A Two-Sided Matching Model of Monitored Finance*

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Abstract

We analyse a two-sided matching model and incentive contracts where expert investors (venture capitalists) with different monitoring capacities are matched with firms with different levels of initial wealth. Firms do not have sufficient start-up capital to cover their project costs and hence, seek external financing. In equilibrium, the matching and the payoffs of the venture capitalists and the firms are determined simultaneously. We also show that, in equilibrium investors with higher monitoring ability invest in firms with lower initial wealth following a negatively assortative matching pattern. Further, a rise in the quality of monitoring, following the entry of new venture capitalists, often enhances overall efficiency, and the minimum and maximum returns to the venture capitalists (weakly) increase.

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1 Introduction

During the last three decades, the US venture capital industry has witnessed a dramatic increase in the fund flowing into new ventures. In the decades 1960 and 70, although a significant number of pension funds invested in new start-up firms, little resources were devoted to monitoring and managing these investments. In the mid-1980's, the overwhelming increase in the supply of venture capital was mainly due to the entry of the *gatekeepers* (the investment advisors) into this industry. These advisors came up with high quality expertise in raising resources for investment in young firms and ability to manage these funds. They monitored the firms' performance closely and became integral parts of the ventures with respect to the decision making.

A typical venture organisation is characterised by limited partnerships, in which the venture capitalists (VCs, henceforth) are general partners who control the fund's activities. Apart from fund raising, these general investors monitor the firms performance and provide their managerial expertise in the management of the fund. Venture capitalists often differ in abilities to monitor, and are often ranked into top-tier, second-tier, ..., bottom-tier, etc. with respect to their monitoring capacities. In the context of investment banking this ranking is explicit.[1] Although in the venture capital industry this ranking is not explicit, experience of an investor (number of previous investments made by him prior to a particular investment) can be taken as an empirical measure of monitoring ability (see Sorensen [26]). Differences in monitoring capacities are important since it is well-recognised that better monitors are able to add more value to the ventures they invest in. Also, venture capitalists can differ with respect to their ability to attract funds which is often taken as a measure of their reputation.

In this paper, we analyse several aspects of venture capital industries, although the model presented here can address other types of financial institutions such as investment banking. First, the venture capitalists (who differed in expertise) have invested in special type of funds which might be distinguished according to better growth prospects, levels of riskiness, possibility of faster maturity before going public, etc. The investment outcomes typically depend on the characteristics of the firms and the VCs. More effective monitors are better positioned in the market in a sense that they have access to better investment opportunities. On the other hand, less wealth-contrained firms are more likely to succeed. Consequently, the market may observe sorting between firms and VCs. In this regard, we ask the following question: What kind of sorting pattern we observe when venture capitalists decide to invest in a new venture? Sorting is important since it influences the venture outcomes. In an empirical analysis of the US venture capital industry over 14 years, Sorensen [26] shows that the market outcomes significantly differ with and without sorting. Second, increase in the quality of expertise to manage fund resulted in immense growth prospect and faster maturity in the young start-ups. How one can explain this gain in efficiency in the venture capital industry? This is particularly important regarding the nature of portfolio in venture funds, since from the second half of 1980, as opposed to the preceeding two decades, the flow of funds was accompanied by greater quality of expertise (following the emergence of the gatekeepers). Although many empirical studies address this question (see Gompers and Lerner [12]), a little theoretical works have been devoted in this respect. Third, we analyse the financial contracts one can observe in the VC-backed funds. This analysis is particularly interesting since, as postulated by Kaplan and Stromberg [15], venture capital contracting can approximate closely the contracts in the investor theory.

We consider a model of *monitored finance* where the market consists of venture capitalists with different abilities to monitor and entrepreneurs/firms with different levels of start-up capital. Analysis of possible sorting in venture capital industries calls for a two-sided matching model. A *matching* describes different possible VC-firm pairs. Each pair writes binding contracts which creates the value of a venture. This value is shared between the parties according to a rule which is imbedded in the terms of the contract. In the equilibrium, the payoffs to the venture capitalists and the firms and the matching are determined simultaneously. A contract specifies that the VC finances the project and receives state-contingent claims on the project return. Each firm operates on its project after it obtains external fund and chooses a non-contractible effort level. Choice of effort influences the probability of having a high return from the project. Firm's liability is limited to its current income. Hence, differences in wealth imply differences in liabilities. This constraint generates moral hazard at the firm level. Monitoring by venture capitalist is aimed at ameliorating this moral hazard.

Monitored finance is a dominant form of financing young projects. One such example is the one we address here. In general, the role of the venture capitalists is a mix of fund management and investment. We have discussed earlier that venture capitalists, apart from raising funds from their client-pool, closely monitor the firms' progress and manage the fund's activities. Relationship banking is another form of monitored finance. Sometimes banks lock themselves into long term relationships with their borrowers (see Rajan [20]). Often sorting through endogenous matching is prevalent in investment banking (see Chen [6]). Also, literature on financial intermediation conclude that when external borrowing by firms is indispensable, capital-poor firms are denied credits by uninformed investors and they have to rely on the informed capital available in the economy (see Hölmstrom and Tirole [13], and Repullo and Suárez [21]). Informed capital owns, in general, better monitoring technology compared to less informed investors. Hence, it is better able to cope with moral hazard at the firm level that arises because of the inability to contract upon all the actions taken by the entrepreneurs.

The two-sided matching game considered here is a one-to-one model. In a limited partnership venture organisation, covenants might oblige the general partners to invest in a limited number of venture funds. Investment in unlimited number of funds may lead to opportunistic behaviour and a venture capitalist can only dedicate little time in the management of a particular fund. Hence, in order to capture what this sort of covenants postulate, we restrict the investment of a venture capitalist in only one firm.[2] In this matching game, we determine the equilibrium matching and payoffs (arising from the contracts) simultaneously. An outcome of this market is a matching function and a set of contracts compatible with the matching. We use *stability* as the equilibrium concept. An outcome is stable (or, is in equilibrium) if there is no VC-firm pair that would be (strictly) better-off by signing a different contract.

We characterise the equilibrium of this market. The equilibrium contracts and matching are determined simultaneously. We consider a matching game where venture capitalists and firms interact for one period. In our model, the equilibrium contracts are optimal debt contracts. We show that in the equilibrium matching, VCs with better capacity to monitor invest in firms with lower wealth (negatively assortative matching). The nature of substitutability between the effectiveness of monitoring and the wealth of the firms guarantee this kind of monotone matching pattern under moral hazard. Becker [3], Shimer and Smith [25], and Legros and Newman [16] give sufficient conditions for assortative matching. These conditions involve substitutability between the characteristics of the matched individuals. When the contracts between venture capitalist and firm involve provision of incentive, the optimality of contracts is not implied by the total surplus maximisation. In the current model, as in Legros and Newman [16], the substitutability in producing as well as in transferring the surplus determines the negatively assortative equilibrium matching pattern.

We then analyse the effects of an increased quality of monitoring expertise on the equilibrium outcomes. If new venture capitalists enter the industry with higher monitoring capacity, often the overall efficiency of the ventures goes up, although this crucially depends on whether the firms form the short-side of the market. The use of a one-to-one matching model implies that rise in the quality of monitoring following the entry of new VCs with higher monitoring capacity results in an exit of less effective monitors. The monitoring capacity associated with each venture improves which in turn improves the efficiency of the venture. This may explain the phenomenon of the venture capital cycle that the venture capital industry in the US has experienced: less investment by pension funds and more investment by venture capitalists in limited partnership venture organiations during the second half of the 1980. As a result of this rise in the supply of more informed capital a large number of start-ups matured faster before going public, and the industry had witnessed higher growth prospects in the young funds. Gompers and Lerner [11] show that monitors with better ability significantly enhance the values of the venture funds. Entry has also significant effect on returns to the VCs. Following an entry of new VCs, the maximum and minimum payoffs of the existing VCs do not decrease.

Several other works consider the presence of two-sided matching in the determination of contracts between principals and agents. Dam and Pérez-Castrillo[9] analyse a pricipal-agent matching market with one-sided heterogeneity. Besley and Ghatak [4] analyse a principal-agent matching model in the presence of motivated agents in organisations. Ackerberg and Botticini [1], and Li and Ueda [17] test empirical models to show the existence of endogenous matching as determinant of contract forms in the contexts of the markets for landlord-tenants and CEO-firms, respectively. Serfes [23] determines assortative matching pattern between risk-sharing and incentive when landlords and tenants are matched. The last two papers address theoretical issues in matching in principal-agent set-up. They ask the question that given the characteristics of principals and agents, and given a set of optimal contracts between them, how should the equilibrium matching pattern look like. They employ the two-sided matching similar to the *college admission problem* developed by Roth and Sotomayor [22]. Unlike these works, we analyse simultaneous determination of matching and contracts (as well as payoffs) in equilibrium. In this sense the current model can be viewed as a generalisation of the *assignment game* of

Shapley and Shubik [24] in which the equilibrium matching is determined along with payoffs of buyers and sellers, rather than taking the payoffs to a pair as given. Sorensen [26] analyses an empirical two-sided model of venture capital similar to that of Roth and Sotomayor [22] and show the influence of assortative matching.

There are other works which analyse matching between venture capitalists and firms where the individuals in a market are matched according to a random matching function. Michelacci and Suárez [18] consider a random matching model where venture capitalists differ in abilities and analyse the relation between monitoring ability and new business creation. They show that the faster the young ventures mature and go public, the quicker the venture capitalists' informed capital is redirected towards the new start-ups. Inderst and Müller [14] develop a model of contracting, bargaining and search to show that the relative scarcity of venture capital affects the pricing, contracting and value creation in start-ups. Both these models, unlike ours, are based on random matching, search and bargaining approach. In our view, a reasonable matching among individuals should rather be treated as endogenous. Our model considers a centralised market where individuals are matched according to a matching function. A decentralised mechanism is considered in Dam and Pérez-Castrillo [9]. The empirical work of Sorensen [26] also supports the view presented in the current work.

The paper is organised as follows. In Section 2, we lay out the basic model. In the following section, we ananlyse optimal contract for a particular VC-firm pair. We describe the matching market in Section 4. In Section 5, we state the main results concerning equilibrium. In the next section, we analyse the effects of entry of new venture capitalists on the equilibrium outcomes. We conclude in Section 7. A formal analysis of the set of stable outcomes and relevant proofs are relegated to the Appendix.

2 The Model

We consider a financial market where there is a set $F = \{f_1, \ldots, f_n\}$ of risk-neutral firms. Frm f_j is endowed with initial wealth w^j, firm $f_{j'}$ has wealth $w^{j'}$, etc. Each firm has an innovative project whose implementation costs 1 monetary unit. We arrange the firms according to their wealth levels in descending order as $w^1 \geq \ldots \geq w^n \geq 0$. Firm's initial wealth is not sufficient to cover the entire project cost, hence each firm seeks external finance.

There is a set $V = \{v_1, \ldots, v_N\}$ of risk-neutral venture capitalists (who are potential investors) enowed with different monitoring capacities $m = (m_1, \ldots, m_N)$. We arrange the venture capitalists with respect to their monitoring capacities in descending order as $m_1 \geq \ldots \geq m_N$.

Venture capitalists and firms are matched in pairs. Whenever matched, a VC-firm pair signs a binding contract and the VC finances the entire project. Firm's wealth works as a collateral in the project even though, the firm does not invest its wealth in the project.[3] We assume that a VC can invest in at most one venture fund due to possible technological rigidities, organisational covenants, etc. When a VC agrees to finance a project, the firm undertakes an effort $e \in [0, 1]$ which influences the idiosyncratic states, *Success* or *Failure*. Effort choice of a firm is not verifiable, and hence not contractible. A dollar invested in a venture yields a return y in the event of success with probability e, and nothing in case of failure. Effort is costly for the firm. Since effort level is not contractible, this gives rise to a moral hazard problem at the firm level. In order to ameliorate this problem, a VC monitors the firm he invests in. If the VC has capacity m_i then he incurs a cost of monitoring m_i. A more effective monitor can make a firm behave more diligently by lowering its cost of effort. Hence, if a firm is monitored by VC v_i then its cost function takes the form $C(e, m) = e^2/2m_i$. This functional form implies that higher monitoring capacity reduces firm's effort-cost for a given level of effort.

3 Contract between a Firm and a VC

A VC-firm pair (v_i, f_j) signs a contract $c_{ij} = (R_{ij}, r_{ij})$ that specifies state-contingent payments to the VC, $R_{ij} > 0$, in case of success and $r_{ij} > 0$, in the event of failure. The corresponding effort chosen is e_{ij}. Given a contract c_{ij}, the expected value of a venture (v_i, f_j) is given by $EV_{ij} \equiv e_{ij}\, y$. The expected utilities of venture capitalist v_i and firm f_j when they sign the contract c_{ij} are, respectively:

$$\begin{aligned} U_i(f_j, c_{ij}) &= e_{ij}R_{ij} + (1 - e_{ij})r_{ij} - m_i\,, \\ u_j(v_i, c_{ij}) &= e_{ij}(y - R_{ij}) - (1 - e_{ij})r_{ij} - \frac{e_{ij}^2}{2m_i}\,. \end{aligned}$$

The choice of effort is not contractible. Let e_{ij} maximise the firm's utility, i.e.,

$$e_{ij} \in \underset{e}{argmax} \left\{ e(y - R_{ij}) - (1-e)r_{ij} - \frac{e^2}{2m_i} \right\}. \qquad (IC_j)$$

This is the *incentive compatibility* constraint of firm f_j. We denote by c^{null}, the *null contract*, under which all individuals consume zero utility. Suppose, firm f_j's reservation payoff is $s^j \geq 0$. A venture capitalist faces a risk-free market rate of interest which is normalised to zero. Venture capitalist's and firm's *individual rationality* are given by the following constraints:

$$\begin{aligned} e_{ij}R_{ij} + (1-e_{ij})r_{ij} - m_i &\geq 0, \\ e_{ij}(y - R_{ij}) - (1-e_{ij})r_{ij} - \frac{e_{ij}^2}{2m_i} &\geq s^j. \end{aligned}$$

Firm's liability is limited to the state-contingent return plus its initial wealth. Limited liability implies

$$R_{ij} \leq y + w^j \ , \qquad (LS_j)$$

$$r_{ij} \leq w^j \ . \qquad (LF_j)$$

The assumption of risk neutrality together with limited liability makes the incentive compatibility constraint costly and hence, it gives rise to *moral hazard* at the firm level. Since firm's utility is (strictly) concave in e, one can replace (IC_j) by the *first order condition* of the firm's maximisation problem as follows:

$$e_{ij} = m_i(y - R_{ij} + r_{ij}). \qquad (IC'_j)$$

We also make the following assumptions.

ASSUMPTION 1 $m_i \in [\frac{1}{y}, 1]$ *for all* i.

A contract for a VC-firm pair must satisfy the individual rationality and limited liability constraints. We club all these natural restrictions into the following definition.

DEFINITION **1** *A contract is* **feasible for a firm** f_j *if it satisfies the restrictions of individual rationality and limited liability.*

Denote by X^j the set of contracts that are feasible for firm f_j. From now on we will concentrate only on feasible contracts. Let Z^j be the subset of feasible contracts that are incentive compatible and consider the following programme.

$$\max_{c_{ij} \in Z^j} U_i(f_j, c_{ij}). \qquad (P_{ij})$$

Let $c_{ij}(s^j)$ be the set of contracts that solve the above maximisation programme. One basic characteristic of these contracts are that they are (constrained) Pareto optimal. In particular, in this context these are incentive compatible debt contracts.[4] Let $U(m_i, w^j, s^j)$ and $u(m_i, w^j, s^j)$ be the payoffs to VC v_i and firm f_j, respectively, generated by $c_{ij}(s^j)$. These payoffs correspond to a point of a concave utility possibility frontier.

The above is a *classic* example of the set of contracts often discussed in the principal-agent literature when only a *given* principal-agent relationship is concerned. In other words, this is the set of feasible and incentive compatible contracts when one abstracts from a *principal-agent market*, and focuses only on a particular (given) relationship. The payoff to the firm is determined entirely by its outside option, and hence exogenous. As we have mentioned earlier that one of the main goals of this paper is to endogenise the reservation utilities (hence, the payoffs) of the firms, in the following sections, we concentrate on a market where many venture capitalists and firms interact, and consequently the payoff of each firm is determined endogenously and influenced by the other VC-firm pairs formed in the economy.

4 Matching

in the previous section we have described the optimal contract between one firm and one venture capitalist. In a market comprising of several firms and VCs, the same contracts may not arise in equilibrium since contract signed by a particular pair imposes externalities on the (optimal) contracts for the other pairs. To this end, a partial equilibrium model of VC-firm contracting (the one described in the previous section) is somewhat inadequate to characterise the market equilibrium. In this section we assume that firms and VCs are matched according to a matching function. A VC being matched with a firm implies that the VC has agreed to invest in the firm's venture. When the pair is formed, and a contract is signed. A contract for a matched pair is influenced by the other pairs being formed according to the matching function. On the other hand, a firm can seek for alternative financier. This implies that the matching depends on the contract terms. In the model of the previous section, given a particular VC-firm pair, the payoff of the firm is completely determined by its (exogenously given) reservation utility. When several VCs and firms are considered, the reservation utility of firms become endogenous. Hence, in this market the matching and the payoffs generated from the contracts (which constitute an outcome) are endogenous too.

In this section we would like to determine the payoffs of all VCs and firms as well as the optimal matching. Simultaneous determination of matching and payoffs calls for defining a relevant outcome for the market. In this model an outcome consists of a matching rule and a corresponding payoff vector.

Definition **2** *A* **matching** *for the market is a mapping μ that assigns a VC (a firm) to a firm (a VC) (we write $\mu(v_i) = f_j$ ($\mu(f_j) = v_i$)), or specifies that a VC (a firm) stays alone (we write $\mu(v_i) = v_i$ ($\mu(f_j) = f_j$)). A one-to-one matching is a mapping μ such that $\mu(v_i) = f_j$ if and only if $\mu(f_j) = v_i$.*

A matching function assigns a firm to a venture capitalist or vice versa. It also allows for the fact that an individual may stay unmatched. The last part implies the restriction to a one-to-one matching. We have discussed earlier that covenants may put limits to the number of firms the venture capitalists invest in. Hence, we assume that a VC can invest only in one firm, or a firm can be financed at most by one VC.

A menu of contracts compatible with a matching μ is a vector of feasible contracts, one for each pair matched according to μ. Each of these feasible contracts generates feasible payoffs U_i for VC v_i and u_j for firm f_j compatible with this particular matching. Let the sets of feasible payoffs to the VCs and the firms be $U = (U_1, \ldots, U_N)$ and $u = (u_1, \ldots, u_n)$.

Definition **3** *An* **outcome** (μ, U, u) *is a matching μ and feasible payoffs compatible with the matching.*

The outcomes of the market we describe here are determined endogenously. This endogeneity has two aspects. First, the contracts signed by the VCs and the firms, as well as their payoffs are endogenous. The second aspect is that the matching itself is endogenous.

In the following section we determine the set of *equilibrium* payoffs for the market as well as which firms are financed by which venture capitalists (the matching pattern). We require that a *reasonable* outcome should be immune to the possibility of being blocked by any VC-firm pair (as well as by any single individual). In other words, an outcome for a market has to be *stable*.

Definition 4 *An outcome* (μ, U, u) *for the market is* **stable** *(or, is in equilibrum) if there is no pair* (v_i, f_j) *and no payoff configuration* (U', u') *generated by a feasible contract* c'_{ij} *such that* $U'_i > U_i$ *and* $u'_j > u_j$.

The above definition implies that no VC-firm pair with a feasible contract can *block* an outcome if it is stable. The restriction of individual rationality implies that no individual firm or VC unilaterally blocks a stable outcome.

5 The Set of Stable Outcomes

In this section we describe the set of stable outcomes of the market.[5] We start by stating a couple of important properties of a stable outcome. First, all the contracts in a stable outcome are (constrained) Pareto optimal. By optimality we mean that there is no possibility of (strictly) improving upon the payoff of any individual in a VC-firm pair without making the other worse-off.

Proposition 1 *All the contracts in a stable outcome are optimal.*

It is worth noting that the optimality of a contract between a venture capitalist and a firm in any stable outcome is guaranteed by the possibility that the same pair can block the initial outcome with a different feasible contract. Hence, a contract signed by a matched pair (v_i, f_j) must maximise the expected utility of one party, taking into account that the other gets at least a certain utility level. One particular class of optimal contracts are discussed in Section 3. The optimality of the contracts in a stable outcome implies that the set of payoffs generated by solving programme (P_{ij}) form a part of the stable outcome. This makes sure that if in a stable outcome if a firm f_j is matched with VC v_i, then $U_i = U(m_i, w^j, s^j)$ and $u_j = u(m_i, w^j, s^j)$.

PROPOSITION **2** *The number of pairs formed equals the number of individuals in the short-side of the market. Only the wealthiest firms and the most effective monitors are matched.*

Given the restriction of one-to-one matching, it is clear that if there are different numbers of VCs and firms everybody cannot be matched. If the firms form the short-side of the market ($n < N$), then only n pairs are formed. All firms and the n most effective monitors are matched. On the other hand, if firms constitute the long-side of the market ($n > N$), only N pairs are formed, all VCs and the wealthiest N firms are matched. Clearly, if there are equal number of firms and VCs, everybody is matched. In this sense, the equilibrium matching is optimal since only the venture capitalists with highest monitoring capacity and the wealthiest firms form pairs, and total value of all the ventures is maximised.

PROPOSITION **3** *In a stable outcome the higher the wealth of a firm, the higher is its payoff. If a firm is unmatched it obtains zero.*

The above proposition is fairly intuitive. If a firm with higher wealth gets lower payoff then, following Proposition 1, the VC who is matched with the firm with lower wealth can always offer a slightly better contract to the wealthier firm and form a blocking pair. This property depends neither on the number of firms and venture capitalists in the market, nor on a specific matching pattern. If all firms had same initial wealth, they would have obtained same payoffs. In the following definition we introduce the concept of *willingness to pay.*

DEFINITION **5** *The* **willingness to pay** *of a venture capitalist v for w against w' is defined as*

$$\Delta U_i(f_j,\, f_{j'}) \equiv U(m_i,\, w^j,\, s^j) - U(m_i,\, w^{j'},\, s^{j'}).$$

The above expression reads as: for given levels of initial wealth, w^j and $w^{j'}$, if venture capitalist v_i is currently engaged with $f_{j'}$, the above quantity is the maximum additional amount he is willing to pay to contract with f_j instead, or this is the maximum extra amount he is willing to pay to keep f_j in case he is currently with this firm rather than $f_{j'}$.

PROPOSITION **4** *In a stable outcome venture capitalists with higher monitoring capacity invest in firms with lower wealth, i.e., the matching is negatively assortative.*

In the above proposition we describe the equilibrium matching pattern. We show that firms with higher wealth obtain credit from VCs with lower capacity of monitoring, i.e., the matching is negatively assortative. In order to prove the above property, we proceed as follows. Consider any two firms such that f_j has higher wealth than $f_{j'}$. First we show that, in a stable outcome, the willingness to pay of the VC matched with f_j for f_j against $f_{j'}$ is higher than that of the VC matched with $f_{j'}$. In other words, the VCs compete for the right to be matched with firms with higher wealth. In equilibrium an VC matched with a firm with higher wealth clearly out bids the partner a firm endowed with lower wealth. Next, we show that the willingness to pay for a wealthier firm is decreasing in monitoring capacity. When contracts require provision of incentives, firms' wealth and monitoring capacity are substitutes. Due to limited liability, moral hazard is more severe with a more wealth constrained firms, and hence the values of these ventures increase with more effective monitoring. This substitutability is equivalent to the condition of decreasing willingness to pay. And a negatively assortative matching is a direct consequence of this property. Also, this is the unique optimal matching in any stable outcome.

We have mentioned earlier that the reservation utilities of firms, and hence the payoffs are determined simultaneously and they are endogenous. We have also stated in Proposition 3 that in a stable outcome wealthier firms get higher payoffs. Since, in a stable outcome the payoffs depends on the reservation utilities (in fact, $u_j = s^j$), from this proposition it is clear that firms' reservation payoffs are not exogenous. Given the negatively assortative matching pattern, one can say more regarding the endogenous determination of the payoffs.

PROPOSITION **5** *Suppose in a stable outcome $l = min\{n, N\}$ pairs are formed. Then this stable outcome exhibits the following property:*

$$\Delta U_{l-j+1}(f_j, f_{j+1}) \geq s^j - s^{j+1} \geq \Delta U_{l-j}(f_j, f_{j+1}), \text{ for all } j = 1, \ldots, l-1.$$

From Propositions 3 and 5, it is clear that the reservation utilities of the firms are endogenous. These are determined from a set of inequalities described in the aforesaid propositions. Consequently, the payoffs of the VCs and the firms are also endogenous since they depend on the values of reservation utilities of the firms. The equilibrium payoff of each firm is bounded below and above. Let $\underline{u} = (\underline{u}_1, \ldots, \underline{u}_n)$ and $\bar{u} = (\bar{u}_1, \ldots, \bar{u}_n)$ be the minimum and maximum payoffs of the firms, respectively. To see the existence of a minimum and a maximum, consider two firms with different wealth levels. If the venture capitalist investing in a firm is willing to contract with a firm with higher wealth, the maximum amount he is willing to give up equals his willingness to pay for the wealthier firm. Hence, at equilibrium the minimum the wealthier firm can obtain is the payoff of the firm with lower wealth plus the aforesaid amount. On the other hand, the maximum the the firm with higher wealth can obtain is the payoff of the firm with lower wealth plus the willingness to pay of the monitor investing in the wealthier firm. It is also the case that the equilibrium payoff to each VC is bounded below and above. In a similar fashion as above, let $\underline{U} = (\underline{U}_1, \ldots, \underline{U}_N)$ and $\bar{U} = (\bar{U}_1, \ldots, \bar{U}_N)$ be the minimum and maximum payoffs of the VCs, respectively.

Finally, it is worth noting that a stable outcome (μ, U, u) is not unique, although the optimal matching is. This is because the (equilibrium) reservation utility of a firm f_j is not unique, and may range over an interval. Further, if one stable outcome (μ, U, u) is better for a firm than another stable outcome (μ, U', u'), then (μ, U, u) is better than (μ, U', u') for all the matched firms and and worse for all the matched VCs. In particular, corresponding to an optimal matching μ, there is an outcome $(\mu, \bar{U}, \underline{u})$ which is the best from the VCs' point of view and worst for the firms. This outcome can be called the *V-optimal* stable outcome. On the other hand, there is a stable outcome $(\mu, \underline{U}, \bar{u})$ which is the best from the firms' point of view and worst for the VCs. This can be called the *F-optimal* stable outcome.[6]

6 Entry of Venture Capitalists

We have discussed earlier that in the mid-1980's the US venture capital industry witnessed immense growth due to the emergence of venture capitalists with high expertise to manage new funds. In this section, we analyse the effects of the entry of new VCs on the market equilibrium. Entry might lead to an increase in the overall quality of monitoring in the industry, and hence this will have significant impact on the funds' performance and payoffs.

Let $EV_j \equiv p_{ij}\, y$ the expected value of a venture/firm f_j when VC v_i and firm f_j are matched in a stable outcome.[7] Fix the number of firms in the market to n. Suppose at the initial equilibrium outcome (μ, U, u), there are N venture capitalists. The analysis in this section crucially depends on the initial number of firms and VCs. The total efficiency is defined by $\sum_j EV_j$.[8]

Suppose that following an entry of new N' venture capitalists the new equilibrium outcome is (μ', U', u'). Following Proposition 5, the new matching is also negatively assortative. If firms forms the short-side of the market at the initial equilibrium ($n \leq N$), entry of at least one VC with better monitoring capacity than that of at least one of the monitors matched initially enhances efficiency. This is stated in the following proposition.

Proposition 6 *If the firms form the short-side of the market and at least one of the entrants has monitoring capacity better than that of the least effective monitor matched in the initial equilibrium, then following an entry total efficiency increases.*

The intuition behind the above proposition is fairly simple. In any stable outcome, prior to or following an entry, the matching is negatively assortative. When firms are in the short-side of the market, there is room for only n pairs. Following Proposition 4 as only the wealthiest firms and the best monitors are matched in a stable outcome, entry of a better monitor always drives the worst monitor out of the matched set. Overall monitoring capacity goes up, and hence total efficiency increases. Obviously, if the entrants posses monitoring capacity less effective than that of the least effective monitor matched in the initial equilibrium, then entry has no effect on the initially stable outcomes and hence total efficiency does not change.

When the firms form the long-side of the market ($n > N$), the impact of entry on total efficiency is ambiguous. In order to see this consider the following example.

Example 1 Consider a market with firms $\{f_1, \ldots, f_5\}$ and VCs $\{v_1, v_2, v_3\}$. In a stable outcome (μ, U, u), the matching is negatively assortative. Hence, $\mu(f_1) = v_3$, $\mu(f_2) = v_2$, $\mu(f_3) = v_1$, $\mu(f_4) = f_4$ and $\mu(f_5) = f_5$. The total efficiency is given by $EV_1 + EV_2 + EV_3$.

Suppose now a new VC v_i enters the market. Then firm f_4 who was initially unmatched gets matched with some VC under the new equilibrium outcome (μ', U', u'). Here we need to consider following four situations.

(i) Suppose $v_1 > v_2 > v_3 > v_i$. Then $\mu'(f_1) = v_i$, $\mu'(f_2) = v_3$, $\mu'(f_3) = v_2$, $\mu'(f_4) = v_1$

and $\mu'(f_5) = f_5$. The value of the firm f_4 goes up since $EV_4 > 0$. But since all other firms are now matched with less effective monitors, their values go down. Hence, the total efficiency goes up only if the gain is sufficiently high in order to offset the loss.

(ii) Suppose $v_1 > v_2 > v_i > v_3$. Then $\mu'(f_1) = v_3$, $\mu'(f_2) = v_i$, $\mu'(f_3) = v_2$, $\mu'(f_4) = v_1$ and $\mu'(f_5) = f_5$. Value of firm f_4 goes up, value of firm f_1 does not change since it is matched with v_3 both under μ and μ', but values of f_2 and f_3 go down since they are now matched with less effective monitors. Hence, total efficiency may or may not increase.

(iii) Suppose $v_1 > v_i > v_2 > v_3$. Then $\mu'(f_1) = v_3$, $\mu'(f_2) = v_2$, $\mu'(f_3) = v_i$, $\mu'(f_4) = v_1$ and $\mu'(f_4) = f_4$. Value of firm f_4 goes up, values of firms f_1 and f_2 do not change since they are matched with the same monitors both under μ and μ', but value of f_3 goes down since it is now matched with a less effective monitor. Hence, total efficiency may or may not increase.

(iv) Suppose $v_i > v_1 > v_2 > v_3$. Then $\mu'(f_1) = v_3$, $\mu'(f_2) = v_2$, $\mu'(f_3) = v_3$, $\mu'(f_4) = v_i$ and $\mu'(f_5) = f_5$. In this case value of f_4 goes up, where as that of the other firms remain unchanged. Hence, total efficiency increases.
Hence, the effect of entry of new venture capitalists on a stable outcome is not unambiguous. The impact depends on whether the firms form the short-side of the market. The above discussion is summarised in the following proposition.

PROPOSITION **7** *If the firms form the long-side of the market and at least one of the entrants has monitoring capacity better than that of the least effective monitor matched in the initial equilibrium, then the effect of entry on total efficiency is ambiguous. Total efficiency goes up only if the monitoring capacities of the entrants are effective enough in order to offset the loss in values of the other ventures (who are matched with less effective monitors) in the new equilibrium*

When initially there are more firms than venture capitalists, entry of a VC with low monitoring capacity does not sufficiently increase the value of the fund he is involved in. Due to negative assortment some firms are matched with less effective monitors leading to a loss of value in these firms. Unless the monitoring capacities of the entrants are high enough in order to offset the loss in values, total efficiency does not increase.

What influence does entry have on the returns (payoffs) to the venture capitalists? Since there is not a unique stable outcome, it is difficult to analyse the influence of entry on any possible combinations of payoffs of the VCs and the firms. We can only have a partial answer in this regard. In order to analyse this we concentrate only on the F-optimum and V-optimmum stable outcomes.[9] If the overall effectiveness of monitoring improves then the minimum and maximum payoffs of the venture capitalists do not decrease. This is summarised in the following proposition.

PROPOSITION **8** *Let $\bar{U}$ be the payoffs of the VCs corresponding to the V-optimal stable outcome and $\underline{U}$ be their payoffs corresponding to the F-optimal stable outcome. If the overall monitoring effectiveness increases, i.e., if only more effective monitors enter the market, then the minimum and maximum equilibrium payoffs of the venture capitalists do not decrease.*

In this economy, the multiplicity of equilibrium payoffs makes it difficult to analyse the effect of entry on all the equilibrium payoffs of the firms and the VCs. Nevertheless, the effect on the payoffs corresponding to the V-optimal and F-optimal stable outcomes are unambiguous. Following an entry of more effective monitors in the market, the overall quality of monitoring improves. Each firm is assigned to a more effective monitor compared to the initial equilibrium. If in the new equilibrium, the monitoring capacity in a fund is strictly higher, this monitor generates higher value (Propositions 6 and 7) in the fund and extracts more surplus. Hence, the maximum (corresponding to the V-optimum) and minimum (corresponding to the F-optimum) equilibrium payoffs of each VC do not decrease.

7 Conclusion

In this paper we model a venture capital industry as a two-sided matching game and characterise the set of stable outcomes. We show that when firms need to raise external fund to finance their projects, in equilibrium, the capital-poor firms have to rely on more informed capital in the market, and consequently they suffer from excessive monitoring. This conforms to the findings of Hölmstrom and Tirole [13], and Repullo and Suárez [21]. Unlike these two works, ours is a model with finite number of individuals and we do not allow for any correlation among the project returns. But the use of matching games to model the financial market allows us endogenise the payoffs of all the participating individuals. We also propose a very simple framework that is able to solve general (competitive) equilibrium models of financial markets characterised by incentive problems. The payoffs of firms with higher wealth are typically higher in equilibrium. It is worth noting that, the results obtained in Proposition 4 are robust to any equilibrium matching patterns. We also show that entry of new venture capitalists with better monitoring capacity may enhance total efficiency.

One limitation of the current paper should be recognised. The model we describe here is essentially a static one which fails to capture the dynamic aspects of a venture capital contracting. In a VC-backed firm, the venture capitalists generally finance the project in consecutive stages. The contracts that emerge in a long term relationship, in reality, can be quite different from a standard one-period optimal incentive compatible contracts. Convertible debt (debt contract in the early stages and equity-like contract in the later stages) is the most common in use. Issues regarding stage financing and the analysis of the above mentioned contract form are beyond the scope of this very stylised model. The novelty of the use of a one-period matching game is that it allows us to determine the payoffs endogenously along with the equilibrium matching.

The current model leaves several avenues for future research. We consider a one-to-one matching game with several venture capitalists and several firms. If the project return are correlated then this would call for more sophisticated contract design and a many-to-one matching model, which would not be a trivial extension of the financial market described in the current paper. The equilibrium in the case with correlated projects would then facilitate to analyse the effects of different kinds of macroeconomic shocks. A one-to-one matching model is a simple way to capture the essence of covenants of a new venture that were discussed earlier. Another extension would be to allow more than one VC to invest in the same firm. Often the issue of non-exclusivity of contracts in the credit market bears important consequences on the properties of market equilibria.[10] Our one-to-one feature essentially implies exclusive contracts. A many-to-one matching set up is able to incorporate where several venture capitalists with different monitoring capacities invest in the same firm. In this case, as Hölmstrom and Tirole [13] interpret, if a more informed investor monitors the firm then it works as certifying the firm's solvency and helps attract external capital into the firm from less informed investors. Finally, one can extend the model by making the venture capitalists capital constrained. This would give rise to a moral hazard problem in the level of monitoring, and it would also be more rational then to consider correlated project returns in a many-to-one setup.

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