=\frac{\partial^2 b}{\partial x^2}+[\frac{2(r-\Delta)}{\sigma^2}-1]\frac{\partial b}{\partial x}-\frac{2r}{\sigma^2}b+\frac{2\theta_d}{\sigma^2 F}$$

Let $\alpha = 2(r - \Delta) / \sigma^2 - 1$, $\beta = 2r/\sigma^2$, $\delta = 2\theta_d/\sigma^2F$, and $b(x, \tau) = \upsilon(x, \tau) + \delta/\beta$.

Since θ_s and θ_d are sufficiently small relative to X, λ is not much affected by the stochastic movement of X. This is a slight approximation but of very little consequences on the actual solution.

The partial differential equation then becomes

$$\upsilon_\tau = \upsilon_{xx} + \alpha\upsilon_x - \beta\upsilon$$

Now, let $\upsilon(x, \tau) = e^{px+q\tau} u(x, \tau)$. Rearranging the above PDE with respect to u,

$$u_\tau = u_{xx} + (2p+\alpha)u_x + (p^2 + \alpha p - \beta - q)u$$

By letting $p = -\alpha/2$ and $q = -\alpha^2/4 - \beta$, we can get the heat equation:

$$u_\tau = u_{xx}$$

And since $b(x, 0) = e^{px} u(x, 0) + \delta/\beta$ and $b(x, 0) = Min(e^x, 1)$,

$$u(x,0) = Min[e^{(1+\frac{\alpha}{2})x} - e^{\frac{\alpha}{2}x}\frac{\delta}{\beta}, e^{\frac{\alpha}{2}x} - e^{\frac{\alpha}{2}x}\frac{\delta}{\beta}]$$

A well-known solution of the heat equation is

$$u(x,\tau) = \frac{1}{2\sqrt{\pi\tau}} \int_{-\infty}^{\infty} u(s,0) e^{-\frac{(x-s)^2}{4\tau}} ds$$

Therefore, using $u(x, 0)$, we can derive the function $u(x, \tau)$ as follows.

$$u(x,\tau) = \frac{1}{2\sqrt{\pi\tau}} \int_{-\infty}^{0} (e^{(1+\frac{\alpha}{2})s} - \frac{\delta}{\beta} e^{\frac{\alpha}{2}s}) e^{-\frac{(x-s)^2}{4\tau}} ds + \frac{1}{2\sqrt{\pi\tau}} \int_{0}^{\infty} (e^{\frac{\alpha}{2}s} - \frac{\delta}{\beta} e^{\frac{\alpha}{2}s}) e^{-\frac{(x-s)^2}{4\tau}} ds$$

Letting $z = (s - x)/(2\tau)^{0.5}$,

$$u(x,\tau) = \frac{1}{2\sqrt{\pi\tau}} \int_{-\infty}^{-\frac{x}{\sqrt{2\tau}}} (e^{(1+\frac{\alpha}{2})(x+z\sqrt{2\tau})-\frac{z^2}{2}} - \frac{\delta}{\beta} e^{\frac{\alpha}{2}(x+z\sqrt{2\tau})-\frac{z^2}{2}} dz + \frac{1}{2\sqrt{\pi\tau}} \int_{-\frac{x}{\sqrt{2\tau}}}^{\infty} (e^{\frac{\alpha}{2}(x+z\sqrt{2\tau})-\frac{z^2}{2}} - \frac{\delta}{\beta} e^{\frac{\alpha}{2}(x+z\sqrt{2\tau})-\frac{z^{2z}}{2}}) dz$$

Rearranging each term in the right hand side,

$$u(x,\tau) = e^{(1+\frac{\alpha}{2})(x+\tau+\frac{\alpha}{2}\tau)}\Phi(d_1) + e^{\frac{\alpha^2}{4}+\frac{\alpha}{2}x}[1-\Phi(d_2)] - \frac{\delta}{\beta}e^{\frac{\alpha^2}{4}\tau+\frac{\alpha}{2}x}$$

$$d_1 = -\frac{x}{\sqrt{2\tau}} - (1+\frac{\alpha}{2})\sqrt{2\tau}$$

$$d_2 = -\frac{x}{\sqrt{2\tau}} - \frac{\alpha}{2}\sqrt{2\tau}$$

Finally, repeating the change of variable technique, we obtain an approximate analytic solution based on the assumption that Δ is sufficiently small,

$$B(X,t) = Fe^{-r(T-t)} - [Fe^{-r(T-t)}\Phi(d_2) - Xe^{-\Delta(T-t)}\Phi(d_1)] + (1-e^{-r(T-t)})\frac{\theta_d}{r}$$

$$d_1 = \frac{-\ln(\frac{X}{F}) - (r-\Delta+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 + \sigma\sqrt{T-t}$$

APPENDIX 2

With the original revenue process,

$$\frac{dR_t}{R_t} = g_t dt + \sqrt{V_t}\, dZ_t$$

since the revenue is non-tradable, if we assume that the stock price is a function of the revenue, R_t, that is, $S_t = f(R_t)$, by the Ito's lemma,

$$dS_t = (g_t f'(R_t) + \frac{1}{2}V_t f''(R_t))dt + \sqrt{V_t} f'(R_t) dZ_t$$

$$= \mu_t dt + \sigma_t dZ_t$$

Since stocks are tradable assets, the risk-neutral measure can be described as follow:

$$dW_t = dZ_t + \left(\frac{\mu_t - r}{\sigma_t}\right)dt$$

Substituting this risk-neutral measure for dZ in the revenue process, we obtain the new revenue process under the risk-neutral measure. (See Baxter and Rennie (1996))

$$\frac{dR_t}{R_t} = \left(g_t - \left(\frac{\mu_t - r}{\sigma_t}\right)\sqrt{V_t}\right)dt + \sqrt{V_t}\,dW_t$$

$$= \left(g_t - \lambda\sqrt{V_t}\right)dt + \sqrt{V_t}\,dW_t$$

Since μ and σ are the expected return and the standard deviation for the stock, under the assumption that λ is a constant, using the CAPM, we approximately estimate λ with the past data as follows:

$$\lambda = (\mu - r)/\sigma = \rho(r_m - r)/\sigma_m$$

where ρ is the correlation between the stock and the market portfolio.