Preliminary and Incomplete

Leverage, Options Liabilities and Corporate Bond Pricing

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This Draft: January 10, 2007

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Abstract

Most of the existing structural models only focus on the simplified liability structure, and ignore some off-balance sheet liabilities. It has been proved by the collapse of some giant financial companies that the ignorance of off-sheet financing could underestimate default risk and lead to an incorrect credit spread. This paper starts from Collin-Dufresne and Goldstein (2001)'s stationary leverage model and impose option liabilities on the firm's capital structure to resolve the drawbacks of CDG (2001) model, which was pointed out in Eom, Helwege and Huang (2003), in explaining credit spread. In the literature, jump diffusion process for firm value is feasible to increase credit spread (e.g.: Zhou (1997)); regime-switching interest rate process (e.g.: Bansanl and Zhou (2002)) is more fitted to spot rate process, and also reflects the impact of business cycle. Therefore, credit spreads under a jump-diffusion model or a regime-switching interest rate model are also examined in this paper by using simulation-based approach. In this paper, we find that the credit spread overestimation problem in CDG (2001) model can be resolved by combining option liabilities and regime-switching interest rate process when it is an investment grade bond; when it is a junk bond, only regime-switching interest rate process is needed because credit spread is more sensitive to the addition of default boundary, and it easily cause credit spread overshooting. However, the widely used jump-diffusion model is not helpful in resolving the overestimation problem of credit spread. Furthermore, vulnerable option values, debt values, and zero-coupon bond values are also examined under different model settings and leverage ratios in this paper.

1. Introduction

In last decade, many contingent claim pricing models have been proposed and implemented in practice, especially the fixed-income pricing model. As the default risk attracts more and more attentions, a correctly priced corporate bond is getting more important for any financial practitioner. However, existing financial model is not rich enough to capture all risks in corporate bond. Although, default risk, liquidity risk, and tax effect have been widely discussed in fixed income literature, but the capital structure effect and off-balance sheet instruments are rarely discussed and taken into consideration in corporate bond pricing. In addition, existing structural model seldom correctly capture the dynamics of corporate bond credit spread. Two major problems for typical structural models are the failure to have a positive credit spread in a very short term[1], and incorrectly estimate the overall level of credit spread.

In the literature, structural model and reduced form model are two types of model which are popular in financial research and industry practice. However, in structural model literature, previous ones only focus on the modeling of asset process or debt process, and almost none of them considered the composition of liabilities. The composition of a firm's liabilities could be quite different and significantly affect the determination of default boundary and corporate bond pricing. For example, firms in retail industry may use a lot of lease financing to reduce the financial burden of fixed cost; firms in airline industry may lease their air crafts for operation, instead of purchasing facilities on their own. The incentive of using lease financing may vary, but it diversifies the composition of a firm's liability structure. In addition, the rapid growth of financial market brings many innovations and also creates many hedging demand to remove financial risk. Therefore, the derivatives trading position in asset or liabilities play important roles in a firm's financial management. The aggregate size of derivatives position by financial firms has been growing dramatically in last decades. Even non-financial firms, especially large ones, tend to use derivatives for hedging or increase investment profit. Recently, some energy trading companies, e.g.: Enron, take huge derivatives position in its liabilities and leads to serious crisis for financial market. Therefore, the ignorance of firm's liability structure could underestimate a firm's default risk; thus, affect the values of related financial claims. On another hand, firms are used to follow stationary leverage ratio strategy to prevent them from exposing to huge financial risk. Collin-Dufresne and Goldstein (2001) (i.e. CDG hereafter) incorporate this feature into

[1] In observation, regardless of how healthy a firm may be or how short the bond may be, there is some level below which credit spread can't fall. This observation has been put into empirical modeling, like Duffee (1999), and empirically supported by Fons (1994), Jones, Mason, and Rosenfeld (1984), and Sarig and Warga (1989).

corporate bond pricing and find that this inclusion can successfully change the Longstaff and Schwartz (1995)'s(hereafter LS) downward sloping credit spread term structure for junk bond to upward sloping credit spread term structure. In addition, CDG model yields a higher credit spread term structure than LS model in investment grade bond and the credit spread is less sensitive to firm value. However, CDG model is still unable to fix the zero short term credit spread problem. In another aspect, Cao and Wei (2001), Episcopos (2004) point out that when the firm has option liabilities, it can successfully solve the zero short term credit spread problem and yield positive credit spread in a very short term. In addition, Zhou (1997) proposed a structural model which models the firm's value as a jump-diffusion process such that it could also solve the short term zero credit spread problem. Hence, one purpose of this paper is to combine CDG (2001) and Cao and Wei (2001) to examine the impact on bond valuation and credit spread. Furthermore, we also follow Zhou (1997) to include jump-diffusion concept into the firm value process and the option underlying asset process to see the impact on relevant results. In addition, Eom, Helwege, and Huang (2003) point out that CDG (2001) usually overestimate the overall level of credit spread, especially the long term one. Therefore, in order to overcome this drawback, we remodel the dynamics of interest rate process by using regime-switching model and examine the improvement of CDG model in the long term.

2. Literature Review

The credit model for pricing corporate bond could be categorized as either structural model or reduced-form model. The structural model starts from Merton (1974), and continued by Black and Cox (1976), Leland (1994), Longstaff and Schwartz (1995), Leland and Toft (1996), CDG (2001), Episcopos (2004) and so on. On another hand, reduced form model starts from the seminal paper, Jarrow and Turnbull (1995), and is further developed by Duffie and Singleton (1997), Duffie and Singleton (1999). The structural model uses contingent claim analysis to price corporate bond and treat it as a barrier option with exogenously given (e.g.: Merton (1974), Longstaff and Schwartz (1995)) or endogenously given (e.g.: Leland and Leland and Toft (1996)) default triggering boundary. In contrast, reduced form models take default as unexpected jump which can't be predicted *ex ante*. Comparing these two models, reduced form model has the advantage of easy implementation and is widely used in trading; however, structural model has more robust economic intuition, but needs more computation.

In previous structural model literatures, default boundary is often assumed exogenously given and constant over time (e.g.: Merton (1974), Longstaff and Schwartz (1995)). In these models, default boundary is assumed to be or a proportion of the face value of outstanding debt; however, in practice, the outstanding debt is changing over time, and seems to have a stationary constant ratio. Goldstein *et.al.* (2001) provides empirical studies and show that firm value is maximized when a firm acts to keep its leverage ratio within a certain band. In addition, CDG (2001) proposes a structural model which incorporates the property of stationary leverage ratio and stochastic interest rate into modeling and finds that it can capture the pattern of credit spread better than previous models.

In another aspect, the corporate bond pricing concerning with different liabilities composition is rarely discussed in the literature. Cao and Wei (2001) proposed a structural model to discuss vulnerable option value and credit spread when the firm has a European call, not only debt in its liability. They found the inclusion of call option into a firm's liability can effectively explain short term credit spread which was ignored in previous literature. Recently, Episcopos (2004) continues the work of Klein and Inglis (2001) to re-examine the credit spread and vulnerable option issue with a written option in the firm's liability. Episcopos (2004)'s result modifies Cao and Wei (2001)'s one that vulnerable option value is higher than regular option value in some cases, which is obviously counter intuitive. In addition, Huang and Yildiray (2006) consider the impact of lease financing on firm's liability and further discuss the impact on capital structure decision and corporate bond pricing by employing endogenous default model proposed by Leland and Toft (1996). The results reveal that lease financing can bring positive impact on corporate bond price when leverage ratio, defined as debt value over firm value, is low because positive tax-shield benefit dominates negative bankruptcy costs. However, lease financing could even make corporate bond riskier, and decrease bond's value when leverage ratio is very high. This stream of research illustrates that the composition of liabilities really matters in debt valuation and point out the impacts from different perspectives. Nonetheless, these researches didn't consider the stationary leverage ratio convention which is widely adopted in practice. However, zero short-term credit spread has been strongly rejected empirically, e.g.: Fons (1994), Jones, Mason, and Rosenfeld (1984), and Sarig and Warga (1989), although some researches have been devoted in solving this problem, but still under a quite limited structure. Besides, existing structural models are documented that they either overestimate true credit spread (e.g: Leland and Toft (1996), CDG (2001)), or underestimate true credit spread (e.g.: Merton (1974)). Therefore, a way to reduce long term credit spread, and increase short term credit spread is a key issue for improving existing structural model (See Eom, Helwege, and Huang (2003)).

As it was pointed out in Huang and Huang (2003), the risk premium for the firm's asset may follow a counter-business cyclical process. It implies that asset risk premium is higher at the bottom of business cycle, and lower at the top of business cycle. However, the property can't be directly functioned into the pricing of corporate bond because the risk premium is removed in risk-neutral pricing. It only indirectly affects the price of corporate bond through affecting the true default probability. Bansal and Zhou (2002) uses Efficient

Method of Moment to estimate the short term interest rate process, and proposed that two factor regime switching model is the most perfectly fitted specification for interest rate model. In addition, they also show that the regime switching property is consistent with business cycles. Therefore, modeling interest process in regime switching setting can directly attack the dynamics of firm's asset in risk neutral world; meanwhile, also preserve the impact of business cycle on the firm's asset process.

3. The Basic Model

In this paper, we follow the spirit of structural approach to construct a corporate bond valuation model which keeps stationary leverage ratio and has options liabilities. Besides, we also consider a stochastic interest rate environment which is consistent with Collin-Dufresne and Goldstein (2001). This setting modifies CDG (2001)'s model, and expect to overcome the zero short term credit spread problem by including additional option liabilities. Before the introduction of model, a few assumptions must be given:

Assumptions:

1. We follow the conventional assumption of structural model, assuming that firm's asset process is not affected by the firm's capital structure. Following CDG (2001), we assume the firm always targets a fixed range of leverage ratio by issuing equity to amortize outstanding debt or issuing new debt in capital market.
2. For brevity, we assume the firm only has debt and a written option in its liability. In written option position, we follow the assumption of Cao and Wei (2001), Episcopos (2004) that the firm only writes European style call option, and the written European call has equal seniority as the debt about claims on the final assets of the firm in the event of bankruptcy.
3. In the market, there are three uncertainties: firm value, underlying asset of the written call option, and interest rate. The detail process of those uncertainties will be defined below
4. For simplicity, we ignore the reorganization procedure after filing for bankruptcy, and assume that once the firm defaults, the firm's asset will be liquidated to pay outstanding claims.

Consider a firm that has liabilities which is composed of debts and a short position on a call option written on another firm's stock with strike price stk and maturity T_c. Following traditional assumptions, we assume firm value-dynamics under risk neutral measure Q follows geometric Brownian motion:

$$\frac{dV_t}{V_t} = (r - \delta_V)\,dt + \sigma_V dz_1^Q(t) \tag{1}$$

where r is the risk free rate, δ_V is the payout ratio, σ_V is the volatility, $dz_1^Q(t)$ is the Brownian motion under risk-neutral measure Q. In addition, the log firm value process, y_t under risk-neutral measure Q, can be represented as

$$dy_t = \left(r_t - \delta_V - \frac{\sigma_V^2}{2} \right) dt + \sigma_V dz_1^Q(t) \tag{2}$$

where r is the risk-free rate and $dz_1^Q(t)$ is the Brownian motion under risk-neutral measure. For targeting leverage ratio at a fixed range, the firm has to dynamically adjust its outstanding debt balance by issuing new debt to amortize or issuing new debt in corporate bond market. In previous research, like Merton (1973), Longstaff and Schwartz (1995), they assume that the default boundary is constant or equal to the face value of outstanding debt. On another hand, some researches, like Cao and Wei (2001), assume default boundary is a proportion of the only zero-coupon bond value. Here, we follow the setting of CDG (2001), assuming that default boundary for debt is a proportion of outstanding debt value. Due to the stochastic property of firm asset value and the linkage between firm value and debt value in this model, the default boundary for bond *per se* is stochastic. Besides, we also follow CDG (2001), assuming that target leverage ratio is also affected by market interest rate. In their paper, stationary leverage ratio refers to that the firm keeps a target leverage ratio, and the target leverage ratio is influenced by interest rate level. Therefore, the log default boundary process for bond can be represented as:

$$dk_t = m\left[y_t - \nu - \phi\left(r_t - \theta \right) - k_t \right] dt \tag{3}$$

Note that the log default boundary stated here is the default boundary used in the CDG (2001) paper. Here, we extend this setting a little bit more and implicitly assume the solution for equation (3) is outstanding debt value; later, we will define another default boundary level which is used in this paper. Here we assume the interest rate process is governed by Vasicek (1977) dynamics which can be represented as:

$$dr_t = \kappa\left(\theta - r_t \right) dt + \eta dz_2^Q(t) \tag{4}$$

where $dz_2^Q(t)$ is a Brownian motion under risk-neutral measure. The explanation about this setting is intuitive. The log default boundary process is a mean reverting process with m as the reverting speed, and $y_t - \nu - \phi\left(r_t - \theta \right)$ as the log-term drift level. The long-term drift level is concerned with log firm value, y_t ,and interest rate process, r_t. Here, ϕ is the diverting speed from long term interest rate, θ and ν is the adjusting term. $y_t - \nu - \phi\left(r_t - \theta \right)$ links firm value with debt default boundary; meanwhile, it also incorporates the impact of interest rate which adjusts the relationship between firm value and debt default boundary. By this setting, we are able to determine the bond's default boundary, K_t, which is also the solution of (3) times a degree of protection for debt liability, φ_1. Here, K_t is also the aggregate debt value for the firm at time t. In another aspect, the firm has an outstanding European call option with maturity T_c and the dynamics of the underlying asset under risk-neutral measure is given as:

$$\frac{dS_t}{S_t} = \left(r_t - \delta_S\right)dt + \sigma_S dz_3^Q(t) \tag{5}$$

In order to be solvent, the firm not only needs to keep the asset value above the default boundary of debt, also needs to satisfy the solvency requirement of written European option. Some of previous researches only consider the solvency requirement of option on the maturity date, e.g.: Klein and Inglis (2001), Episcopos (2004); however, in practice, the firm might default before option's maturity date because the firm is unable to satisfy the requirement of bond covenant. In some cases, if the option writer is a regulated financial institution, it is required to satisfy the capital requirement for OTC derivatives positions by implementing Value at Risk approach; therefore, the firm needs to frequently evaluate its option positions and prepare some solvency capital for that. On another hand, if the option is not an OTC product, instead, is a product traded in an exchange, it is natural that a proportion of the option's market value must be maintained for margins requirement. Hence, we modify the assumptions of Cao and Wei (2001), but we only follow the assumption that the default boundary of the option liability is based on the option's market value. Thus, the default boundary for option liability can be represented as a degree of protection for the option liability, φ_2, times option's market value. For brevity, C_t also can be regarded as a call option value under stochastic interest rate and the stochastic parts of S_t and r_t are correlated, i.e. $dz_3^Q(t)dz_2^Q(t) = \rho_{Sr}dt$. It has a closed form and can be represented as following (about the derivation, please see Appendix A):

$$C_t = S_t e^{-\left(\delta_S + \sigma_{T-t}^{*}\sigma_S\rho_{Sr}\right)\left(T_c - t\right)} N\left(d_1\right) - strkB\left(t,T\right)N\left(d_2\right)$$

Since the discounted price under risk neutral measure is consistent with the true price, therefore, we can use this formula to determine the default boundary for option position when we do simulation in next section. In summary, before option mature, the default boundary dynamics of the firm is governed by (3) and (5); on option maturity, the default boundary is governed by debt's default boundary plus the exercise value of the written call. After option's maturity, the default boundary is only governed by bond's default boundary dynamics. Therefore, we can represent the default boundary, $v\left(t\right)$, as:

$$v(t)=\begin{cases}\varphi_1 K_{\tau^*}+\varphi_2 C_{\tau^*} & \text{if } \tau^*<T_C\\ \varphi_1 K_{T_C}+\left(S_T-stk,0\right)^+ & \text{if } \tau^*=T_C\\ \varphi_1 K_{\tau^*} & \text{if } \tau^*>T_C\end{cases}$$

where, τ^* is the default time. This setting is consistent with Cao and Wei (2001) except that the default boundary of debt is linked to asset value and interest rate, rather than just the theoretical value of zero coupon bonds. It should be noted that when at option maturity, the default boundary for option is not a proportion of the option value; instead, it is the exercise value of the option because the issuer is obligated to finish the settlement.

Since the default boundary is determined, the distribution rule after default must be decided in order to clarity the claim rights of every party after default. For simplicity, we only consider the case that the written option is OTC traded because the exchange traded option is concerned with the guarantee of exchange; therefore, it even complicates the claim structure. In this case, once the firm defaults and satisfies events of default defined in ISDA master contract (2003), the defaulting party is obligated to settle with non-defaulting party by the early termination payment terms defined in ISDA master contract. In practice, it is most likely to settle the unexpired call option at market quote convention; meanwhile, the defaulting firm also needs to compensate another party the loss occurred due to its default. Therefore, this distribution rule is more like the ”Market Value Rule” described in Cao and Wei (2001), and it is summarized in Table 3. In Table 3, the payoffs of bondholder, equity holder, and option **holder are listed based on different scenarios. Also, different recovery rates are assigned to respective claim holders. In this table, we assume that when the firm defaults, its asset will be liquidated to pay the claims of bondholder and option holder. Since, in our setting, when default occurs, the total asset value is less than the claims of bondholders and option holders. Hence, nothing could be recovered for shareholder. This scheme is more flexible than the** one used in CDG model which only assumes constant recovery rate. Empirically, Altman (1989), Betker (1992), Franks and Torous (1994), and Altman and Bencivenga (1995) show the default recovery rates differ across bonds in cross-section and time-series. This evidence supports the stochastic recovery rate assumption used in this paper; however, against other models which take constant recovery assumption. Under this setting, it modifies Cao and Wei (2001)’s structure to narrow down the research target; meanwhile, it includes CDG’s stationary leverage assumption.

4. Extended Model

In the literature, several papers consider the unexpected shocks into the modeling of firm value process. Zhou (1997) compromises the feature of structural model and reduced model to come up with a model which includes the unexpected default into regular diffusion structure. In this paper, they found that the inclusion of jump factor can successfully resolve the short term zero credit spread problem, and also come out some credit spread pattern which is consistent with empirical studies, but can't be carried out by pure diffusion model. Huang and Huang (2003) extended the work of Zhou (1997) and model the jump size as double exponential process. However, in Huang and Huang (2003), only constant asset risk premium and constant default boundary are considered. Besides, Merton (1976) also shows that financial asset returns usually follow a jump-diffusion process, rather than just diffusion processes. In addition, he also applies jump-diffusion model to option pricing and derives a close form solution for European option. In this section, we follow Zhou (1997) and Merton (1976) to let the firm value and underlying asset of the firm's option liability follow jump-diffusion processes. Moreover, the stationary leverage ratio assumption is still preserved which is ignored in Zhou (1997), and Huang and Huang (2003). However, the analytic solution of this setting is still unavailable; therefore, a Monte Carlo experiment will be conducted to examine the impacts. In this structure, (1) can be rewritten as:

$$\frac{dV_t}{V_t} = \left(r - \delta_V - \lambda_V \zeta_V\right)dt + \sigma_V dz_1^Q(t) + \left(\Pi_V - 1\right)dY_V \tag{6}$$

where dY_V is a Poisson process with intensity parameter λ_V, Π_V is the jump amplitude and an *i.i.d.* log-normal random variable, such that $\ln \Pi_V$ follows

$$N\left(\mu_\pi^V, \left(\sigma_\pi^V\right)^2\right); \text{ and } \zeta_V = E\left[\Pi_V - 1\right] = e^{\left(\mu_\pi^V + \left(\sigma_\pi^V\right)^2/2\right)} - 1.$$

Similarly, the underlying asset of option liability also follows a jump diffusion process which can be represented as:

$$\frac{dS_t}{S_t} = \left(r - \delta_S - \lambda_S \zeta_S\right)dt + \sigma_S dz_3^Q(t) + \left(\Pi_S - 1\right)dY_S \tag{7}$$

where dY_S is a Poisson process with intensity parameter λ_S, Π_S the jump amplitude and is an *i.i.d.* log-normal random variable, such that $\ln \Pi_S$ follows

$$N\left(\mu_\pi^S, \left(\sigma_\pi^S\right)^2\right); \text{ and } \zeta_S = E\left[\Pi_S - 1\right] = e^{\left(\mu_\pi^S + \left(\sigma_\pi^S\right)^2/2\right)} - 1.$$

It should be noted that the jump impact on firm's asset value return or underlying asset return are assumed to be independent in our model. Usually, the unexpected jumps of asset and underlying asset are uncorrelated, but if the underlying asset of option is in the same industry as the firm or the underlying asset is issued by the firm's subsidiary, they are more likely to be correlated. For simplicity, in this simulation, we exclude this possibility, but the case of correlated jumps can be extended easily.

5. Model with Regime-Switching Interest Rate Process

Conventionally, the interest rate process in structural model is only modeled as one-factor interest rate model, e.g.: one-factor Vasicek or one-factor CIR. The regime-switching property of short interest rate has been proved in a few empirical studies, e.g.: Hamilton (1989), Garcia and Perron (1996), Bansal and Zhou (2002) and so on. As it was pointed out in Bansanl and Zhou (2002), the regime-switching process for interest rate is not only a statistical evidence; there are also economic reasons to believe that regime-switching is important to understanding the behavior of yield curve, e.g. business cycle has impacts on the whole structure of interest rate. Unfortunately, none of the existing literature considers the regime-switching interest rate process into credit modeling. In corporate bond pricing, the pricing job is done in risk-neutral world, and the asset risk premium is removed because of the change of measure. In Huang and Huang (2003), they calibrate their model to match the empirical cumulative default probability, and it also provides an indirect channel for the firm's asset risk premium process to take effects. In other words, if some additional calibrations are not carried out, it is hopeless to incorporate real economy effect, e.g.: business cycle or asset risk premium, which is counter business cyclical, into corporate bond pricing unless we directly model the interest rate process. Hence, we employ the estimations done by Bansal and Zhou (2002) and the well-fitted two factor regime switching Cox-Ross-Ingersoll interest rate model to calculate the credit spread again. In Bansal and Zhou (2002) model, transition matrix for regime switching, the first two moments, and the market price of risk for interest rate processes in each regime are estimated. Therefore, this estimation can help us to transform the process under real probability into risk neutral probability. In addition, Bansal and Zhou (2002) also show that the pattern of regime switching is consistent with business cycle. By incorporating regime-switching interest rate model, the impact of business cycle still can be preserved in the asset value process under risk neutral world; moreover, the default boundary is still influenced by interest rate process because it is driven by interest rate. Suppose the interest rate process follows a two-state Markov regime shifts, and the transition probability matrix of a Markov chain can be represented as:

$$\Theta = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} \tag{8}$$

where, π_{ij} represents the transition probability from state i to state j, and $\sum_{j=0,1} \pi_{ij} = 1$. With this Markov Switching specification, the interest rate process is surely influenced by the states, and relevant parameters change with states. Following Bansal and Zhou (2002), we can specify the two interest rate *factor processes* as following:

$$dx_t^i = \kappa_i^{CIR}\left(\theta_i^{CIR} - x_t^i\right)dt + \sigma_i^{CIR}\sqrt{x_t^i}dz_{2,i}(t) \qquad i=\{1,2\} \tag{9}$$

where index *i* represents the factor process identity. We can represents (9) with subscripts to show that interest rate process changes with states; also, we can re-write (9) in discrete form as:

$$x_{t+1}^i - x_t^i = \kappa_{S_{t+1},i}^{CIR}\left(\theta_{S_{t+1},i}^{CIR} - x_t^i\right)dt + \sigma_{S_{t+1},i}^{CIR}\sqrt{x_t^i}dz_{2,i}(t) \qquad i=\{1,2\} \tag{10}$$

By this specification, $\kappa_{S_{t+1}}^{CIR}$, $\theta_{S_{t+1}}^{CIR}$, $\sigma_{S_{t+1}}^{CIR}$ are all regime-dependent on state S_{t+1}, and the market price of risks for interest process, $\lambda_{S_{t+1}}^{CIR}$ is also regime-dependent on state S_{t+1}. By this setting, the short interest rate level at t+1 is also affected by the state at t+1, which illuminates the state-dependency property. In Bansel and Zhou (2002), they use Efficient Method of Moment (EMM) to estimate the monthly factors for this interest rate process and Markov Switching structure. Since all operations are working in risk-neutral world, (10) needs to be changed to risk-neutral measure, which can be represented as:

$$x_{t+1}^i - x_t^i = \kappa_{S_{t+1},i}^{CIR}\left(\theta_{S_{t+1},i}^{CIR} - \frac{\lambda_{S_{t+1},i}^{CIR}\sigma_{S_{t+1},i}^{CIR}}{\kappa_{S_{t+1},i}^{CIR}}\sqrt{x_t^i} - x_t^i\right)dt + \sigma_{S_{t+1}}^{CIR}\sqrt{x_t^i}dz_{2,i}^Q(t) \tag{11}$$

where, $dz_2^Q(t)=dz_2(t)\text{-}\lambda_{S_{t+1}}^{CIR}dt$. With these two factor process under risk neutral measure, by CIR (1985), Chen and Scott (2003), the nominal interest rate dynamics can be represented as the sum of those two factor processes, i.e.: $r_t=x_t^1+x_t^2$. Based on this property, a new Monte Carlo simulation can be conducted to compare with previous results.

6. Pricing Corporate Debt and Vulnerable option

Suppose a coupon bond with promised coupon payment C at dates t_j, where $t_N \equiv T_b$, where N is the number of periods from 0 to T_b, and a promised principal payment P. By LS (1995), CDG (2001), the price of coupon bond can be written as:

$$\begin{aligned} P^T\left(l_0, C_0\right) &= C\sum_{j=1}^{N} E^Q\left\{e^{-\int_0^{t_j} r_s ds}\left[\mathbf{I}_{(\tau^*>t_j)} + \varpi_{coup}^b\left(D_{\tau^*}, C_{\tau^*}\right)\mathbf{I}_{(\tau^*\le t_j)}\right]\right\} \\ &\quad + P\int_0^{T_b} E^Q\left\{e^{-\int_0^{\tau^*} r_s ds}\left[\mathbf{I}_{(\tau^*>u)} + \varpi^b\left(D_{\tau^*}, C_{\tau^*}\right)\mathbf{I}_{(\tau^*\le u)}\right]\right\} du \\ &= C\sum_{j=1}^{N} E^Q\left\{e^{-\int_0^{t_j} r_s ds}\left[1 - \left(1 - \varpi_{coup}^b\left(D_{\tau^*}, C_{\tau^*}\right)\right)\mathbf{I}_{(\tau^*\le t_j)}\right]\right\} \\ &\quad + P\int_0^{T_b} E^Q\left\{e^{-\int_0^{\tau^*} r_s ds}\left[1 - \left(1 - \varpi^b\left(D_{\tau^*}, C_{\tau^*}\right)\right)\mathbf{I}_{(\tau^*\le u)}\right]\right\} du \end{aligned} \tag{12}$$

where, $\varpi^{b}_{coup}\left[D_{\tau^*},C_{\tau^*}\right]$ is the recovery of coupons when the firm default, $\varpi^{b}\left[D_{\tau^*},C_{\tau^*}\right]$ is the recovery rate of principal. Equation (12) assumes that whenever the firm defaults, the firm will liquidate its asset and distribute the residual value to pay bondholder's principal. However, in LS (1995), CDG (2001), the principal payment is only paid on bond maturity date, even default happened before maturity date. About the coupon payment, equation (12) has the same assumption as LS (1995), CDG (2001), assuming that coupon is only paid on coupon payment date, and future coupon payments are lost when the firm defaults. In our model, the recovery rate of coupon and principal depend on the distribution rule, and the rule is related to debt value and call value at default. Therefore, this model not only relaxes the constant default boundary assumption, but also includes the stochastic recovery rate property into this setting. We follow the assumption in CDG (2001), assuming that the recovery rate of coupon is very close to 0 (See Helwege and Turner (1999)). Therefore, the coupon bond price can be represented as:

$$
\begin{aligned}
P^{T}\left(l_0,C_0\right) &= C\sum_{j=1}^{N} E^{Q}\left[e^{-\int_0^{t_j} r_s ds}\left[1-\mathbf{I}_{\left(\tau^*\le t_j\right)}\right]\right] \\
&\quad + P\int_0^{T_b} B\left(0,\tau^*\right)E^{T}\left[1-\left(1-\varpi^{b}\left[D_{\tau^*},C_{\tau^*}\right]\right)\mathbf{I}_{\left(\tau^*\le u\right)}\right]du
\end{aligned}
\tag{13}
$$

where, $E^{T}[\bullet]$ is the expectation under forward measure T, $B\left(0,\tau^*\right)$ is the risk-free zero coupon bond price under Vasicek process up to default time point τ^*, and the general form of this item can be represented as

$$
B(t,T) = e^{A^{(T-t)}\text{-}r_0 B^{(T-t)}} \quad \text{where, } B^{(T-t)} = \tfrac{1-e^{-\kappa(T_b-t)}}{\kappa},
$$

$$
A^{(T-t)} = \left(\tfrac{\eta^2}{2\kappa^2}-\tfrac{\alpha}{\kappa}\right)\left(T_b-t\right)+\left(\tfrac{\alpha}{\kappa^2}-\tfrac{\eta^2}{\kappa^3}\right)\left(1-e^{-\kappa(T_b-t)}\right)+\tfrac{\eta^2}{4\kappa^3}\left(1-e^{-2\kappa(T_b-t)}\right),
$$

and α is equal to $\kappa\theta$ plus market price of risk.

On another hand, the vulnerable option formula is similar to the above one, considering stochastic interest rate and default probability into valuation. The option price at time t can be represented as following expression when it is default free:

$$
C_t = E^{Q}\left[e^{-\int_t^{T_c} r_s ds}\left(S_{T_c}-strk\right)^{+} \mid \Im_t\right]
$$

where, $\Im_t$ is the filtration up to time t. When the call option is not default free, we can express option price as:

$$C^T\left(l_0,P_0\right)=B\left(0,T_c\right)E^T\left[\mathbf{I}_{\left(\tau^*>T_c\right)}\left(S_{T_c}-strk\right)^+\right]+\int_0^{T_c}E^Q\left[e^{-\int_0^{\tau^*}r_s ds}\varpi^c\left(D_{\tau^*},C_{\tau^*}\right)C_{\tau^*}\mathbf{I}_{\left(\tau^*\le u\right)}\right]du$$

where, $\left(S_{T_c}-strk\right)^+=Max\left(S_{T_c}-strk,0\right)$, $\varpi^c\left(D_{\tau^*},C_{\tau^*}\right)$ is the call option's recovery rate; again, it is stochastic and depends on default boundary and distribution rule. In the expression, we still assume that whenever the firm defaults, the recovered option *market value* is paid to option holder right after the firm defaults and liquidate its asset.

Although preliminarily the coupon bond value and vulnerable option value can be expressed as a discounted expectation, $B\left(0,T\right)$ times an expectation under forward measure T, but in our model, there are three Gaussian Markov processes (firm value process, short interest rate process, and stock price process) and it is not feasible to derive a closed form expression for the first passage time probability under forward measure T. In addition, these three uncertain factors also involve in the determination of recovery rate for principal in respective asset class. Therefore, it is more appropriate to use Monte Carlo simulation to compute debt value, vulnerable option value and credit spread under this structure.

7. Numerical Method

In this section, Monte Carlo simulation procedures are conducted to carry out bond value, vulnerable option value and credit spread. In addition, those results are also compared with the values carried out by baseline models and see the performance. In our model, the default time point, τ^*, not only determines the default probability, but also determines the payoffs for bondholders and option holders when default occurs. Therefore, in order to match the time point for calculating default payoffs and the time point for default event, we conduct a simulation based on the "continuous monitoring rule", which implies that whenever default happens before the end of period, the payoffs for bondholders and option holders are paid out immediately. Based on this rule, the recoveries for bondholders and option holders depend on the distribution rule at the default time point. The simulation steps can be summarized as following:

Steps: (Simulation under stationary leverage ratio and option liabilities)

1. Use Cholesky decomposition[2] to simulate three correlated standard normal random variables, $\left[\varepsilon_1,\varepsilon_2,\varepsilon_3\right]$.

[2] Cholesky decomposition is a procedure for decomposing a symmetric positive definite matrix to a product of one upper triangular matrix, L, and one lower triangular matrix, U, where $L=U'$. The application of Cholesky decomposition on simulation, readers can refer Hull (2000), p408-409.

2. Discretize the random processes of firm value, debt value, interest rate level, and option's underlying value as below

$$\Delta V_t = V_t \left(r - \delta_V\right) \Delta t + \sigma_V V_t \sqrt{\Delta t} \varepsilon_1 \tag{14}$$

$$\Delta k_t = m\left[y_t - \nu - \phi\left(r_t - \theta\right) - k_t\right] \Delta t \tag{15}$$

$$\Delta S_t = S_t \left(r_t - \delta_S\right) dt + \sigma_S S_t \sqrt{\Delta t} \varepsilon_2 \tag{16}$$

$$\Delta r_t = \kappa\left(\theta - r_t\right) \Delta t + \eta \sqrt{\Delta t} \varepsilon_3 \tag{17}$$

3. Simulate those random processes in weekly basis, i.e. $\Delta t = 1/52$, and at the beginning of every period, compare the simulated value with the assumed bankruptcy boundary
4. If the value is still above the default boundary, the process keep going to the next step; if the value is below default boundary, based on the distribution rule when the firm defaults, respective payoffs are assigned to bondholders and option holders, and this simulation is terminated.
5. Continue the simulation up to bond's maturity date, and discount all payoffs by the given spot interest rate process to get the present values of contingent claims for bond holder and option holder.
6. Start another round of simulation, and follow the pervious five steps.
7. Sum up all simulated present values of contingent claims and divide them by the number of simulations.

On another hand, we also simulated the relevant values when firm value and stock price follow jump-diffusion processes. For clarity, the simulation procedure can be stated again as below:

Steps: (Simulation under stationary leverage ratio, option liabilities, and jump-diffusion process)

1. Use Cholesky decomposition to simulate three correlated standard normal random variables, $\left[\varepsilon_1, \varepsilon_2, \varepsilon_3\right]$.
2. Discretize the random processes of firm value, debt value, interest rate level, and option's underlying value as below

$$\Delta y_t = \left(r_t - \delta_V - \lambda_V \zeta_V - \frac{\sigma_V^2}{2}\right) \Delta t + \sigma_V \sqrt{\Delta t} \varepsilon_1 + h_V \pi_V \tag{18}$$

where, $\pi_V \sim N\left(\mu_\pi^V, \left(\sigma_\pi^V\right)^2\right)$, and $h_V = \begin{cases} 0 & \text{with prob. } 1 - \lambda_V \Delta t \\ 1 & \text{with prob. } \lambda_V \Delta t \end{cases}$, the simulation for normal random variable, π_V, can be easily carried out by using the build-in function of MATLAB™. The Bernoulli random variable, h_V , can be carried out by numerical procedure which is stated in Appendix B. Other processes are as following:

$$\Delta k_t = m\left[y_t - \nu - \phi\left(r_t - \theta\right) - k_t\right]\Delta t$$

$$\Delta r_t = \kappa\left(\theta - r_t\right)\Delta t + \eta\sqrt{\Delta t}\varepsilon_3$$

$$\Delta \ln(S_t) = \left(r - \delta_S - \lambda_S\zeta_S - \frac{\sigma_S^2}{2}\right)dt + \sigma_S\sqrt{\Delta t}\varepsilon_2 + h_S\pi_S \tag{19}$$

where, $\pi_S \sim N\left(\mu_\pi^S, \left(\sigma_\pi^S\right)^2\right)$, and $h_S = \begin{cases} 0 & \text{,with prob. } 1\text{-}\lambda_S dt \\ 1 & \text{,with prob. } \lambda_S dt \end{cases}$

In this case, we assume the underlying asset of option liability also follows jump-diffusion process. We apply the pricing concept of Merton (1976), combined with the closed form solution of call option under stochastic interest rate. A closed form solution for European call option under stochastic interest rate and jump-diffusion process can be obtained and it is given in Appendix. Although we have closed form solution for the call option, the initial underlying asset price still needs to be simulated for the initial input of that closed form solution. Therefore, we still need to use expression (19) to simulate underlying asset price path.

3. Simulate those random processes in weekly basis, i.e. $\Delta t = 1/52$, and at the beginning of every period, compare the simulated value with the assumed bankruptcy boundary
4. If the value is still above the default boundary, the process keep going to the next step; if the value is below default boundary, based on the distribution rule when the firm defaults, respective payoffs are assigned to bondholders and option holders, and this simulation is terminated.
5. Continue the simulation up to bond's maturity date, and discount all payoffs by the given spot interest rate process to get the present values of contingent claims for bond holder and option holder.
6. Start another round of simulation, and follow the pervious five steps.
7. Sum up all simulated present values of contingent claims and divide them by the number of simulations.

In the simulation, we conduct 10,000 times simulation for every case. We preliminarily set the duration of option contract at 6 months, but we allow the term of corporate bond increase from 4 years to 20 years. In the yield spread calculation, we follow the procedure of CDG (2001), compute the yield to maturity for defaultable bond and undefaultable bond, then take the difference as the yield spread. Obviously, it is a more robust measure for yield spread than the one defined as the difference between the yield to maturity of defaultable bond and risk free treasury rate. It is because the first one calculates the spread based on the same ground, and this way can exclude the impacts of other factors on yield spread, like tax, liquidity. Hence, this measure purely reflects default risk. If we use the second one, which is the difference of the YTM for defaultable bond and YTM for risk free treasury yield, this measure may contain the impacts of factors other than default risk.

In this simulation, the baseline example follows the parameter settings given in CDG (2001) except the correlation coefficients between interest rate process and firm value process and option related parameters. Instead, we adopt the correlation coefficient assumptions from Cao and Wei (2001) for the three stochastic factors. In these parameter assumptions, we should note that the final recovery rate for debt holder is $\varphi_1 \gamma_1 = 0.44$, which is consistent with the CDG (2001), and LS (1995). The reason for this ratio is that when the firm defaults, it must have reached the default boundary which is $\varphi_1 K_{\tau^*}$ for bond holders; meanwhile, when default happened, there are some bankruptcy costs and make bond holder lose some contingent claim value. Therefore, in the end, bond holder only can recover $\varphi_1 \gamma_1$ of original debt value. In the baseline case, we assume $\varphi_1 = 1$, hence, γ_1 is an approximation for the recovery rate ϖ^b applied to coupon bond's principal. In the extended model, we follow Zhou (1997)'s assumptions, and all relevant parameters are given in table 1

Regarding the regime-switching model, the simulation steps are very similar to the previous two simulations. The only difference is the simulation about regime switching structure. When the regime structure changes, relevant parameters for interest rate factor processes need to be changed because it is in different regimes. In addition, the market price of risk for each factor also needs to be changed in order to match the economic changes which are caused by regime changes. In brief, the simulation steps can be summarized as following:

Steps: (Simulation under stationary leverage ratio, option liabilities, and regime-switching interest rate process)

1. Repeat step1 and step 2 in previous simulations. The interest rate process follows the one given in expression (11). However, in Bansel and Zhou (2002), it was implicitly assumed that the uncertainty terms of interest rate process in either regime are not correlated. Following this assumption, we don't specify a correlated structure between the Brownian motions in those two regimes.
2. At the beginning of every period, determine what state of regime the interest process will be in this period by simulation. This simulation is as following
3. Suppose the state now is state 1, the transition structure is $\begin{cases} \text{state } 0 \quad \text{with prob. } \pi_{10} \\ \text{state } 1 \quad \text{with prob. } \pi_{11} \end{cases}$,

where $\pi_{10} + \pi_{11} = 1$; this structure is similar to a Bernoulli random variable.

4. Generate a uniform random variable by MATLAB™. If this number is greater than π_{11}, the next state will transit to state 0; if this number is less than or equal to π_{11}, the state will still be state 1. When the state now is 0, the same rule applies.
5. When state changes, related parameters for interest rate process need to be changed to the values correspondent to that state.
6. Simulate those random processes in *monthly basis* i.e. $\Delta t = 1/12$, and at the beginning of every period, compare the simulated value with the assumed bankruptcy boundary. Here, the time increment is different from previous simulation in order to match the estimated parameter values given in Bansal and Zhou (2002). In that paper, they use *monthly* treasury yield data to estimate parameters, as pointed out in their paper, the estimated parameters are monthly basis.
7. If the value is still above the default boundary, the process keep going to the next step; if the value is below default boundary, based on the distribution rule when the firm defaults, respective payoffs are assigned to bondholders and option holders, and this simulation is terminated. It should be noted that the option default boundary in this simulation still follows the settings given in previous simulation, i.e.: the closed form solution for option liability is obtained under stochastic interest rate process following Vasicek process. Based on this setting, it is more reasonable to compare with the results obtained from previous simulations. In addition, the closed form solution for option position is a theoretical value, and it can be calculated by using any interest rate process only if it is reasonable. Second, a closed form solution for European option under regime-switching CIR interest rate process is not available yet. Hence, we just keep original setting to calculate option's default boundary value.
8. Continue the simulation up to bond's maturity date, and discount all payoffs by the given spot interest rate process to get the present values of contingent claims for bond holder and option holder.
9. Start another round of simulation, and follow previous five steps.
10. Sum up all simulated present values of contingent claims and divide them by the number of simulations.

9. Results

Following the steps listed in the last section, the debt values, coupon bond values, vulnerable option value and credit spread on obtained by using Monte Carlo simulation. In the simulations, the parameter values are based on table1 and table 2 except that the correlation coefficient between firm asset and interest rate uncertainty is set to -0.25, instead of -0.2

which is used in CDG (2001) for comparison purpose. We take the results with the parameters used in CDG (2001) as the benchmark to discuss how different model settings improve the performance of CDG (2001) in explaining credit spread. In addition, we will also discuss the impact of model settings on the value of vulnerable options.

9.1 Credit Spreads

In Figure 1, we graph the credit spreads of investment grade bond (leverage ratio=15%) and junk bond (leverage ratio=65%) under different models. In investment grade bond, the Jump-diffusion model (Jump, hereafter) shows higher credit spreads than the credit spreads of the other two models. The credits spread of regime-switching model (RS, hereafter) has lowest credit spreads. When the bond is in investment grade, Jump model can increase the credit spread for a short term bond and fix the zero credit spread problem. This observation is consistent with Zhou (1997) and the empirical findings documented in Fons (1994), Jones, Mason and Rosenfeld (1984), Sarig and Warga (1989). However, from this Figure1.1, we know that RS model is unable to generate positive credit spread for short term bond, but indeed, RS model can produce a smaller credit spread for long term bond than CDG model in low leverage ratios. On another hand, when it is a junk bond (See Figure 1.1), the credit spreads under Jump Model still leads among the credit spreads of these three models. In this case, RS model still generate a smaller credit spread than CDG model no matter how long the maturity is. This feature enable RS model to generate a better fit than CDG model for credit spread in long term bond in terms of the suggestion of Eom, Helwege, and Huang (2003). As it was point out in Eom, Helwege, and Huang (2003), a way to reduce long term credit spread, and increase short term credit spread is a key issue for improving existing structural model. Therefore, RS model can function to improve the later drawback.

In Figure 2, we figure the credit spreads of investment grade bond and junk bond under different models with option liabilities. In this Figure2.1, we assume initial underlying asset value for option liability is 100, and the strike price of this option liability is 80. Initially, the firm has option liability which is around 20% of initial firm asset value. When the bond is an investment grade bond, the credit spreads generally are upward with maturity. Because of the option liability, the credit spreads are enhanced when maturity is less than 6 years with the assumption that the maturity of option liability is only 6 month. Although the option liability is in a big proportion of firm value, the firm is very unlikely to default in a short term because the firm value is still distant from default boundary; hence, option liability only has limited impacts on the firm’s default probability and credit spreads in a short term. Regarding the junk bond case, the credit spreads under these three models are all downward sloping, which is different from the results of CDG (2001), but consistent with Zhou (1997). In CDG (2001), the credit spread is always upward sloping without option liability no matter it is junk bond or investment grade bond. However, Helwege and Turner (1995) empirically argue that credit spread curves of many B-rated bonds are upward-sloping, but some B-rated bonds really have downward-sloping credit spreads. The inclusion of option liability somehow makes theoretical credit spread more consistent with empirical observations. It is noted that the credit spread of CDG and the credit spread of Jump are close in junk bond. When a firm is highly levered, default is mostly due to the diffusion part of firm value process, rather than the jump part. Hence, the unexpected shock doesn't have such a high marginal impact to default probability as that in low leverage case. In addition, the option liability is also valued by using jump-diffusion model which reduces the value of call option and default boundary. Accordingly, the increase of credit spread due to the unexpected impact of firm value is partially offset by the decrease of credit spread which is brought by the reduction of option liability value. Hence, the negative impact of jump is not as significant as that in investment grade bond. In junk bond figure (see figure 2.2), credit spreads given by RS model are still less than that of CDG model. It shows that RS model is still able to resolve the overestimation problem of credit spread in CDG model even when the firm has option liability and high leverage ratio, but unable to resolve the short term underestimation problem for credit spread in this case. Moreover, in figure 2.2, we can observe that the credit spreads of those cases with option liability are much higher than the credit spread of base case. It is due to that high leverage ratio and option liabilities have led to relatively high default probabilities; the inclusion of jump part makes the firm even more likely to hit default boundary and default its debt. Therefore, it leads to such a great difference between the credit spreads of base case and the credit spreads of other cases in figure 2.2.

In Figure 3, we compare credit spreads with different models and different liability structures. The purpose of this table is to see how RS model or Jump model can improve the drawbacks of credit spreads in CDG base case after we include different levels of option liability. In investment bond case (see figure 3.1), credit spreads given by Jump model is always larger than the credit spread in CDG base case no matter the inclusion of option liability. Accordingly, Jump model is helpful in resolving short term credit spread underestimation problem in CDG model, but in overall level, the credit spreads generated by Jump model is strictly greater than CDG model, which implies that empirically, Jump model overestimate credit spreads even more severely than CDG model, and hopelessly to resolve the drawbacks embeded in CDG model. On another hand, RS model in investment grade bond performs well and help resolving the drawbacks of CDG model. In figure 3.2, we can see that RS model after including option position can generate a higher short term credit spread and a relative lower long term credit spread compared with the credit spread in CDG base case. The result is exactly the solution for the drawbacks we have been mentioning. Therefore, we can conclude that in investment grade bond, or we can say a reasonable low leverage bond, the RS model, which combines stationary leverage ratio, option liabilities, and regime-switching interest rate process can have a better empirical performance based on the studies of Eom, Helwege, and Huang (2003). In another aspect, in junk bond (see figure 3.2), the credit spreads of RS model with option liability are larger than credit spreads in CDG base case. In high leverage, RS model with option liability can't play a role in reducing long term credit spread because in high leverage, even a small value of option liabilities can greatly increase the default probability. The Jump model also overshoots the credit spread compare with CDG base case. Hence, the inclusion of option liability in Jump or RS model is a theoretical reasonable setting in high leverage case, but may not be empirically valid. In general, the problem of CDG model in high leverage case is the overestimation of credit spread in all maturities. The credit spread underestimation problem in short term bond is not severe in this case. Based on this observation, when it is a junk bond, or we can say a bond its issuer has high leverage ratio, it is feasible to use ordinary RS model without any option liability to model the credit spread of junk bond, and this setting can effectively resolve the credit spread overestimation problem caused by CDG model. However, in our numerical results, even the option's exercise price is 100; the option value still amounts to around 6.5, which is around 6.5% of initial firm value. For theoretical robustness, it may be still feasible to put option liability into RS model and generate reasonable credit spread patterns only if the option liability is not too much.

Figure 4 graph the credit spreads under different models and different initial interest level. The existence of option liabilities result in a "smile" type credit spreads pattern because the short term option liabilities greatly enhance the default probability in a short run. It has been documented in Merton (1974), LS (1995), CDG (2001) that interest rate level and credit spread have inverse relationship. Longstaff and Schwartz (1995) explain this relationship based on the effect of interest rate on risk neutral drift term. We also can think risky debt is a risk-free debt plus a shorting of put option. When the interest level is high, put option level decreases; then risky debt value increases, and leads to smaller credit spread. Empirically, Collin-Dufresne, Goldstein, and Martin (2001), Davies (2004) support the inverse relationship results. However, Morris, Neale and Rolph (1998) obtain the opposite result, supporting the positive relationship between interest rate level and credit spread. In figure 4, we got a consistent result with CDG (2001), showing an inverse relationship between interest level and credit spread. In our model, it has option liabilities, which is also affected by interest rate level. Due to the fact that call option value is positively correlated with interest rate level, therefore, the default boundary for option liabilities increases. In figure 4, we can see that sometimes, the credit spreads in high interest rate level are higher than the credit spreads in lower interest rate level. This is because the positive impact of credit spread which is brought by the increase of option liabilities value dominates the negative impact of credit spread which is due to the increase of risk neutral drift rate.

9.2 Vulnerable Option, Defaultable Debt, and Defaultable Coupon Bond

Table 4 shows debt value, option value and coupon bond value with and without default risk. In this table, the firm doesn't have option liabilities. The vulnerable option (defaultable option) listed in this table is the option whose default depends on a firm, but when the firm defaults, nothing can be recovered for this vulnerable option. For example, investment bank may issue a European call option, and its underlying asset is the stock of company X. There is another company B decides the default of this call option. Whenever the company B defaults, the call option contract terminates, and nothing can be recovered. The undefaultable option values in this table are calculated based on the closed form solution listed in Appendices. It should be noted that the closed form solution for call option under jump-diffusion process is based on Merton (1976), and the volatility term is corrected by the jump impact. Because of this difference, in this table, we can observe that in Jump Model, default option value is higher than undefaultable option value. Although, Cao and Wei (2001) claim that vulnerable option value sometimes can be greater than regular option value, which was against by Episcopos (2004). After we conduct a simulation procedure which is consistent with the simulation for defaultable option in Jump Model, we find that defaultable option value is still less than undefaultable option value for compensating default risk. In other cases, the results are as expected; the default option values are always less than undefaultable option values. In table 4, the effect of leverage on the vulnerable option value is not very significant because the maturity term of the option is short and the option doesn't increase the default boundary in this case. In table 5, the firm has option liabilities and the option position greatly increase the default boundary especially in high leverage case. Hence, in table 5, we can see a more significant difference between defaultable option and undefaultable option values. In this case, vulnerable option values are still less than undefaultable option values.

The debt values in table 4 and table 5 are aggregate values; the coupon bond values in these two tables are calculated by setting coupon equal to 0.75, and principal equal to 1. The coupon bond values are computed by the formulas in section 6. In table 4, we can see that defaultable debt values and coupon bond values are less than the ones without default risk. The default premium (the difference between the prices of undefaultable asset and defaultable asset) is greater when the firm uses more leverage, reflecting that higher leverage increases default probabilities. In table 5, the inclusion of option liabilities increases the default boundary, hence, increases default probabilities, and reduces debt values and coupon bond values even more than the reduction implied in table 4. In these tables, the default risk premium implicit in jump-diffusion model is not as significant as we think. Although the jump part in the firm's asset process increases default risk, but the jump part of the option's underlying asset decreases option liability value and default boundary, inducing the decrease of default probability. Therefore, these two effects offset, and end up with an insignificant impact of default premium.

10. Conclusions

Although several credit risk models have been proposed, but none of them combine stationary leverage ratio property (e.g.: CDG (2001)) and the firm's off-balance sheet financing (e.g.: Cao and Wei (2001)) together to explain credit spread or other contingent claim values. This paper uses simulation approaches to discuss the impacts of stationary leverage ratio and option liabilities together on credit spread, vulnerable option value, debt value and coupon bond value. Furthermore, we extend the model to include jump-diffusion process into the firm's asset process and option's underlying asset process. We find that it can increases credit spread most significantly and lower contingent claim values, somehow, not as significant as we expect. This modeling is consistent with Zhou (1997) which contains unexpected random shocks, but empirically, jump-diffusion setting may overestimate credit spread even more severely.

In addition, we modified the interest rate process used in modeling to regime-switching process for directly including business cyclical impact into risk-neutral drift rate. Empirically, the improvement has been validated by Bansal and Zhou (2002). In our study, the inclusion of regime-switching property can effectively decrease credit spread. If we combine regime- switching property and option liabilities together, in low leverage ratio, it can increase short term credit spread and decrease long term credit spread relative to the prediction of CDG model. It is the expected improvement as pointed out in Eom, Helwege, and Huang (2003)'s empirical study. However, in high leverage case, it is better to use regime-switching model without option liabilities to prevent from overestimation problem compared with CDG model. It is due to that when leverage ratio is high; credit spread is highly sensitive to the addition of option liabilities, and it will easily cause overestimation of credit spread. We also examine the relationship between interest rate level and credit spread. Our results are consistent with CDG (2001), LS (1995) even after including option liabilities.

Regarding vulnerable option value, it has consistent relationship with credit spread because credit spread is a proxy for default risk and default premium, which is reflected in the difference between undefaultable and defaultable contingent claim values. As expected, jump-diffusion model leads to the greatest discount of contingent claims, followed by CDG model, and then regime-switching model. In high leverage, the discount is even more severe because the default probability is highly sensitive when default boundary is high. It also implies that the ignorance of option liabilities may lead to serious overestimation of vulnerable option value.

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A. Appendix

We have the following two processes for the underlying asset process and short interest rate:

$$\frac{dS_t}{S_t} = \left(r_t - \delta_S\right)dt + \sigma_S dz_3^Q(t) \tag{A−1}$$

$$dr_t = \kappa\left(\theta - r_t\right)dt + \eta dz_2^Q(t) \tag{A−2}$$

$$dz_3^Q(t)dz_2^Q(t) = \rho_{Sr}dt \tag{A−3}$$

by Theorem 6.4.3 in Chap6, Shreve (2004), the discounted bond price can be represented as :

$$B\left(t,T\right) = e^{A^{(T-t)} - \eta B^{(T-t)}} \tag{A−4}$$

where, $B^{(T-t)} = \frac{1-e^{-\kappa(T_b - t)}}{\kappa}$, $A^{(T-t)} = \frac{\eta^2}{2}\int_t^T \left(\frac{1-e^{\kappa(T_b - s)}}{\kappa}\right)^2 ds - \kappa\theta\int_t^T \left(\frac{1-e^{\kappa(T_b - s)}}{\kappa}\right)ds$.

Furthermore, $A^{(T-t)}$ can be represented as:

$$A^{(T-t)} = \left(\frac{\eta^2}{2\kappa^2} - \frac{\alpha}{\kappa}\right)\left(T_b - t\right) + \left(\frac{\alpha}{\kappa^2} - \frac{\eta^2}{\kappa^3}\right)\left[1 - e^{-\kappa(T_b - t)}\right]$$
$$+\frac{\eta^2}{4\kappa^3}\left[1 - e^{-2\kappa(T_b - t)}\right]$$

where, α is equal to $\kappa\theta$ plus constant market price of interest rate risk.

Under measure Q , the stochastic differential equation (S.D.E) for $B\left(t,T\right)$ can be represented as:

$$dB\left(t,T\right) = r_t B\left(t,T\right)dt + \sigma_{T-t}^* B\left(t,T\right)dz_2^Q(t) \tag{A−5}$$

where, $\sigma_{T-t}^* = \frac{\eta}{\kappa}\left[1 - e^{-\kappa(T-t)}\right]$, furthermore, we can transform (B-1) to the process under forward measure T[3], and we can represent it as:

$$dS_t = \left(r_t - \delta_S - \sigma_{T-t}^*\sigma_S\rho_{Sr}\right)S_t dt + \sigma_S S_t dz_3^T(t)$$

and, the forward price at time T_c can be represented as:

$$\frac{S_{T_c}}{B\left(T_c,T_c\right)} = \frac{S_t}{B\left(t,T_c\right)}e^{-\left(\delta_S + \sigma_{T-t}^*\sigma_S\rho_{Sr} + \frac{1}{2}\sigma_S^2\right)(T_c - t) + \sigma_S z_3^T(T_c - t)}$$

where, $For_S\left(t,T_c\right) = \frac{S_t}{B\left(t,T_c\right)}$ is the forward price from t to T_c. Now, by Theorem 9.4.2 in Shreve (2004), the call option price under stochastic interest rate can be represented as:

$$C_t = S_t e^{-\left(\delta_S + \sigma_{T-t}^*\sigma_S\rho_{Sr}\right)(T_c - t)}N\left(d_1\right) - strkB\left(t,T\right)N\left(d_2\right)$$

[3] About the introduction of forward measure, please refer to Chap 9, Shreve (2004).

where, $N(\bullet)$ is c.d.f. of standard normal distribution, and

$$d_1 = \frac{\ln\left(\frac{For_S(t,T_c)}{strk}\right) - \left(\delta_S + \sigma^*_{T-t}\sigma_S\rho_{Sr} - \frac{1}{2}\sigma_S^2\right)(T_c - t)}{\sigma_S\sqrt{T_c - t}}, \text{ and } d_2 = \frac{\ln\left(\frac{For_S(t,T_c)}{strk}\right) - \left(\delta_S + \sigma^*_{T-t}\sigma_S\rho_{Sr} + \frac{1}{2}\sigma_S^2\right)(T_c - t)}{\sigma_S\sqrt{T_c - t}}$$

When underlying asset follows the jump-diffusion process, a closed form solution still can be obtained by applying Merton (1976) jump-diffusion call option formula and replacing the volatility of the underlying asset with the corrected one. The formula can be represented as

$$C_t^{jump} = \sum_{j=0}^{\infty} \frac{e^{-\lambda_S T_C}}{j!} C_t\left(\sigma_S{}^j, \ \bullet\right)$$

where, $\sigma_S{}^j = \sqrt{z^2 + \alpha^2\left(\frac{j}{T_C}\right)}$, $\alpha = \sqrt{\frac{\zeta\sigma_S^2}{\lambda_S}}$, $z = \sqrt{\sigma_S^2 - \lambda_S\alpha^2}$. ζ is the proportion of volatility which is accounted by jump part

B. Appendix

Simulation procedure for Bernoulli random variable

Suppose the probability mass function of the Bernoulli random variable X is given as:

$$P(X) = \begin{cases} 0 & \text{with prob. 1-p} \\ 1 & \text{with prob. p} \end{cases}, \quad p \in (0,1)$$

First, a uniform random variable $U \in [0,1]$ is simulated by using any software package. Suppose the simulated uniform random variable value is u, if u is less than or equal to p, the simulated Bernoulli random number is assigned to be 1. If u is greater than p, the simulated Bernoulli random number is assigned to be 0.

Table 1: Parameter Values for Basic Model and Extended Model: In this table, parameters needed are listed according to the values used in previous papers. About bond related parameters, they are mostly from CDG (2001) paper; about option related parameters, they are from Cao and Wei (2001); about jump-diffusion model, parameters are from Zhou (1997).

Parameter	Parameter Value
Bond Related Parameters (Source: CDG (2001))	
C	0.75
P	1
r_0	0.06
θ	0.06
κ	0.1
η	0.015
δ_V	0.03
σ_{V}	0.2
m	0.18
ν	0.6
ϕ	2.8
φ_1	1
φ_2	1
γ_1	0.44
α	0.06
Option Related Parameters (Source: Cao and Wei (2001))	
γ_2	1
S_0	100
σ_S	0.2
stk	100
δ_S	0.03
ρ_{SA}	0.3
ρ_{Sr}	-0.4
ρ_{Ar}	-0.25
Jump-diffusion Parameters (Source: Zhou (1997))	
μ_π^V	0
$(\sigma_\pi^V)^2$	0.25
λ_V	0.05
μ_π^S	0
$(\sigma_\pi^S)^2$	0.25
λ_S	0.05
ζ	0.25

Table 2: Parameter Values for 2-factors Regime-Switching Model (Source: Bansal and Zhou (2002)): This table is from quoted from the estimated parameter values in Bansal and Zhou (2002). In this table, the parameter subscripts $\{i,j\}$ represent the parameter value at regime *i*, factor *j*., the transition probability π_{ij} represents the transition probability from state *i* to state *j*

Parameter	Parameter Value
Factor 1, Regime 0	
$\theta_{0,1}^{\mathrm{CIR}}$	0.00437
$\kappa_{0,1}^{\mathrm{CIR}}$	0.03388
$\sigma_{0,1}^{\mathrm{CIR}}$	0.00421
$\lambda_{0,1}^{\mathrm{CIR}}$	0.01155
Factor 1, Regime 1	
$\theta_{1,1}^{\mathrm{CIR}}$	0.00273
$\kappa_{1,1}^{\mathrm{CIR}}$	0.04375
$\sigma_{1,1}^{\mathrm{CIR}}$	0.00629
$\lambda_{1,1}^{\mathrm{CIR}}$	-0.03145
Factor 2, Regime 0	
$\theta_{0,2}^{\mathrm{CIR}}$	0.00076
$\kappa_{0,2}^{\mathrm{CIR}}$	0.01501
$\sigma_{0,2}^{\mathrm{CIR}}$	0.00392
$\lambda_{0,2}^{\mathrm{CIR}}$	-0.02079
Factor 2, Regime 1	
$\theta_{1,2}^{\mathrm{CIR}}$	0.00540
$\kappa_{1,2}^{\mathrm{CIR}}$	0.00344
$\sigma_{1,2}^{\mathrm{CIR}}$	0.00409
$\lambda_{1,2}^{\mathrm{CIR}}$	-0.02287
Transition Probability	
π_{00}	0.94990
π_{11}	0.91993

Table3: Payoffs of different scenarios when the firm defaults (Distribution Rules). This table illustrates the final payoffs of call holder, bondholder, and shareholder in different cases and scenarios. In this table, $\mathrm{A}(\mathrm{t})$ is the asset value at time t, γ_1 is the recovery rate associated with debt, γ_2 is the recovery rate associated with option.

Senario1: default prior to option's maturity $\tau^* < T_C$	
	Payoff
Call holder	$\left(\frac{C_{\tau^*}}{C_{\tau^*}+D_{\tau^*}}\right)A(\tau^*)\gamma_2$
Bondholder	$\left(\frac{D_{\tau^*}}{C_{\tau^*}+D_{\tau^*}}\right)A(\tau^*)\gamma_1$
Shareholder	0
Scenario 2: default at option's maturity $\tau^* = T_C$	
	Payoff
Call holder	$\left(\frac{C_{\tau^*}}{C_{\tau^*}+D_{\tau^*}}\right)A(\tau^*)\gamma_2$
Bondholder	$\left(\frac{D_{\tau^*}}{C_{\tau^*}+D_{\tau^*}}\right)A(\tau^*)\gamma_1$
Shareholder	0
Scenario 3: default after option's maturity $\tau^* > T_C$	
	Payoff
Call holder	0
Bondholder	$A(\tau^*)\gamma_1$
Shareholder	0

Table 4: Debt, Option, Coupon Bond Values without option liabilities under different models and initial leverage ratios: the models used in this table are CDG model, Jump Model, and RS model. In each model, we have results under 15%, 65% leverage ratios. The initial stock price is 100, and the exercise price for call option is 100; other parameters are given in table 1 and table 2. Here, the default-free option value is obtained by applying the closed form solution listed in Appendix.

	CDG, 15%					
Maturity	undefault debt	default debt	undefaultopt	default_opt	undefaultcoupon	defaultcoupon
4	23.43	23.43	6.63	6.27	6.05	6.05
8	27.00	26.86	6.63	6.19	10.04	10.01
12	27.08	26.57	6.63	6.33	13.20	13.01
16	25.27	24.41	6.63	6.25	15.78	15.16
20	23.03	22.15	6.63	6.20	17.84	16.70
	CDG, 65%					
Maturity	undefault debt	default debt	undefaultopt	default_opt	undefaultcoupon	defaultcoupon
4	47.70	45.09	6.63	6.28	6.01	5.64
8	38.25	36.06	6.63	6.15	10.02	8.78
12	31.86	30.71	6.63	6.17	13.20	10.91
16	27.43	27.28	6.63	6.07	15.77	12.45
20	23.93	24.74	6.63	6.22	17.84	13.47
	Jump, 15%					
Maturity	undefault debt	default debt	undefaultopt	default_opt	undefaultcoupon	defaultcoupon
4	23.51	23.48	6.23	6.52	6.04	6.04
8	27.47	27.21	6.23	6.45	10.04	9.97
12	27.80	27.11	6.23	6.52	13.20	12.87
16	26.52	25.48	6.23	6.60	15.79	14.95
20	25.00	23.99	6.23	6.63	17.83	16.28
	Jump, 65%					
Maturity	undefault debt	default debt	undefaultopt	default_opt	undefaultcoupon	defaultcoupon
4	47.89	44.76	6.23	6.67	6.00	5.53
8	38.91	36.38	6.23	6.63	9.99	8.51
12	33.18	31.78	6.23	6.40	13.19	10.54
16	28.82	28.59	6.23	6.65	15.77	11.92
20	25.59	26.20	6.23	6.84	17.82	12.86
	RS, 15%					
Maturity	undefault debt	default debt	undefaultopt	default_opt	undefaultcoupon	defaultcoupon
4	23.76	23.76	6.63	6.22	6.07	6.07
8	28.12	28.08	6.63	6.30	10.15	10.14
12	29.34	29.02	6.63	6.17	13.49	13.39
16	28.48	27.80	6.63	6.23	16.28	15.91
20	26.66	25.75	6.63	6.27	18.64	17.84
	RS, 65%					
Maturity	undefault debt	default debt	undefaultopt	default_opt	undefaultcoupon	defaultcoupon
4	48.34	46.35	6.63	6.37	6.04	5.76
8	39.75	38.06	6.63	6.17	10.13	9.14
12	34.52	33.47	6.63	6.32	13.48	11.62
16	30.70	30.30	6.63	6.33	16.27	13.41
20	28.01	28.05	6.63	6.20	18.63	14.85

Table 5: Debt, Option, Coupon Bond Values with option liabilities under different models and initial leverage ratios: the models used in this table are CDG model, Jump Model, and RS model. In each model, we have results under 15%, 65% leverage ratios. The initial stock price is 100, and the exercise price for call option is 80; other parameters are given in table 1 and table 2. Here, the default-free option value is obtained by applying the closed form solution listed in Appendix.

	CDG, 10080, 15%					
Maturity	undefault debt	default debt	undefaultopt	default_opt	undefaultcoupon	defaultcoupon
4	23.38	23.28	21.63	21.13	6.05	6.01
8	27.05	26.80	21.63	20.99	10.05	9.96
12	26.92	26.30	21.63	21.04	13.22	12.93
16	25.27	24.32	21.63	20.81	15.78	15.04
20	23.00	22.08	21.63	21.16	17.84	16.60
	CDG,10080, 65%					
Maturity	undefault debt	default debt	undefaultopt	default_opt	undefaultcoupon	defaultcoupon
4	47.70	37.97	21.63	20.89	6.01	3.30
8	38.25	33.08	21.63	20.84	10.02	5.16
12	31.86	30.08	21.63	20.76	13.20	6.38
16	27.43	27.96	21.63	20.58	15.77	7.36
20	23.93	26.47	21.63	20.91	17.84	7.96
	Jump10080, 15%					
Maturity	undefault debt	default debt	undefaultopt	default_opt	undefaultcoupon	defaultcoupon
4	23.48	23.26	21.69	21.02	6.04	5.97
8	27.52	27.06	21.69	21.06	10.03	9.85
12	28.04	27.13	21.69	20.89	13.21	12.75
16	26.56	25.34	21.69	21.01	15.78	14.76
20	24.59	23.50	21.69	21.54	17.82	16.17
	Jump10080, 65%					
Maturity	undefault debt	default debt	undefaultopt	default_opt	undefaultcoupon	defaultcoupon
4	47.89	38.08	21.69	20.62	6.00	3.34
8	38.91	33.34	21.69	20.59	9.99	5.07
12	33.18	30.68	21.69	20.70	13.19	6.31
16	28.82	28.53	21.69	20.72	15.77	7.00
20	25.59	27.44	21.69	21.00	17.82	7.52
	RS10080, 15%					
Maturity	undefault debt	default debt	undefaultopt	default_opt	undefaultcoupon	defaultcoupon
4	23.77	23.70	21.63	21.09	6.07	6.04
8	28.13	28.01	21.63	21.26	10.15	10.10
12	29.24	28.89	21.63	21.21	13.49	13.35
16	28.59	27.89	21.63	21.02	16.28	15.85
20	26.69	25.73	21.63	21.03	18.63	17.73
	RS10080, 65%					
Maturity	undefault debt	default debt	undefaultopt	default_opt	undefaultcoupon	defaultcoupon
4	48.40	39.76	21.63	20.58	6.04	3.73
8	39.83	34.74	21.63	20.55	10.13	5.86
12	34.44	31.65	21.63	20.53	13.47	7.43
16	30.85	29.80	21.63	20.53	16.27	8.64
20	27.42	27.84	21.63	20.44	18.63	9.53

Figure 1: Credit spreads of investment grade bond and junk bond under different models: In this figure, parameter values are given in table1 and table2, and the initial leverage ratio is 15% for investment grade, 65% for junk bond.

Figure 1.1

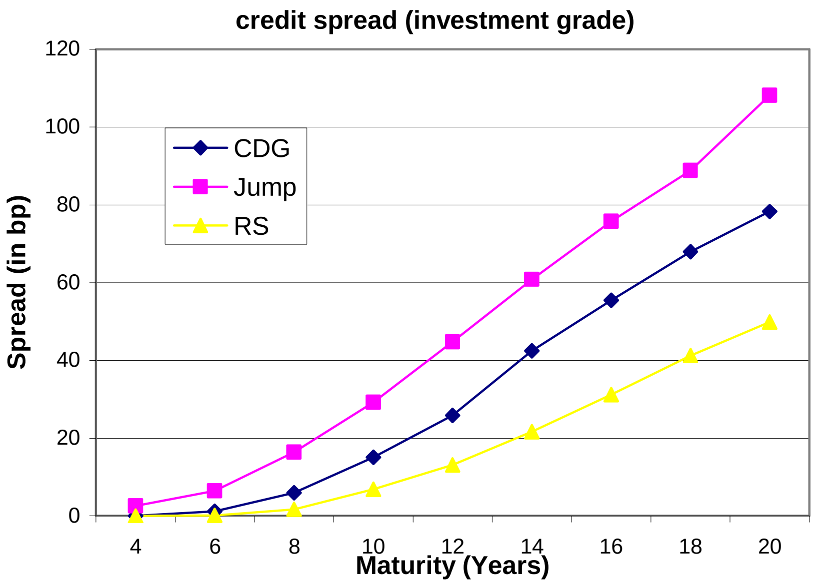

Figure 1.2

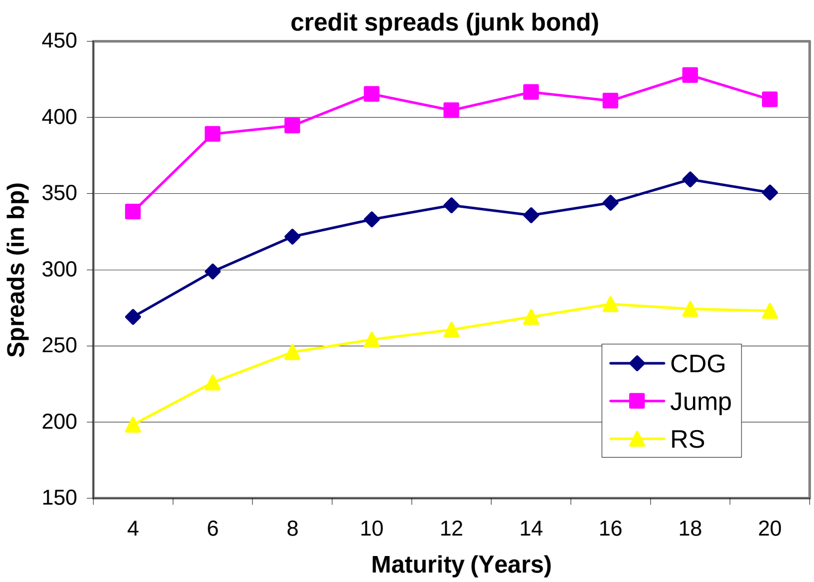

Figure 2: Credit spreads of investment grade bond and junk bond under different models and with option liabilities: In this figure, parameter values are given in table1 and table2, and the initial leverage ratio is 15% for investment grade, 65% for junk bond. In the option liability, initial stock price (S_0) is 100, and the strike price (stk) for this European call option is 80.

Figure 2.1

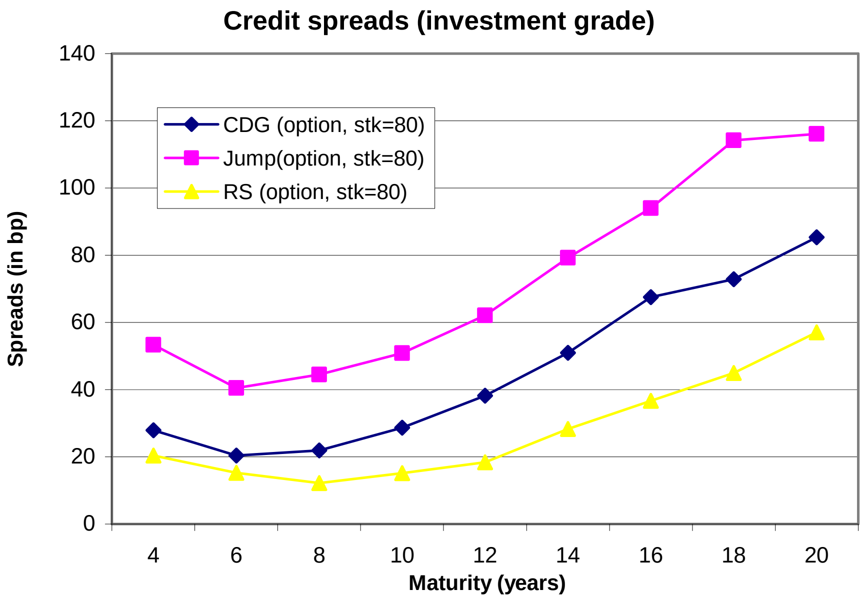

Figure 2.2

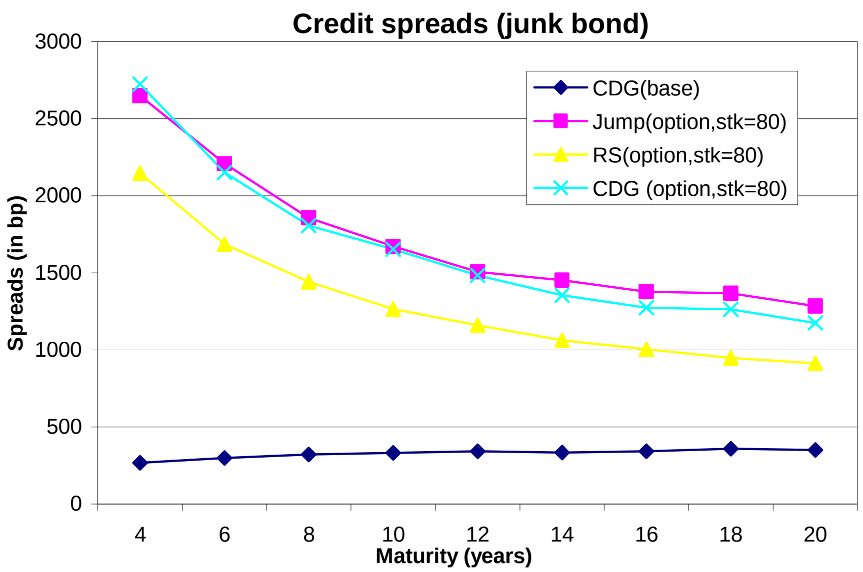

Figure 3: Credit spreads comparison with different models and different liability structure: In this figure, parameter values are given in table1 and table2, and the initial leverage ratio is 15% for investment grade, 65% for junk bond. In the option liability, initial stock price (S_0) is 100, the exercise price is listed in the legend of respective figure.

Figure 3.1

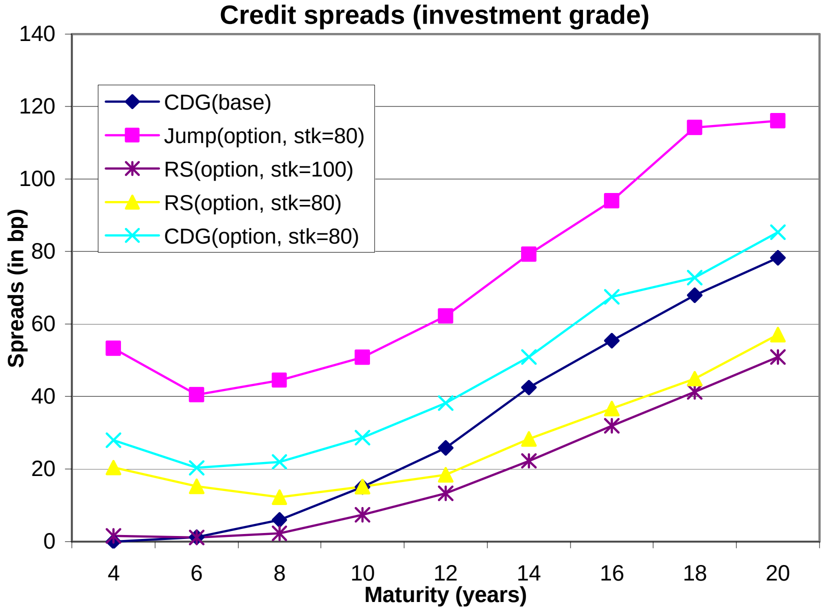

Figure 3.2

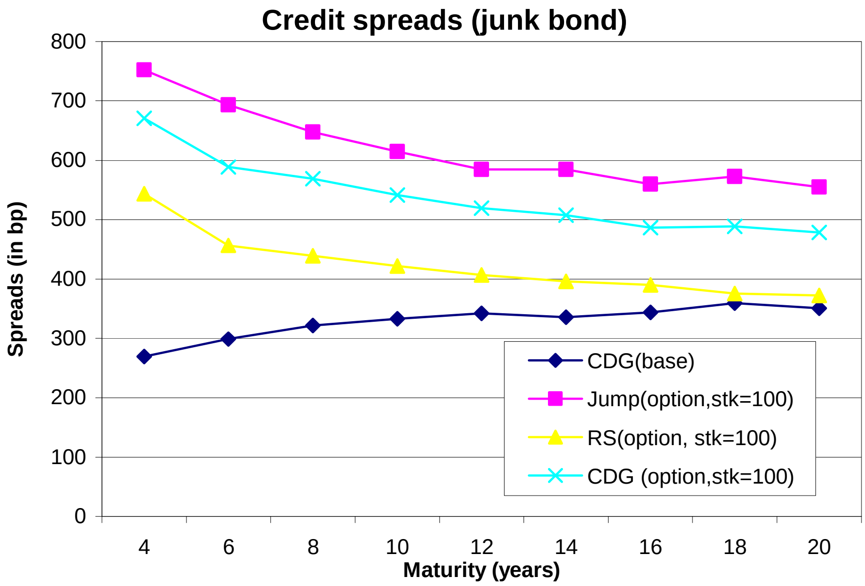

Figure 4: Credit spreads comparison with different models and different initial interest rate level: In this figure, parameter values are given in table1 and table2, and the initial leverage ratio is 35%. In the option liability, initial stock price (S_0) is 100, the exercise price is 80.