

Extending the LYONs Pricing Model

John Harding

Shantaram P. Hegde

Ravi Shanker Mateti*

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Liquid Yield Option Notes (LYONs) are callable, puttable and convertible bonds. For most of the LYONs in existence today, their convertibility into common stock depends on the path followed by the issuing company's stock price. The McConnell and Schwartz (1986) model is not well suited to price these kinds of LYONs because their model does not account for the dependency of the embedded conversion option on the path followed by the underlying stock price. We extend their basic model to explicitly account for path dependency and test our model using a fairly recent sample of LYONs.

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* Contact author: rmateti@business.uconn.edu

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Abstract

Liquid Yield Option Notes (LYONs) are callable, puttable and convertible bonds. For most of the LYONs in existence today, their convertibility into common stock depends on the path followed by the issuing company's stock price. The McConnell and Schwartz (1986) model is not well suited to price these kinds of LYONs because their model does not account for the dependency of the embedded conversion option on the path followed by the underlying stock price. We extend their basic model to explicitly account for path dependency and test our model using a fairly recent sample of LYONs.

I. Introduction

Liquid Yield Option Notes (LYONs) were created by Merrill Lynch for its clients Waste Management, Inc. and Staley Continental, Inc. in 1985. LYONs are complex securities – they are callable, puttable, and convertible bonds. Given its complexity, arriving at the “correct” price of a LYON is not a simple matter. McConnell and Schwartz (1986) proposed a simple model to price LYONs. The model was used to test how well the model’s prices compared with the observed market prices of the LYONs issued by Waste Management, Inc. The results were good.

The LYON priced by the McConnell and Schwartz (MS) model has a simple convertibility feature – it can be converted into the common stock of the company at a fixed ratio at any time, may be after an initial period, no matter what the behavior of the stock price has been in the past. However, most of the LYONs in existence today do not have such a simple convertibility feature. For example, one of the conditions under which the LYONs issued by General Mills are convertible into 13.0259 shares of common stock per \$1,000 principal amount at maturity in any fiscal quarter (and only during such fiscal quarter) is when the closing price of General Mills’ common stock for at least 20 trading days in a period of 30 consecutive trading days ending on the last trading day of the immediately preceding fiscal quarter is greater than an amount equal to the “accreted conversion price” per share¹ of the common stock multiplied by the applicable percentage. The LYONs issued by CSX Corporation are convertible on any day into 17.7461 shares per \$1,000 principal amount at maturity if the closing price of the common stock for at least 20 trading days in a period of 30 consecutive trading days ending on the trading day before the day of surrender is more than the applicable percentage of the accreted conversion price per share of its common stock at that preceding day.

From above we see that convertibility of these LYONs into common stock depends on the path of the issuer’s stock price. In turn, whether the LYON is convertible or not

¹ Accreted conversion price is the sum of the issue price plus accrued original issue discount, divided by the conversion rate.

influences its price and thus we see that price of the LYON is influenced by the path of the stock price. The MS model is not applicable to these kinds of LYONs. The model needs to be extended to take care of this “path dependency”. The LYONs in existence today have lots of features, like contingent interest, besides the convertibility conditions mentioned above, which makes the security even more complex and difficult to analyze. However, there have been no other papers since McConnell and Schwartz (1986) addressing the increasing complexity of LYONs even though there are a large number of issues in existence, 172 as on Jan 10, 2005. The focus of this paper is to extend the MS model to price LYONs whose convertibility is contingent of the behavior of stock prices.

II. Extensions of the MS model

For the sake of completeness, we first reproduce the MS model. The model is based on the following assumptions:

- (a) The price of the LYON depends on the issuing firm’s stock price and time, $L(S, t)$.
- (b) The term structure of interest rates is flat and known.
- (c) The dynamics of the issuer’s stock price can be represented by geometric Brownian motion,

$$dS = [\mu S - D(S, t)]dt + \sigma_s S dz_s$$

where μ is the drift rate of the process, $D(S, t)$ is the total amount of dividends paid at time t , and σ_s is the instantaneous volatility of stock returns.

Using the standard arbitrage arguments, the price of the LYON must satisfy the differential equation

$$[rS - D(S, t)]L_s + L_t + 1/2\sigma_s^2 S^2 L_{ss} - rL = 0$$

where r is the known, constant interest rate and the subscripts of L represent partial derivatives.

The prices are given by the solution to the above partial differential equation subject to the boundary conditions which reflect the call, put, and conversion features of the security. They also reflect the optimal call strategy by the issuer and optimal conversion and redemptions strategies by the holder of the security.

The Maturity Condition

On the day of maturity, the value of the LYON must be greater of the conversion value and the face value of the LYON:

$$L(S, T) = \text{Max}(C_r S, F)$$

where C_r is the number of shares of the issuer's common stock into which the LYON can be converted, F is the face value of the LYON, and T is the maturity date.

The Conversion Condition

At all times, the value of the LYON must be greater than or equal to its conversion value:

$$L(S, t) \geq C_r S$$

The Redemption (or Put) Condition

At any of the redemption dates (t_p), the value of the LYON must be greater than or equal to the prevailing redemption price:

$$L(S, t_p) \geq P(t_p)$$

Where $P(t_p)$ is the redemption (or put) price at time t_p .

The Call Condition

At all times the value of the LYON must be less than or equal to the greater of the call price and the conversion value:

$$L(S, t) \leq \text{Max}\{C(t), C_r S\}$$

where $C(t)$ is the call price of the LYON at time t .

From the above model we see that it does not have any conditions to capture the nature of path dependencies mentioned earlier. We extend the MS model in different ways to take care of different convertibility clauses. We also modify the MS assumption that the price of the LYON depends only on the stock price and time, and assume that it also depends on certain path dependent variables. All other assumptions and relationships of the MS model are retained but add new relationships to take care of path dependencies.

First kind of convertibility.

Convertibility of the LYON depends on behavior of stock price during the last 30 trading days of the immediately preceding fiscal quarter

An example of a LYON which has this kind of convertibility clause is that issued by General Mills. Besides other conditions under which conversion can take place, the LYON is convertible into 13.0259 shares of common per \$1,000 principal amount at maturity if as of the last day of the immediately preceding fiscal quarter, the closing price of the common stock for at least 20 trading days in a period of 30 consecutive

trading days ending on the last trading day of such fiscal quarter is more than the specified percentage of the accreted conversion price per share of the common stock.

From the above conversion clause, the LYON can either be convertible or not convertible during an entire fiscal quarter. We introduce a new variable to capture the convertibility or non-convertibility of the LYON into common stock. The price of the LYON is certainly dependent on whether or not it is convertible into the common stock of the company. If it is convertible we would expect the price of the LYON to be higher than in the case when it is not convertible. Also, whether or not the LYON is convertible in the immediately following fiscal quarter depends on the number of times the closing price of the stock has been above the specified threshold during the last 30 trading days of the current fiscal quarter. Other things being equal, if the LYON is convertible in the next quarter, we would expect the price of the LYON to be higher than if it were not convertible. We introduce two new variables to reflect these two pieces of information about the LYON. In our extended model, the price of the LYON is determined by the current stock price (S), a new binary variable con which gives information regarding whether or not the LYON is convertible in the present fiscal quarter ($con = 1$ if convertible & $con = 0$ if not convertible), and another new variable k , which gives information about the number of days the closing stock price of the stock has been above the specified threshold during the last 30 consecutive trading days of the current fiscal quarter ($q = 0$ at times before the start of the last 30 trading days in the current fiscal quarter, and q equals the number of days the closing stock price has been above the specified threshold during the last 30 consecutive trading days of the current fiscal quarter).

So, the price of the LYON can be expressed as a function of S , con , q , and t : $L(S, con, q, t)$. In the time period of the current fiscal quarter before the start of the last 30 trading days, there is no change in the values of the variables con , and q since the value of the convertibility variable, con , remains constant throughout the entire quarter and q equals zero before the start of the last 30 trading days of the current fiscal quarter. Because of this, changes in the price of the LYON during this time period is only because of changes

in the stock price and time. Therefore, the changes in the price of the LYON during this e period can be represented by the Black-Scholes PDE like in the MS model:

$$[rS - D(S, t)]L_s + L_t + 1/2\sigma_s^2 S^2 L_{ss} - rL = 0$$

Though the PDE governing the changes in the prices is the same, now the price of the LYON, L, does not depend just on stock price and time but also on the convertibility variable (con), and the variable q which represents the number of days the threshold has been crossed. During the last 30 trading days of the current quarter, the value of the convertibility variable continues to remain the same, but q can change at the end, and only at the end, of every day during the last 30 trading days of the fiscal quarter. The value of q does not change during the day but only at the end of the day depending on whether the closing stock price exceeded the specified threshold. So, during the day, the above PDE still represents the changes in the prices of the LYON. At the end of the day, the value of q can change depending on the closing stock price and the price of the

$$L(S(t_i), \text{con}, k, t_i^-) = L(S(t_i), \text{con}, F(S(t_i), k), t_i)$$

where t_i is the time at the close of a trading during the last 30 trading days in a fiscal quarter,

t_i^- is the time just before the closing of a trading day,

$S(t_i)$ is the closing price of stock at time t_i

k is the number of days just before till time t_i the stock price exceeded the threshold

$F(S(t_i), k)$ is the function which gives the number of days the closing price has exceeded the threshold price, S^* , up and until time t_i

$$F(S, k) = k+1, \quad \text{if } S \geq S^*$$

$$= k, \quad \text{if } S < S^*$$

The above condition is satisfied because the price of the LYON just before time t_i is equal to the price at time t_i .

Whether or not the LYON is convertible in the next quarter depends on whether or not the stock price has exceeded the threshold on 20 days or more during the last 30 trading days of the current fiscal quarter. This condition holds irrespective of the convertibility status of the LYON during the current quarter. At the close of the last trading day of a fiscal quarter, the following relationship has to hold,

$$L(S(t_e), \text{con}, k, t_e) = L(S(t_e), G(\text{con}, k), 0, t_e^+)$$

where t_e is the close of the last trading day of a fiscal quarter,

t_e^+ is the first instant of time of the first trading day of the next fiscal quarter

$G(\text{con}, k)$ is a function which gives the status of convertibility in the next fiscal quarter:

$$\begin{aligned} G(\text{con}, k) &= 1, \quad \text{for } k = 20, 21, \dots, 30, \text{ \& for } \text{con} = 0, 1 \\ &= 0, \quad \text{for } k = 0, 1, \dots, 19, \text{ \& for } \text{con} = 0, 1 \end{aligned}$$

The convertibility status in the next fiscal quarter does not depend on the convertibility status during the current fiscal quarter. It depends only on whether not the stock price has been above the specified threshold on 20 trading days or more in the last 30 consecutive trading days of the current quarter.

Second kind of convertibility

Convertibility status of a LYON on any day depends on the behavior of the stock price during the immediately preceding last 30 consecutive trading days

An example of a LYON having this kind of convertibility is that issued by CSX Corporation. The LYONs issued by CSX Corporation are convertible into 17.7461 shares per \$1,000 principal amount if the closing price of the common stock for at least 20 trading days in the last 30 consecutive trading days ending on the trading day before the day of submission of LYON for conversion is more than the applicable percentage of the accreted conversion price per share of its common stock. We discuss two ways of handling this kind of convertibility.

Approach 1

We first discuss the path dependent function which determines the number days the closing price of stock has exceeded the threshold in the immediately preceding 30 trading days. Let this function be denoted by $F(S(t_i), k)$, where $S(t_i)$ is the stock price at time t_i and k is the number of times the stock price exceeded the threshold in the last 30 trading days till time just before time t_i .

$$\begin{aligned} F(S(t_i), k) &= k + 1, \text{ if } S \geq S^* \text{ and } S(t_{i-29}) < S^* \text{ and} \\ &= k, \text{ if } S \geq S^* \text{ and } S(t_{i-29}) \geq S^* \\ &= k, \text{ if } S < S^* \text{ and } S(t_{i-29}) < S^* \\ &= k - 1, \text{ if } S < S^* \text{ and } S(t_{i-29}) \geq S^* \end{aligned}$$

From above we see that for $S \geq S^*$, $F(S(t_i), k)$ is either equal to $k + 1$, or k depending on whether $S(t_{i-29}) < S^*$ or $S(t_{i-29}) \geq S^*$ respectively. Similarly, for $S < S^*$, $F(S(t_i), k)$ is either equal to k or $k-1$ depending on whether $S(t_{i-29}) < S^*$, or

$S(t_{i-29}) \geq S^*$. This ambiguity creates a problem in implementing the finite difference method. We resolve this ambiguity when implementing the jump condition

$$L(S(t_i), k, t_i^-) = L(S(t_i), F(S(t_i), k), t_i)$$

at the close of every trading day using the following approximation:

For $S(t_i) \geq S^*$,

$$L(S(t_i), k, t_i^-) = \binom{28}{k-1} / \binom{29}{k} \cdot L(S(t_i), k, t_i) + \binom{28}{k} / \binom{29}{k} \cdot L(S(t_i), k+1, t_i),$$

and

for $S(t_i) < S^*$,

$$L(S(t_i), k, t_i^-) = \binom{28}{k-1} / \binom{29}{k} \cdot L(S(t_i), k-1, t_i) + \binom{28}{k} / \binom{29}{k} \cdot L(S(t_i), k, t_i),$$

where

$\binom{28}{k-1} / \binom{29}{k}$ is the probability that $S(t_{i-29}) \geq S^*$,

$\binom{28}{k} / \binom{29}{k}$ is the probability $S(t_{i-29}) < S^*$

The above probabilities are got under the assumption that the closing price at the end of each day is equally likely to have crossed the threshold.

Approach 2

In this approach we approximate the path dependent function by two other functions. We call them the over-approximation function and the under-approximation function. We expect that the price of the LYON using the over-approximation would be an upper bond of the true theoretical price of the LYON, and the price got using the under-

approximation function would be a lower bound of the true theoretical price. So we get a range of values within which the true theoretical price of the LYON is expected to lie.

The over- approximation function, $F^O(\cdot)$ is defined as follows:

$$\begin{aligned} F^O(S(t_i), k) &= k + 1, \text{ if } S(t_i) \geq S^*, \\ &= k, \text{ if } S < S^* \end{aligned}$$

Under-approximation function, $F^U(\cdot)$:

$$\begin{aligned} F^U(S(t_i), k) &= k, \text{ if } S(t_i) \geq S^*, \\ &= k - 1, \text{ if } S < S^* \end{aligned}$$

The jump conditions when using the over-approximation function is as follows:

$$\begin{aligned} L(S(t_i), k, t_i^-) &= L(S(t_i), k + 1, t_i), \quad \text{for } S(t_i) \geq S^* \\ &= L(S(t_i), k, t_i), \quad \text{for } S(t_i) < S^* \end{aligned}$$

The jump conditions using the under approximation function:

$$\begin{aligned} L(S(t_i), k, t_i^-) &= L(S(t_i), k, t_i), \quad \text{for } S(t_i) \geq S^* \\ &= L(S(t_i), k - 1, t_i), \quad \text{for } S(t_i) < S^* \end{aligned}$$

III. Empirical Results

We got all the data necessary to do our study from Bloomberg. This includes information on the issues of LYONs, the interest rates, dividend yields, etc. We test our extended model by pricing 4 issues of LYONs. We used our model to price the LYONs as at the close of Jan.10, 2005. The companies issuing LYONs in our sample are International Game Technology, General Mills, CSX Corporation, and International Paper Company. LYONs issued by International Game Technology and General Mills are convertible into common stock if in the last 30 trading days of the immediately preceding conversion period the closing stock price has been above the specified threshold for that quarter for at least 20 trading days. Conversion can also take place on the occurrence of events like when the credit rating assigned to long term senior debt by both Moody's and Standard & Poor's is below specified levels, or if the LYONs have been called for redemption by the company. We did not incorporate these features in our model because we feel that by doing so the model would become unduly complex and the resulting gains in accuracy may not compensate adequately the additional complexity of the model. These remarks hold good for pricing the other three LYONs as well. The convertibility feature of the LYONs issued by General Mills is similar to those issued by International Game Technology. The LYONs issued by CSX Corporation are convertible at any time if during the immediately preceding 30 trading days, the stock price of the company has been above specified thresholds for at least 20 trading days. The LYONs issued by International Paper Company too have this convertibility feature. The details of the issues of the LYONs in our study and the results of our extended models are as follows.

International Game Technology (IGT)

IGT issued LYONs on Jan 29, 2003, which are due on Jan 29, 2033. The LYONs were issued at a price of \$592.91, which represents a yield to maturity of 1.74%, continuously compounded. The credit rating assigned by S&P is BBB, and by Moody's is Baa2. For these ratings, the yield to maturity on Jan. 10, 2005, the day for which the prices are

being estimated, was 5.9%. The average dividend yield for this company for the last one year was 1.33%.

These LYONs can be redeemed at the option of the holder on the following days and following prices: On Jan 29, 2006 for \$624.76, on Jan 29, 2008 for \$646.91, on Jan 29, 2013 for \$705.87, on Jan 29, 2018 for \$770.17, on Jan 29, 2023 for \$840.32, and on Jan 2028 for \$916.86. On maturity, the holder gets the face value of \$1,000 if not called by the company, or converted or redeemed by the holder earlier.

On or after Jan 29, 2006, the company may redeem all or part of the LYONs issue at a price equal to original issue price plus the accrued original issue discount. In other words, when the LYON can be called, the call price is given by $\$592.91 \cdot \exp(0.0174t)$, where \$592.91 is the original issue price, t is the length of time in years from the issue date to the time when the LYON is called, and 0.0174 is the continuously compounded yield to maturity at the time of issue.

Holders may convert their holdings of LYONs into 21.704 shares of common stock per \$1,000 principal amount if the conditions mentioned above are satisfied. The threshold the stock price has to exceed is 120% of the accreted conversion price. Accreted conversion price equals $(\$592.91 / 21.704) \cdot \exp(0.0174t)$, where \$592.91 is the original issue price, 21.704 is the conversion ratio and 0.0174 is the continuously compounded yield to maturity at the time of issue.

Using our extended model, we priced this LYON using as on Jan 10, 2005. The price we get from the model is \$712.52 and the observed market price on this date was \$730.00. A priori, we expect the model's price to be lower than the observed price because certain features like contingent interest payable to the holders under certain circumstances, the ability to convert into stock in some instances even if the stock price conditions are not met, etc. add value to the investors but these have been left out of the model for the reasons mentioned above. However, we note that the model's price and the observed price are very close, a difference of only 2.4% between the two prices.

General Mills

General Mills issued LYONs on Oct. 28, 2002, which are due on Oct. 28, 2022. The LYONs were issued at a price of \$671.65, which represents a yield to maturity of 2.00%. The credit rating given by S&P is BBB+ and by Moody's is Baa2. On Jan 10, 2005 the yield corresponding to these classes was 5.9% and the dividend yield for the last one year was 2.38%.

These LYONs can be redeemed at option of the holder on the following days and following prices: On Oct. 28, 2005 for \$713.00, on Oct. 28, 2007 for \$741.96, on Oct. 28, 2012 for \$819.68, Oct. 28, 2017 for \$905.50.

The company may call the LYONs at any time on or after Oct. 28, 2005. The call price equals the original issue price plus the accrued original issue discount. The call prices can be calculated just as was done for IGT.

The LYON is convertible into 13.0259 shares of common stock per \$1,000 principal amount in any fiscal quarter if the stock price for at least 20 trading days in a period of 30 consecutive trading days ending on the last day of the immediately preceding fiscal quarter exceeds the specified percentage of the accreted conversion price. This percentage starts at 125% at the time of issue and falls by 0.25% every six months.

The observed market price on Jan. 10, 2005 was \$713.80 and our model's price is \$686.15, a difference of 3.8% between the two prices.

CSX Corporation

CSX Corporation issued LYONs on Oct. 30, 2001, which are due on Oct. 30, 2021. These LYONs were issued at a price of \$819.14, which represents a yield to maturity of

1%. The credit given by S&P is BBB and that by Moody's is Baa2. The yield to maturity for this class of securities on Jan 10, 2005 was 6%. The average dividend yield for the past year is 1.03%.

These LYONs can be redeemed by the holder on Oct. 30, 2003, Oct. 30, 2004, Oct. 30, 2006, Oct. 30, 2008, Oct. 30, 2011, and Oct. 30, 2016 for \$835.64, \$861.06, \$878.43, \$905.12, and \$951.46 respectively.

The company may call the LYONs at any time on or after Oct. 30, 2008. The LYON can be called at a price equal to the original price, \$819.14, plus the original issue discount.

These LYONs convertible into 17.1761 shares of common stock per \$1,000 principal amount at any time if during the immediately preceding 30 trading days, the stock price of the company has been above specified thresholds for at least 20 trading days. The threshold is set at 120% of the accreted conversion price on the day of issue and this percentage drops by 0.50% on every year.

The observed market price on Jan 10, 2005 was \$866.3 and the model's price \$835.90, a difference of 3.5% between the two prices.

International Paper Company

International Paper Company issued LYONs on June 20, 2001 which are due on June 20, 2021. The issue price was \$475.66, which gives a yield to maturity of 3.75%. The credit rating given by S&P was BBB and Moody's was Baa2. The yield to maturity for this class of securities was 5.9% on Jan 10, 2005. The average dividend yield for the last year was 2.43%.

The holder can redeem the LYONs on June 20, 2004, June 20, 2006, June 20, 2011, and June 20, 2016 for \$531.74, \$572.76, \$689.68, and \$830.47 respectively.

The company may call the LYONs at any time on or after June 20, 2006. The LYON can be called at a price equal to the original price, \$475.66, plus the original issue discount.

These LYONs convertible into 9.5111 shares of common stock per \$1,000 principal amount at any time if during the immediately preceding 30 trading days, the stock price of the company has been above specified thresholds for at least 20 trading days. The threshold is set at the time of issue at 120% of the accreted conversion price, and this percentage declines by 0.256% at the end of each semi-annual period over the life of the LYONs to 110%.

The observed market price on Jan 10, 2005 was \$550.20 and the model's price \$528.08, a difference of 4.0% between the two prices

IV. Work-in-progress

We plan to test our model on a much larger sample of LYONs, about 50 to 60, and draw some meaningful inferences by relating the accuracy of the model to the terms and conditions of the issue and the characteristics of the issuing company. We also wish to perform a few sensitivity analyses.

V. Conclusion

We see that the model's prices are quite close to the observed market prices. More importantly, we note that the model's prices are consistently below the market prices. This was expected because besides being callable, puttable, and convertible, the LYONs in existence today have many more features such as contingent interest becoming payable to the investor on the occurrence of certain events, the ability to convert the LYONs into common stock not depending on the performance of the stock price, etc. These features

are mostly beneficial to the investors and are likely to be reflected in the prices of the LYONs. However, from the modeling point of view, trying to incorporate these secondary features increases the complexity of the model many fold. Depending on what level of accuracy is desired, one has to weigh the likely benefits in terms of greater accuracy of the model with the costs of additional complexity introduced into model by incorporating the secondary features and also the problem of estimating additional inputs.

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