INDEX ARBITRAGE WITH THE KOSPI 200 FUTURES

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ABSTRACT

This paper explores index arbitrage strategies using the Korea Stock Price Index (KOSPI) 200 futures and the KOSPI 200 cash. I examine the ex post arbitrage profitability where arbitrage positions are taken at the moment that the mispricing between the futures and cash prices occurs. I also examine the ex ante arbitrage profitability where arbitrage positions are formed as suggested by the initial mispricing, but at the following sets of prices. My results suggest that the KOSPI 200 futures market is extremely inefficient. Arbitrage profits and opportunities persist over time, contrary to previous experiences in other countries. The underpricing phenomenon was particularly strong. The ex ante arbitrage still yielded profits, implying that the market is economically inefficient. I also find the time-to-expiration effect in arbitrage opportunities. I insist that the inactive reverse cash-and-carry arbitrage due to short sale restrictions causes the KOSPI 200 futures to remain underpriced.

INDEX ARBITRAGE WITH THE KOSPI 200 FUTURES

This paper explores index arbitrage strategies using the Korea Stock Price Index (KOSPI) 200 futures and the KOSPI 200 cash. The KOSPI 200 futures rank among the most remarkable financial innovations the Korean securities markets have witnessed in recent years. Since it was launched on May 3, 1996 at the Korea Stock Exchange (KSE), the KOSPI 200 futures contract has evolved into one of the most viable stock index futures contracts in the world.[1]

Many market participants and regulators in Korea believe that the KOSPI 200 futures tend to be underpriced in relation to the KOSPI 200 cash. In fact, backwardation occurs quite frequently, where the KOSPI 200 cash price exceeds the KOSPI 200 futures price.[2] This persistent mispricing between the futures and cash is usually ascribed to the lack of index arbitrage, particularly the reverse cash-and-carry arbitrage of buying underpriced futures and shorting overpriced cash, due to difficulty in the short sale of stocks. While numerous studies addressed the issue of index arbitrage in the U.S. and Japanese markets, for example, Brennan and Schwartz (1990), Chung (1991), Klemkosky and Lee (1991), MacKinlay and Ramaswamy (1988), and Neal (1996) for the U.S. markets, and Brenner, Subramanyam, and Uno (1989), and Lim (1992) for the Japanese markets, the *Korean* markets have received scant attention by academicians despite their dramatic growth.

I determine whether arbitrage between the KOSPI 200 futures and cash is profitable from the standpoint of member firms of the KSE.[3] I first establish a theoretical framework for arbitrage strategies.

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[1] The average daily trading volume of the KOSPI 200 futures was 3670, 11137, 61279, 69078, 81604, and 128431 contracts in 1996, 1997, 1998, 1999, 2000, and 2001, respectively, while that of the S&P 500 futures was 77169, 84641, 124724, 107156, 89158, and 91375 contracts during the same years.

2 Out of 359,100 time-matched observations that are examined in this study , 152,608 observations, which are more than 42%, exhibit backwardation with mean futures price of 68.08 and mean cash price of 69.02. The remaining 206,492 observations show futures prices higher than cash prices, where mean futures price is 85.02 and mean cash price is 84.04.

[3] The major group of arbitrageurs in Korea consists of securities companies that are the KSE members. Most institutional investors such as commercial banks, merchant banks, and insurance companies are not active in index

Specifically, I derive no-arbitrage bounds on the KOSPI 200 futures price in terms of the KOSPI 200 cash prices and the transaction costs that include market impact costs, and taxes.[4] I examine the ex post arbitrage profitability where arbitrage positions are taken at the moment that the mispricing between the futures and cash prices occurs. I examine whether arbitrage frequencies and profits have diminished over time as the KOSPI 200 futures market becomes more stabilized and due to the learning effects of market participants. I also examine the ex ante arbitrage profitability where arbitrage positions are formed as suggested by the initial mispricing, but at the following sets of prices available considering the time needed for execution.

This paper addresses several other issues associated with index arbitrage. Specifically, I examine whether the time to futures expiration impacts the degree of mispricng. I also test if daily trading volume generated by arbitrage trades is positively related with the number of daily observations that are diagnosed as mispriced by my index arbitrage model and determine if the actual practice of arbitrage is consistent with the theoretical cost-of-carry model.

I use transaction prices for the KOSPI 200 futures and the KOSPI 200 cash during the May 1996 – August 2001 period to explore index arbitrage strategies. The results of the paper will be interesting to market regulators and practitioners as well as academicians, since the KOSPI futures market is a rapidly growing segment of the global financial market. The results may bear important policy implications such as relaxation of short sale restrictions to promote more active reverse cash-and-carry arbitrages.

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arbitrage, although asset management companies are involved in index arbitrage with the trading aid of securities companies.

[4]When a stock is sold, 0.3% of the sale proceeds must be paid as trading taxes (.15%)and special taxes (.15%) in Korea. I call these taxes as special sales taxes in this paper.

The paper will also propose a realistic and practical way of executing arbitrage strategies for both domestic and foreign investors.

The remainder of the paper is organized as follows. In Section I, I establish a theoretical arbitrage model that provides no-arbitrage pricing band of the KOSPI futures within which profitable arbitrage is not possible due to transaction costs. In Section II, I describe my data in detail that include transaction prices for the KOSPI 200 futures and cash, along with the data for interest rates and index dividends. In Section III, I conduct both ex post and ex ante analyses of index arbitrage and report empirical results. The impact of several trading variables on index arbitrage opportunities and other related issues are explored in Section IV, and Section V concludes the paper.

I. Arbitrage Strategies

A. Theoretical Futures Price

When I consider arbitrage between the KOSPI 200 futures and cash, I assume that the daily marking-to-market effects are negligible. For now, I also ignore transaction costs involved in arbitrage strategies. Let t and T be the present time and the expiration time of the futures contract, respectively. As shown in Table I, a futures contract can be bought directly in the futures market (Strategy A) or indirectly in the cash market (Strategy B). Similarly, a futures contract can be shorted directly in the futures market (Strategy C) or indirectly in the cash market (Strategy D).

Since Strategy A and Strategy B have the same cash flows at time T, their cash flows at time t must be the same to preclude arbitrage:

$$0 = -S_t + (F_t + D_T) / (1 + r(T-t)/365)$$

$$\rightarrow \quad F_t = S_t\,(1 + r(T-t)/365) - D_T \qquad (1a)$$

where S_t = the KOSPI 200 cash price at t

S_T = the KOSPI 200 cash price at T

F_t = the KOSPI 200 futures price at t

r = the annualized call interest rate which is reported daily by the Bank of Korea

T-t = the number of days to expiration

D_T = the sum of total cash dividends and payoffs from reinvestment of dividends of the 200 stocks in the KOSPI 200 that is converted to an index unit, where D_T is estimated and reported daily by the Korea Stock Exchange (KSE) on the basis of previous dividends paid by the 200 stocks during the period from t to T in the last year[5]

Similarly, since Strategy C and Strategy D have the same payoffs at T, their cash flows at t must be the same to preclude arbitrage:

$$0 = S_t - (F_t + D_T) / (1 + r(T-t)/365)$$

$$\rightarrow \quad F_t = S_t\,(1 + r(T-t)/365) - D_T \tag{1b}$$

If we assume that the borrowing rate in Strategy B is equal to the lending rate in Strategy D, then the theoretical price obtained from Strategies A and B and that obtained from Strategies C and D are equal to each other. However, if we consider transaction costs, we can derive a theoretical no-arbitrage pricing band within which arbitrage yields losses due to transaction costs.

B. No-Arbitrage Pricing Band

Since this paper examines arbitrage opportunities for member firms, trading commissions are not included in the transaction costs of trading futures and cash. Member firms pay fees to the KSE and the Korea Securities Depository (KSD). The KSE fee and the KSD fee are currently .0008% and .0003% of trade amount for futures trading, respectively. When they trade cash, i.e. a portfolio of stocks, member firms pay fees to the KSE and KSD, which are .008% and .0055% of trade amount, respectively. Since

[5] For the 5/3/96 – 7/5/97 period, the KSE computed and announced D_T daily without considering reinvestment of dividends. Since 7/7/97 on which the KOSPI 200 options were launched, the KSE have incorporated payoffs of dividend reinvestment in D_T. For the sake of convenience, I use D_T as they were provided by the KSE for the whole test period, from 5/3/96 to 8/31/01.

these fees are not large enough to impact arbitrage strategies, I ignore them in establishing the arbitrage model.[6]

The market impact cost of buying at asked price and selling at bid price constitutes some portion of transaction costs. Transaction price occurs at either bid price or asked price. For example, when an arbitrageur wants to buy futures, she can buy at the current transaction price if that price coincides with the asked price. However, if the current transaction price is the bid price, then the arbitrageur must buy at the asked price that is one tick higher than the transaction price. Similarly, when the arbitrageur wants to sell futures, she can sell either at the current transaction price if it is the same as the bid price, or at the bid price that is one tick lower than the transaction price. The arbitrageur can generally buy at the current transaction price or at the price one tick higher, and can sell at the current price or one tick lower. Therefore, when transaction prices are used in the analysis, the market impact cost must be equal to 0 or one tick, or on the average one half of a tick.[7]

The tick size for the KOSPI 200 futures is .05 of index unit. I assume that an arbitrageur buys at the transaction price plus .025, and that she sells at the transaction price minus .025. The same logic is applied to cash trading. For example, if the KOSPI 200 cash is traded at 100, then I assume that the arbitrageur can buy at 100.25 and sell at 99.75. This is shown in Table II in terms of (F_t + .025), (F_t – .025), (S_t + .025), (S_t – ,025), (S_T + .025), and (S_T – .025).

Another major portion of transaction costs is taxes. In Korea, the government imposes taxes when traders sell stocks. The tax rate is .3% of trade amount, which is very high compared to the KSE and KSD fees. I incorporate these special sales taxes in the arbitrage model as shown in Table II. Consider Starategy B in Table II. When an arbitrageur unwinds her long cash position at time T, she

[6] In fact, most major arbitrageurs in Korea do not consider these costs when they decide to undertake arbitrage positions.

[7] Previous studies also used one half of the bid-ask spread as the market impact cost. For example, see. Klemkosky and Lee (1991), Lee and Nayar (1993), and Stoll and Whaley (1989).

needs to pay the sales taxes, which is $.003S_T$. Also, in Strategy D, when an arbitrageur takes a short position in cash at time t, she pays the sales tax, which is represented by $.003S_t$.

When opening a futures account, traders are required to deposit cash by the amount of 30,000,000 won (approximately $30,000).[8] Since this one-time deposit can be fulfilled by interest-bearing marketable securities, where 80% of the market value of these securities is counted towards the deposit, forgone interest on the deposit is not considered as part of transaction costs. Also, the futures initial margin is 15% of trade amount, of which 10% can be fulfilled by marketable securities and the remaining 5% requires cash deposit. We ignore forgone interest on this cash initial margin for simplicity. Forgone interest on any further cash deposit in response to margin calls is also ignored.

Considering the market impact costs and the special sales taxes as the transaction costs involved in arbitrage strategies, upper bound and lower bound of no-arbitrage pricing band can be obtained as follows.

Proposition 1. To eliminate the existence of arbitrage profits, the following condition establishes an upper bound for the actual KOSPI futures price, F_t:

$$F_t \le (S_t + .025)\,(1 + r(T-t)/365) - D_T + .05 + .003S_T = F_H \qquad (2)$$

Proof: If the actual futures price is overpriced, this calls for a short hedged position in the index futures to be undertaken. Specifically, as shown in Table II, I can short a futures contract directly (Strategy C) and buy it indirectly (Strategy B) to make an arbitrage profit. This is called a cash-and-carry arbitrage. After including transaction costs, to prevent arbitrage profits in the case of short hedge, if the net cash flow of Strategies C and B at t is zero, the net cash flow at T from Strategies C and B must be less than or equal to zero:

$$F_t - (S_t + .025)\,(1 + r(T-t)/365) + D_T - .05 - .003S_T \le 0$$

$$\rightarrow \quad F_t \le (S_t + .025)\,(1 + r(T\text{-}t)/365) - D_T + .05 + .003S_T \qquad \text{Q.E.D.}$$

8 For simplicity, I assume that the exchange rate is 1,000 Korean Won per 1 U.S. Dollar.

Proposition 2. To eliminate the existence of arbitrage profits, the following condition establishes a lower bound for the actual KOSPI futures price, F_t:

$$F_t \geq (S_t - .025 - .003S_t)\,(1 + r(T-t)/365) - D_T - .05 = F_L \qquad (3)$$

Proof: If the actual futures price is underpriced, this calls for a long hedged position in the index futures to be undertaken. Specifically, as shown in Table II, I can buy a futures contract directly (Strategy A) and short it indirectly (Strategy D) to make an arbitrage profit. This strategy is the reverse cash-and-carry arbitrage. After including transaction costs, to prevent arbitrage profits in the case of long hedge, if the net cash flow of Strategies A and D at t is zero, the net cash flow at T from Strategies A and D must be less than or equal to zero:

$$-F_t - (S_t - .025 - .003S_t)\,(1 + r(T-t)/365) - D_T - .05 \leq 0$$

$$\rightarrow \qquad F_t \geq (S_t - .025 - .003S_t)\,(1 + r(T-t)/365) - D_T - .05 \qquad \text{Q.E.D.}$$

From Propositions 1 and 2, if the actual futures price is located between the upper bound (F_H) and the lower bound (F_L), then an arbitrage position, either long hedge or short hedge, with that contract leads to losses due to transaction costs. No-arbitrage pricing band can therefore be used to judge whether a potentially profitable arbitrage position is taken with a given futures contract at time t. If the actual futures price exceeds the upper bound (F_H), then a short hedge (Strategies C & B) needs to be taken. On the other hand, a long hedge (Strategies A & D) is profitable for a futures contract whose price is below the lower bound (F_L). To determine whether a given futures contract is mispriced at time t, the actual futures price (F_t) needs to be compared with F_H and F_L. F_H contains S_T, and since it is not observable at time t, I use S_t as proxy for $S_{T.}$

II. Data

The test period covers from May 3, 1996, on which the KOSPI 200 futures contract was launched by the KSE, to Auguest 31, 2001. The data set consists of transaction price data for the KOSPI 200 futures and the KOSPI 200 cash, call rates, index dividends, and daily cash trading volume for both the cash-and-carry arbitrage and the reverse cash-and-carry arbitrage.

A. Intraday Price Data for the KOSPI 200 Futures and Cash

I obtained from the KSE the transaction price data for the KOSPI 200 futures for the test period. This database indicates precise transaction times in detail up to 1/100 second. I sampled transaction prices at 1- minute intervals. If there were more than one trade within a 1-minute interval, then the last price in that interval was selected. For example, if trade prices exist at 10:30:24, 10:30:39, and 10:30:54 during the interval between 10:30 a.m. and 10:31 a.m., then the price at 10:30:54 is chosen as the 10:31 price. The KOSPI 200 futures contract expires in March, June, September, and December, and the expiration day is the second Thursday of expiration months. Since the nearby contract is the most liquid and it is heavily used in arbitrage strategies, I focus on the analysis of the nearby contract only and ignore all other deferred contracts.

I obtained from the KSE the minute-by-minute KOSPI 200 cash price data, and matched them with the futures data at 1-minute intervals. The KOSPI 200 cash market, i.e., the Korean stock market, closes at 3:00 p.m. (11:30 a.m. on Saturdays until 12/05/98), while the KOSPI 200 futures market closes at 3:15 p.m. (11:45 a.m. on Saturdays). Therefore, when I matched futures and cash prices at 1-minute intervals, I excluded the futures prices between 3:00 p.m. and 3:15 p.m. (between 11:30 a.m. and 11:45 a.m. on Saturdays) from the sample. Furthermore, there is a clearing process from 2:50 p.m. and 3:00 p.m. in the cash market, and all the cash prices during this period is the same as the 3:00 p.m. price, while the futures price occurs continuously. Therefore, the prices between 2:50 p.m. and 3:00 p.m. are excluded from the sample.[9]

[9] For the 5/3/96 – 12/5/98 period, both markets opened at 9:30 a.m., closed from 11:30 a.m. to 1:00 p.m. for lunch break, and reopened at 1:00 p.m. until it closed for the day. Beginning on 12/07/98 they opened at 9:00 a.m. and closed from 12:00 p.m. to 1:00 p.m. for lunch break. Since 5/22/00, there has been no lunch break.

B. Interest Rates

The Korea Stock Exchange (KSE) uses the latest 91-day CD rate that is reported by the Korea Securities Dealers Association (KSDA) when computing the theoretical price of the KOSPI 200 futures. The reason is that this interest rate is the most representative of the short-term interest rate in the money market in Korea. However, in practice arbitrageurs are composed of securities companies and limited group of institutional investors such as asset management companies. These investors heavily trade in the call money and call loan markets, where the minimum trade volume is 0.1 billion won, or approximately $100,000.

I obtained both daily CD rates and call rates from the Bank of Korea. A preliminary analysis of these two rates shows that there is no significant difference between the two, and I use call rates in this study, since they are more relevant and popular in executing arbitrage strategies. Assuming that interest rates are constant during the trading day, I use daily interest rates as minute-by-minute rates. I matched the KOSPI 200 futures and cash prices and call rates at 1-minute intervals during the entire sample period.

III. Violations of No-Arbitrage Pricing Conditions

Attention is now turned to the issue of index arbitrage opportunities. As long as the actual futures price lies within the boundaries of no-arbitrage band, it will be considered as fairly priced. If the actual futures price is above the upper bound or below the lower bound, then it is mispriced and profitable arbitrage strategies can be executed.

A. Ex Post Results

An ex post arbitrage strategy assumes that arbitrage position can be formed at the moment that a futures contract is identified as mispriced. I use no-arbitrage conditions established in Section I in order to classify every KOSPI futures price that is time-matched with its counterpart KOSPI cash price into Aoverpriced≅, Aunderpriced≅, and Afairly priced≅.

A futures contract that is initially identified as mispriced by the arbitrage model may not necessarily lead to profits due to the tracking error of a basket portfolio.[10] I assume zero tracking error for simplicity. I examine the ex post profitability of index arbitrage between the KOSPI 200 futures and cash, where arbitrage position is undertaken at the moment of mispricing. In Table III, the results are reported by each year in terms of the frequency of mispricing, i.e., the number of overpriced, underpriced, and fairly priced observations.[11] Table III also reports the magnitude of mispricing, both mean and median arbitrage profits for overpriced and underpriced observations.

As shown in Table III, the overall results are surprising. Out of 359,100 time-matched observations, 52% were turned out underpriced, 24% were overpriced, and only 24% were fairly priced, which suggests that the KOSPI 200 futures and cash markets are extremely inefficient. Both mean and median arbitrage profits for mispriced observations are significant at the .01% level, with the underpricing profit far exceeding the overpricing profit, 1.2184 versus .5969.[12]

Has the number of mispricing decreased over time as the KOSPI 200 futures market matures? In general, as market participants become more familiar with the newly introduced futures contract and the market becomes more stable and efficient, profitable arbitrage opportunities tend to diminish.[13] This phenomenon exists to some extent in the KOSPI 200 futures market. In 1997 and 1998, the number of fair pricing was 10~12%, and at least it went up to 30~33% in 1999, 2000, and 2001. However, the overpricing and underpricing frequencies do not show gradual decrease over years. The overpricing frequency was particularly high in 1999 where almost 50% of all observations were overpriced, while the

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[10] The number of stocks in the basket portfolio that would mimic the KOSPI 200 cash varies widely across arbitrageurs. In Korea, typically 120 – 180 stocks are used by most arbitrageurs for the tracking portfolio.
[11] The Jun 96, Sept 96, and Dec 96 contracts were included in the 1996 data for the 5/03/96-12/12/96 period. The 1997 data include the Mar 97 through Dec 97 contracts for the 12/13/96 – 12/11/97 period. The 1998, 1999, and 2000 data cover the 12/12/97 – 12/10/98, 12/11/98 – 12/9/99, and 12/10/99 – 12/14/00 periods, respectively. The 2001 data cover from 12/15/00 to 8/31/01.
[12] For example, if the KOSPI 200 futures price is 70 or 35,000,000 won (=70 × 500,000 won) and 30 contracts are used for arbitrage for the amount of 1,050,000,000 won, approximately 9,000,000 won and 18,000,000 won are yielded by cash-and-carry and reverse cash-and-carry arbitrages, respectively.
[13] For example, see Figlewski (1984) among others.

underpricing frequency was over 70% in 1998 and 2001. There are only 14 overpriced observations out of 60009 in 2001. The Korean markets do not conform to the cost-of-carry model for these periods. Actually, in Korea there was a strong bull market in 1999 following a severe bear market in 1998, and in 2001, the stock market was depressed with global economic down trends. Market sentiments might have helped the futures market deviate from the full carry market; the market participants in the futures market might overreact to both the bull and bear stock markets.

I conduct statistical tests to determine whether the degree and frequency of overpricing are different from those of underpricing; t-test for mean difference, Brown-Mood test for median difference, and z-test for frequency difference. The results are summarized in Table IV. The overall results show both mean and median profits for underpriced observations are significantly greater than those for overpriced observations at the .01% level. This is true for individual years; only in 2001 the difference between overpricing and underpricing is significant at the 1% level, and in other years at the .01% level. Table IV shows that the underpring frequency is significantly higher than the overpricng frequency in each year at the .1% level except that the opposite is true in 1999. In Korea, underpricing seems to dominate overpricing both in degree and frequency.

How about the degree of mispring over time? Has the mean arbitrage profit decreased over years? Table III exhibits a clear tendency of sharp decline in mean arbitrage profit in the case of underpricing; it was as high as 2.3017 in 1997 and fell to .2971 in 2001. On the other hand, the degree of overpricing does not show any declining pattern. To formally test if arbitrage profits have decreased over time and to determine which years have more and which have less than average profit, I estimate a regression in which the dependent variable is $\Pi_t - E(\Pi_t)$ where Π_t is the arbitrage profit at time t and $E(\Pi_t)$ is the mean arbitrage profit over the sample period. I define D_{jt} to 1 if time t is in year j, j = 1996, 1997, …,2001, and zero otherwise. The regression format is as follows:

$$\Pi_t - E(\Pi_t) = \sum_{j=1996}^{2001} \alpha_j D_{jt} + e_t \qquad (5)$$

I do not include an intercept term in the regression in order to capture changes in arbitrage profit across years. By comparing the regression coefficients, I now can determine which year has either significantly higher or lower than average arbitrage profit. Table V shows the regression results.

Panel C in Table V shows that arbitrage profit with mispriced observations is the second highest in 1996 and the highest in 1997, and it decreases year by year from 1997 through 2001, with the lowest in 2001. This evidence is encouraging; the degree of mispring tends to go down over time. The results for underpriced observations in Panels B are not distinguishable from the Panel C results. However, the overpring results in Panel A are quite different with substantilly higher arbitrage profit in 1999 compared to the rest of years, and as described above, the overpricing frequency is also highest in 1999. F-values in Panels A, B, and C indicate that arbitrage profits significantly vary across years.

B. Ex Ante Results

Whenever a violation of the no-arbitrage bounds occurs, this acts as a signal to initiate transactions to exploit it. I assume that transactions cannot be conducted at the prices that violate the no-arbitrage bounds but instead at the next time-matched set of prices for the KOSPI 200 futures and cash. The arbitrage profits from these "after-the-violation" trades are termed ex ante profits.[14] Ex ante arbitrage is more consistent with actual practice of arbitrage than ex post arbitrage, since any lag time between initial mispricing and actual execution is considered. If the ex post arbitrage profits diminishes sharply when they are tested on an ex ante basis, then I may claim that the markets are economically efficient. The results for these ex ante tests appear in Table VI.

I use three lags for ex ante tests. Lag1, Lag2, and Lag3 represent the first, second, and third time-matched sets of prices following the initial mispricing. The mean and median lag time for Lags 1,

[14] For example, Klemkosky and Resnick (1980) and Rhee and Chang (1992) conduct ex ante tests for the equity options market and the foreign exchange market, respectively.

2, and 3 are 1, 2, and 3 minutes, respectively, implying that the KOSPI 200 futures market is highly liquid. Futures trades occur at least once in each 1-minute interval. In Korea, it normally takes a couple of minutes to trade the KOSPI 200 futures and several minutes for a basket portfolio in executing arbitrage strategies. Therefore, Lag3 may be considered more consistent with reality.

As shown in Table VI, the ex post mean and median profits are all significantly positive for both overpricing and underpricing. That is, if trades are consummated at the actual prices that violate the no-arbitrage bounds, arbitrage opportunities appear to be positive and significant. Surprisingly, the ex ante profits are also all positive and significant by the mean and the median. This is true for even the longest time interval of 3 minutes after the actual violation. Also, as time elapses after the violation, the ex ante profits declines only slightly. This evidence implies that any mispricing between the KOSPI 200 futures and cash markets is not corrected soon enough after the mispricing, and profitable ex ante arbitrage opportunities persist. The main finding that the KOSPI 200 futures and cash markets are extremely inefficient still remains valid.

V. Other Issues

Now, several other issues related to index arbitrage are examined. Specifically, I examine whether the degree of mispricng is impacted by the time to futures expiration. I also test if daily arbitrage trading volume in the stock market is positively related with the number of mispriced observations identified by the index arbitrage model.

A. Time-to-Expiration Effect

A futures price can be considered to represent the expected future spot price at futures expiration. As the futures expiration day approaches, the futures price can forecast the future spot price more accurately so that the degree of mispricing lessens, but at the same time futures volatility tends to increase which in turn induces more profitable arbitrage opportunities.[15] To determine whether the degree of mispricing, , which is measured as the ex post arbitrage profit, is impacted by the number of days to expiration, , I estimate and report the following regression model:

$$\Pi_t = \beta_0 + \beta_1 \, NDM_t + e_t \tag{5}$$

where Π_t is the ex post arbitrage profit at time t and NDM_t denotes the number of days to maturity.

The regression results in Table VII show that arbitrage profits increase with time to expiration for both overpriced and underpriced observations. When testing for each year, three years (1997, 1999, 2000) for overpricing and five years (1996, 1997, 1998, 2000, and 2001) for underpricing exhibit the time-to-expiration effect.

B. Arbitrage Trading Volume

The KSE publishes cash trading volume that is generated by index arbitrage trades. I obtained this daily data set of arbitrage trading volume, both in won and shares, from the KSE.[16] My arbitrage model identifies profitable arbitrage opportunities whenver the KOSPI 200 futures and cash prices deviate from the full carry prices. Do arbitrageurs in practice then exploit these opportunities? Since the cost-of-carry arbitrage entails the purchase of an index portfolio and the sale of overpriced futures, I expect the cost-of-carry arbitrage cash volume to increase with the frequency of overpricing on a given day. Similarly, the reverse cost-of-carry arbitrage of shorting stocks and buying underpriced futures induce the reverse cost-of-carry arbitrage cash volume to be positively correlated with the frequency of underpricing. On the other hand, I expect a negative correlation between the frequency of overpricing and the reverse cost-of-carry arbitrage cash volume and between the frequency of underpricing and the cost-of-carry arbitrage cash volume.

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[15] For example, see Fama and French (1987), Lee and Jo (1996), and Stoll and Whaley (1990).

16 Since the KSE data set covers only from 11/25/96, the correlation analysis below is conducted for the 11/25/96 – 8/31/01 period.

I compute the number of overpriced observations ($OVER_t$) and the number of underpriced observations ($UNDER_t$) on a daily basis. The KSE daily data set consists of the cost-of-carry cash volume ($CVOL_t$) and the reverse cash-and-carry cash volume ($RVOL_t$). In Table VIII, I report correlation coefficients between these variables. Results for cash volume in Korean Won amount are shown in Panel A and for cash volume in shares in Panel B.

As expected, in Panels A and B, the correlation coefficient between $OVER_t$ and $CVOL_t$ is significantly positive, and the correlation coefficient between $UNDER_t$ and $CVOL_t$ is significantly negative, both at the .01% level. This evidence implies that the cash-and-carry arbitrage is viable in the KOSPI 200 futures and cash markets. Arbitrageurs appear to exploit overpriced futures by buying the underlying stocks and selling the futures. However, the correlation coefficients between $OVER_t$ and $RVOL_t$ and between $UNDER_t$ and $RVOL_t$ are close to and insignificantly different from zero, implying that, when the KOSPI 200 futures is underpriced, arbitrageurs do not actively participate in the reverse cash-and-carry arbitrage strategies.

As a matter of fact, most arbitrageurs in Korea are not enthusiastic about the reverse cash-and-carry arbitrage, mainly because the short sale of the underlying stocks is too costly. Even when backwardation occurs, i.e. the cash price exceeds the futures price, arbitrageurs hesitate to short the underlying stocks. Currently, arbitrageurs borrow stocks from large pension funds or asset management companies through brokers such as the KSD (Korea Securities Depository) or the KSFC (Korea Securities Finance Corporation) at 7% per year. This borrowing cost should be treated as transaction costs associated with short sale, and the lower bound of no-arbitrage band should become much lower for arbitrageurs. Furthermore, these funds consist of approximately 50 largest capitalization stocks, whereas arbitrageurs need 120 to 180 stocks to mimic the KOSPI 200 cash. Therefore, the tracking error associated with short sale is very high, which sharply lessens arbitrage profits. Also, in Korea, securities companies do not maintain stocks in street name, so that those stocks cannot be used for short sale by arbitrageurs.

In Korea, for example, even under a severe backwardation condition where basis (= futures price – cash price) is –1, arbitrageurs are hesitant to short stocks. They worry that basis may increase shortly, and wait for not just a couple of minutes or hours, but for a couple of days until they are completely assured that the severe backwardation persists. High borrowing costs and large tracking errors hinder arbitrageurs from engaging in the reverse cash-and-carry arbitrage, which then leads to frequent and persistent underpricing of futures. A new policy that relaxes short sale restrictions and promotes more active reverse cash-and-carry arbitrages appears to be in desperate need.

VI. Conclusions

This paper has explored index arbitrage opportunities in the KOSPI 200 futures and cash markets during the May 1996 – August 2001 period on an ex ante as well as ex post basis. I find that these markets are extremely inefficient. Both the degree of mispricng and the frequency of mispring are significantly high and do not diminish even when the markets have grown substantially in recent years. The underpricing phenomenon appears to be more prevalent in the KOSPI 200 futures market. Arbitrage profits tend to decrease as the expiration day of futures approaches. My results also suggest that the cash-and-carry arbitrage with overpriced futures is relatively viable. However, the reverse cash-and-carry arbitrage with underpriced futures is virtually inactive.

The basic picture that emerges from my research is a persistent underpricing of the KOSPI 200 futures. This is in sharp contrast to the other stock index futures markets in the world, where the markets are generally inefficient only in early stages and become efficient over time. I believe that short sale of stocks in Korea is currently too restrictive and costly. To provide market participants with fairly priced futures, a new and innovative policy that can encourage the reverse cash-and-carry arbitrage needs to be devised and implemented desperately.

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Table I
The Relationship between Futures Price and Cash Price

Details of cash flows for buying a futures contract directly (A) and indirectly (B) and for shorting a futures contract directly (C) and indirectly (D) are described with the following symbols:

S_t =	the KOSPI 200 cash price at t
S_T =	the KOSPI 200 cash price at T
F_t =	the KOSPI 200 futures price at t
r =	the annualized call interest rate which is reported daily by the Bank of Korea
T-t =	the number of days to expiration
D_T =	the sum of total cash dividends and payoffs from reinvestment of dividends of the 200 stocks in the KOSPI 200 that is converted to an index unit, where D_T is estimated and reported daily by the Korea Stock Exchange (KSE) on the basis of previous dividends paid by the 200 stocks during the period from t to T in the last year

Strategy	Cash flow at t	Cash flow at T
A: Buy a futures	0	$S_T - F_t$
B: Buy an index Borrow	$-S_t$ $(F_t + D_T)/ (1 + r(T-t)/365)$	$S_T + D_T$ $-F_t - D_T$
C: Short a futures	0	$F_t - S_T$
D: Short an index Lend	S_t $-(F_t + D_T)/ (1 + r(T-t)/365)$	$-S_T - D_T$ $F_t + D_T$

Table II
Arbitrage Strategies with Overpriced and Underpriced Futures

Details of cash flows for the cash-and-carry arbitrage strategy with overpriced futures and the reverse cash-and-carry arbitrage strategy with underpriced futures are described with the following symbols,where the transaction costs include the market impact cost, which equals one half of a tick (.05 in index unit), and the special sales taxes (.3% of trade amount):

- S_t = the KOSPI 200 cash price at t
- S_T = the KOSPI 200 cash price at T
- F_t = the KOSPI 200 futures price at t
- r = the annualized call interest rate which is reported daily by the Bank of Korea
- T-t = the number of days to expiration
- D_T = the sum of total cash dividends and payoffs from reinvestment of dividends of the 200 stocks in the KOSPI 200 that is converted to an index unit, where D_T is estimated and reported daily by the Korea Stock Exchange (KSE) on the basis of previous dividends paid by the 200 stocks during the period from t to T in the last year

Cash-and-carry arbitrage strategy (C & B) with overpriced futures	Cash flow at t	Cash flow at T
C: Short a futures B: Buy an index Borrow	0 $-(S_t + .025)$ $S_t + .025$	$(F_t - .025) - S_T$ $(S_T - .025) + D_T - .003S_T$ $-(S_t + .025)\ (1 + r(T-t)/365)$
Reverse cash-and-carry arbitrage strategy (A & D) with underpriced futures	Cash flow at t	Cash flow at T
A: Buy a futures D: Short an index Lend	0 $(S_t - .025) - .003S_t$ $-(S_t - .025 - .003S_t)$	$S_T - (F_t + .025)$ $-(S_T + .025) - D_T$ $(S_t - .025 - .003S_t)\ (1.+.r(T\text{-}t)/365)$

Table Ⅲ
Violations of No-Arbitrage Pricing Conditions for the KOSPI 200 Futures

Violations of the following no-arbitrage conditions are reported for the 5/3/96 – 8/31/01 period, where overpriced observations are higher than the upper bound (F_H), underpriced observations are lower than the lower bound (F_L), and fairly priced observations are within no-arbitrage boundaries:

$$(S_t - .025 - .003S_t)(1 + r(T-t)/365) - D_T - .05 \leq F_t \leq (S_t + .025)(1 + r(T-t)/365) - D_T + .05 + .003S_T$$

The arbitrage profit is computed as $(F - F_H)$ and $(F_L - F)$ for overpriced and underpriced observations, respectively. *** indicates significance at the .01% level in a two-tailed t-test for mean profit and a signed rank test for median profit.

Year	No. of obs.	Fairly priced	*Overpriced*			Underpriced		
		No. of obs.	No. of obs.	Mean profit	Median profit	No. of obs.	Mean profit	Median profit
1996	34367	8632 (25%)	5216 (15%)	.4849*** (110.89)	.4634*** (6802968)	20519 (60%)	1.6613*** (156.04)	1.2189*** (105260000)
1997	56097	5837 (10%)	18728 (33%)	.4922*** (239.95)	.4794*** (87689178)	31532 (56%)	2.3027*** (276.62)	2.1913*** (2487570000)
1998	58448	7035 (12%)	8038 (14%)	.5732*** (104.33)	.4372*** (16154372)	43375 (74%)	1.4695*** (280.50)	1.2765*** (470360000)
1999	70504	21199 (30%)	34124 (48%)	.6820*** (198.77)	.5117*** (291120000)	15181 (22%)	1.2385*** (116.87)	.7914*** (57619486)
2000	79675	26338 (33%)	19073 (24%)	.5882*** (129.78)	.3172*** (90949601)	34264 (43%)	.7576*** (243.75)	.6396*** (293510000)
2001	60009	18013 (30%)	14 (0%)	.1025*** (2.47)	.0303*** (52.5)	41982 (70%)	.2971*** (277.39)	.2545*** (440630000)
Total	359100	87054 (24%)	85193 (24%)	.5969*** (317.46)	.4566*** (1814500000)	186853 (52%)	1.2184*** (419.71)	.7645*** (8728600000)

Table Ⅳ
Difference in Arbitrage Profits and Frequencies between Overpriced and Underpriced Observations

Statistical tests are conducted to determine whether the degree and frequency of overpricing are different from those of underpricing; t-test for mean difference, Brown-Mood test for median difference, and z-test for frequency difference. Test statistics are reported. ** and *** indicate significance at the .1% and 01% levels, respectively.

Year	Mean(μ)	Median(M)	Overpricing Ratio (P_{over})
	$\mu_{over} = \mu_{under}$	$M_{over} = M_{under}$	$P_{over} = .5$
1996	-102.22***	3122.73***	-95.39***
1997	-211.17***	23958.04***	-57.11***
1998	-118.06***	5105.24***	-155.85***
1999	-49.96***	827.87***	85.31***
2000	-30.82***	2490.99***	-65.78***
2001	-4.69**	7.15**	-204.79***
Total	-179.72***	8374.80***	-194.91***

Table V
Index Arbitrage Opportunities for the KOSPI 200 Futures and Cash over Time

To test if arbitrage profits have decreased over time and to determine which years have more and which have less than average profit, the following regression model is estimated for the 5/3/96 – 8/31/01 period:

$$\Pi_t - E(\Pi_t) = \sum_{j=1996}^{2001} \alpha_j D_{jt} + e_t$$

where Π_t is the arbitrage profit at time t and $E(\Pi_t)$ is the mean arbitrage profit over the sample period. D_{jt} is 1 if time t is in year j, j = 1996, 1997, …,2001, and zero otherwise. *** indicates significance at the .01% level.

No. of obs.	D_{1996} (α_{1006})	D_{1997} (α_{1997})	D_{1998} (α_{1998})	D_{1999} (α_{1999})	D_{2000} (α_{2000})	D_{2001} (α_{2001})	F-value	Adj. R^2
Panel A: Overpricing								
85193	-.1274*** (-16.94)	-.1200*** (-30.24)	-.0390*** (-6.44)	.0698*** (23.73)	-.0240*** (-6.10)	-.5097** (-3.51)	309.32***	.0213
Panel B: Underpricing								
186853	.4429*** (60.21)	1.0843*** (182.71)	.2511*** (49.62)	.0201* (2.35)	-.4608*** (-80.93)	-.9213*** (-179.13)	13018.7***	.2948
Panel C: Mispricing								
272046	.3961*** (61.97)	.6013*** (131.47)	.3025*** (66.90)	-.1735*** (-37.57)	-.3298*** (-74.28)	-.7298*** (-145.87)	8968.26***	.1651

Table VI
The Ex Ante Arbitrage Profitability for the KOSPI 200 Futures and Cash

Assuming that transactions cannot be conducted at the prices that violate the no-arbitrage bounds but instead at the next time-matched set of prices, ex ante arbitrage profits, both mean and median, are computed for the 5/3/96 – 8/31/01 period. Ex ante arbitrage is more consistent with actual practice of arbitrage than ex post arbitrage, since any lag time between initial mispricing and actual execution is considered. Lag1, Lag2, and Lag3 represent the first, second, and third time-matched sets of prices following the initial mispricing. *** indicates significance at the .01% level.

	Lag1	Lag2	Lag3
Panel A: Overpricing			
No. of obs.	84533	83863	83206
ex post mean profit	.5968***	.5968***	.5967***
ex post median profit	.4566***	.4566***	.4565***
lag time	1 minute	2 minutes	3 minutes
ex ante mean profit	.5877***	.5817***	.5760***
ex ante median profit	.4558***	.4554***	.4545***
Panel B: Underpricing			
No. of obs.	185601	184357	183115
ex post mean profit	1.2165***	1.2144***	1.2123***
ex post median profit	.7630***	.7612***	.7588***
lag time	1 minute	2 minutes	3 minutes
ex ante mean profit	1.2124***	1.2077***	1.2032***
ex ante median profit	.7621***	.7588***	.7552***

Table VII
The Impact of the Time to Expiration on the Degree of Mispricing

To determine whether the degree of mispricing is impacted by the number of days to expiration, the following regression model is estimated for the 5/3/96 – 8/31/01 period:

$$\Pi_t = \beta_0 + \beta_1 \text{ NDM}_t + e_t$$

where Π_t is the ex post arbitrage profit at time t and NDM_t denotes the number of days to maturity. * and *** indicate significance at the 5% and .01% levels, respectively.

Year	No. of obs.	Intercept(β_0)	NDM(β_1)	Adj. R^2
Panel A: Overpricing				
1996	5216	.4867***	-.0001	-.0002
1997	18728	.4276***	.0016***	.0208
1998	8038	.5665***	.0003	.0000
1999	34124	.2388***	.0088***	.1297
2000	19073	-.1936***	.0159***	.3731
2001	14	.1209	-.0030	-.0761
Total	85193	.2633***	.0076***	.1300
Panel B: Underpricing				
1996	20519	-.7372***	.0449***	.5039
1997	31532	1.2608***	.0215***	.1324
1998	43375	.6213***	.0176***	.1747
1999	15181	1.2810***	-.0011*	.0003
2000	34264	.3218***	.0087***	.1794
2001	41982	.0851***	.0038***	.1463
Total	186853	.5931***	.0125***	.0640

Table VIII

Correlation Matrices for the Arbitrage Cash Trading Volume and the Frequency of Mispricing

The number of overpriced observations ($OVER_t$) and the number of underpriced observations ($UNDER_t$) for the KOSPI 200 futures were computed on a daily basis. The cost-of-carry arbitrage cash trading volume ($CVOL_t$) and the reverse cash-and-carry arbitrage cash trading volume ($RVOL_t$) were obtained from the KSE. Correlation coefficients between these variables are reported for the 11/25/96 – 8/31/01 period. Results for cash volume in Korean Won amount are shown in Panel A and for cash volume in shares in Panel B. *** indicates significance at the .01% level.

	Cash-and-carry arbitrage Cash trading volume ($CVOL_t$)	Reverse cash-and-carry arbitrage Cash trading volume ($RVOL_t$)
Panel A: Trading volume (won)		
No. of overpricing ($OVER_t$)	.3406***	-.0044
No. of underpricing ($UNDER_t$)	-.2081***	-.0292
Panel B: Trading volume (shares)		
No. of overpricing ($OVER_t$)	.2803***	-.0528
No. of underpricing ($UNDER_t$)	-.1523***	.0489