# Dissertation Proposal for "Skewness and Co-skewness in Bond Returns"

I-Hsuan Ethan Chiang[1]

August 15, 2007

**Abstract**

This paper explores skewness and co-skewness in discrete-horizon bond returns. Using data for 1976–2005, we find bond skewness is comparable to that in equities, varies with the holding period and varies over time. Speculative-grade bonds and collateralized securities have substantial negative skewness. Co-skewness against the market portfolio is priced differently in various bond sectors: taking a unit of co-skewness risk is rewarded with 0.43% and 2.47% per month for corporate bonds and collateralized securities, respectively. Co-skewness risk helps explain the cross section of expected bond returns even when other state variables such as inflation, real activity, or short term interest rates are included.

# 1 Introduction

This paper explores the skewness and co-skewness properties of discrete holding period bond returns. We ask three empirical questions: are bond returns skewed? Does the skewness matter in explaining the cross section of expected returns? Does it still matter when other variables affecting investment opportunities are taken into account?

We find that U.S. bond returns display asymmetric distributions, and their co-skewness against risk factors helps explain the cross sectional variation in returns in some bond sectors. For a corporate bond taking a unit of co-skewness risk, its co-skewness risk premium is 0.43% per month. Taking a unit of co-skewness risk in collateralized security is rewarded with 2.47% per month. We propose a new multibeta-coskewness model focusing on the higher co-moments that become relevant for discrete holding period returns. When state variables such as real activity, inflation, or short term interest

[1]PhD Candidate at Carroll School of Management, Boston College, 140 Commonwealth Avenue, Chestnut Hill, Massachusetts 02467-3808; phone: (617) 552-2024, fax: (617) 552-0431, email: chiangih@bc.edu.

rates are considered, the models with co-skewness terms capture on average about 65% of the cross sectional variation in the sample of bond portfolio returns.

Kraus and Litzenberger (1976, 1983) advocate a "three-moment" asset pricing model, in which both covariance and co-skewness, which is the contribution to portfolio skewness, drive cross sectional variation in expected asset returns. Kraus and Litzenberger (1976), Friend and Westerfield (1980), Barone-Adesi (1985), Lim (1989), Harvey and Siddique (1999, 2000), and Smith (2006)find mix empirical evidence for the three-moment model using equity market data. However, the empirical literature on skewness and co-skewness pricing has largely ignored bond reutns.

This paper studies skewness and co-skewness in discrete holding period bond returns. In continuous-time models, instantaneous bond returns typically follow Itô processes and contain no skewness or co-skewness. However, investors care about discrete holding period returns since they can not realistically rebalance their portfolios continuously. Therefore it is important to examine whether discrete bond returns have skewness and co-skewness.

The fundamental characteristics of bonds motivate asymmetry in bond returns. Unlike stock prices, bond prices face an upper bound: the maximum price of a bond is the sum of future promised cash flows when the interest rate approaches to zero. Furthermore, default risk (e.g. in high yield bonds) or underlying optionality (e.g. in prepayable mortgages) may create asymmetry in bond returns. Thus, bond returns are skewed by nature. We are interested in what asset pricing implications third moments in bonds carry and empirically investigate them in this paper.

# 2 Third Moments and Asset Prices

## 2.1 Co-skewness and Equilibrium Asset Prices

Kraus and Litzenberger (1976) derive the equilibrium implication of skewness preference. They consider the optimal portfolio problem of the representative expected utility maximizing investor with a utility function characterized by the first three moments of wealth. They show that in equilibrium

$$\mathbf{E}(r_i) = \lambda_m \beta_{m,i} + \eta_m \gamma_{m,i},$$

for all $i$, where $r_i$ is the return of asset $i$ in excess of risk free rate, $\beta_{m,i}$ is asset $i$'s beta against the market portfolio, $\gamma_{m,i}$ is asset $i$'s co-skewness against the market portfolio, and $\lambda_m$ and $\eta_m$ are the risk premiums for beta and co-skewness risks, respectively. The co-skewness term is defined as $\gamma_{m,i} = \mathbf{cov}[r_i, (r_m - \mu_m)^2]/\mathbf{E}[(r_m - \mu_m)^3]$ where $r_m$ is the market portfolio excess return with mean $\mu_m$.

Co-skewness can be interpreted as asset $i$'s scaled marginal contribution to the portfolio skewness. An asset with *positive* co-skewness makes the distribution of the market portfolio *more* skewed in the direction depending on the market portfolio skewness. If the market portfolio is negatively skewed, the investor requires risk premium to include this asset in his portfolio.

## 2.2 A Multibeta-Coskewness Model

We introduce a new multibeta-coskewness model in this paper, featuring (1) multifactor setting, and (2) higher moments, and is an extension of Merton (1973), Kraus and Litzenberger (1976), and Cox, Ingersoll, and Ross (1985):

$$\mathbf{E}(r) = \lambda_w \beta_w + \lambda_s \beta_s + \eta_w \gamma_w + \eta_s \gamma_s + \eta_{ws} \gamma_{ws},$$

where

$$\lambda_w = -\frac{\bar{J}_{ww}\sigma_w^2}{\mathbf{E}(J_w)}, \qquad \beta_w = \frac{\mathbf{cov}(r, w)}{\sigma_w^2},$$
$$\lambda_s = -\frac{\bar{J}_{ws}\sigma_s^2}{\mathbf{E}(J_w)}, \qquad \beta_s = \frac{\mathbf{cov}(r, s)}{\sigma_s^2},$$

$$\eta_w = -\frac{\bar{J}_{www}\mathcal{M}_w^3}{2\mathbf{E}(J_w)}, \qquad \gamma_w = \frac{\mathbf{cov}[r, (w - \mu_w)^2]}{\mathcal{M}_w^3},$$
$$\eta_s = -\frac{\bar{J}_{wss}\mathcal{M}_s^3}{2\mathbf{E}(J_w)}, \qquad \gamma_s = \frac{\mathbf{cov}[r, (s - \mu_s)^2]}{\mathcal{M}_s^3},$$

$$\eta_{ws} = -\frac{\bar{J}_{wws}\mathbf{E}[w - \mu_w, (w - \mu_w)(s - \mu_s)]}{2\mathbf{E}(J_w)},$$
$$\gamma_{ws} = \frac{\mathbf{cov}[r, (w - \mu_w)(s - \mu_s)]}{\mathbf{E}[w - \mu_w, (w - \mu_w)(s - \mu_s)]}.$$

where $r$ is a vector of asset excess returns with mean $\mu_r$, $J$ is the indirect utility function of wealth $w$ and another state variable $s$, with means $\mu_w$ and $\mu_s$, variances $\sigma_w^2$ and $\sigma_s^2$, and third central moments $\mathcal{M}_w^3$ and $\mathcal{M}_s^3$, respectively.

# 3 Empirical Results

## 3.1 Data and Univariate Analysis

We use monthly bond index returns, whenever available, for the period January 1976 to December 2005, totaling 360 months. We consider four categories of bond portfolios, including 12 Fama Treasury bond portfolios, 14 Lehman Corporate bond indexes, 9 Lehman Government-Related bond indexes, and 16 Lehman Securitized portfolios, a maximum total of 51 bond portfolios.

Univariate analysis shows that, over the sample period, safer bonds have positive skewness, while defaultable bonds and collateralized securities have substantial negative skewness. The skewness in monthly bond returns is comparable to that in equities. Like stock skewness, bond skewness varies with the holding period and varies over time.

## 3.2 Time Series Tests for Co-skewness

We use a time series test to assess the significance of co-skewness. For each bond portfolio $i$, we consider the following moment conditions for GMM estimation,

$$\begin{aligned} \mathbf{E}(r_m - \mu_m) &= 0, \\ \mathbf{E}[r_{i,t} - c_{0i} - c_{1i} r_{m,t} - c_{2i}(r_{m,t} - \mu_m)^2] &= 0, \end{aligned}$$

where $r_m$ is the market index excess return with mean $\mu_m$ and standard deviation $\sigma_m$. When $c_{2i}$ is non-zero, co-skewness is significant. Our empirical results reject the null hypothesis that bond returns are linear in the market portfolio return. Instead, we find evidence that bond returns are non-linear in market, and most of them are convex in market. The strong non-linearity motivates us to test whether co-skewness is priced in bond returns.

## 3.3 Cross Sectional Tests for Co-skewness Pricing

We use cross sectional regressions to assess the importance of co-skewness in explaining the cross section of expected bond returns. For each time $t$, we use all available test asset data to run the following cross-sectional regression,

$$r_{i,t} = a_t + \lambda_{m,t}\beta_{m,i} + \eta_{m,t}\gamma_{m,i} + \varepsilon_{i,t},$$

where $\beta_{m,i}$ is bond portfolio $i$'s beta against market, and $\gamma_{m,i}$ is bond portfolio $i$'s co-skewness against market. We find that adding co-skewness improves the model fit, but the effect is merely marginal. Co-skewness risk premium is about 0.13% per month. While the co-skewness risk premium is not statistically significant, the amount is economically substantial compared to the risk premium on bond market index (0.23% per month). Including the co-skewness term to the market beta pricing model improves the explanatory power. The average cross-sectional adjusted $R^2$ for single factor beta-pricing model is 0.32, while it improves to 0.49, an above 50% improvement, after co-skewness term is added. Subsample analysis draws similar conclusions.

## 3.4 Tests for Market Segmentation

We test whether there is market segmentation in co-skewness pricing and measures co-skewness premiums in different markets. We find that, taking a unit of co-skewness risk is reward with 0.43% and 2.47% per month for corporate and securitized portfolios, respectively, and both premiums are statistically significant. In the subsamples, co-skewness is also priced in the securitized sector in the first and third subperiods.

Alternatively we consider bond and stock markets as two separate markets. We use the 25 Fama-French size and book-to-market sorted portfolios and the 51 bond portfolios as the test assets. Our results show that co-skewness risk is priced in stocks, and taking a unit of co-skewness risk is rewarded with 0.39% per month, or 4.68% per annum, while it is not priced in bonds in aggregate.

## 3.5 Tests for Multifactor Beta and Co-skewness Pricing

We test the multifactor version of three-moment model using five state variables: inflation, real activity, short term interest rate, credit spread, and term premium. When state variables such as inflation, real activity, and short rate are considered, models with third co-moment terms have smaller pricing errors and on average capture about 65% of the cross sectional variation in expected bond returns. However, higher moments of Fama and French (1993) bond "common factors" provide little improvement.

# 4 Concluding Remarks

This paper empirically tests the skewness and co-skewness in discrete bond returns. While a large literature explores the skewness and co-skewness in equity returns, no one has yet focused on that in bond returns.

This paper tries to answer three empirical questions: Do bond returns contain skewness? Does it matter in explaining the cross sectional variation in expected bond returns? When state variables other than market are taken into consideration, what asset pricing implications does skewness make?

Using monthly data for 1976–2005, we find bond skewness has similar magnitude to that of stocks. When different holding horizons are considered, the skewness of holding period returns of bonds has similar pattern to that of stocks. The skewness of bond returns is time-varying, with very different pattern from that of stock returns.

Using time series and cross sectional regressions, we find the one-factor CAPM is rejected and the model with third co-moment has better performance. The co-skewness risk premium is different for various bond sectors. We find the co-skewness premium is more pronounced in corporate bonds and collateralized portfolios. When state variables such as inflation, real activity, and short rate are considered, we find models with third co-moments have smaller pricing errors and on average capture about 65% of the cross sectional variation in expected bond returns. We conclude that bond returns have substantial skewness and their co-skewness is priced.

# References


[1] Barone-Adesi, G., 1985, Arbitrage equilibrium with skewed asset returns, *Journal of Financial and Quantitative Analysis* 20, 299-313.

[2] Fama, E.F., and K.R. French (1993), Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3-56.

[3] Fama, E.F., and J.D. MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607-636.

[4] Friend, I., and R. Westerfield, 1980, Co-skewness and capital asset pricing, *Journal of Finance* 35, 897-913.

[5] Harvey, C.R., and A. Siddique, 1999, Autoregressive conditional skewness, *Journal of Financial and Quantitative Analysis* 34, 465-487.

[6] Harvey, C.R., and A. Siddique, 2000, Conditional skewness in asset pricing, *Journal of Finance* 55, 1263-1295.

[7] Kraus, A., and R.H. Litzenberger, 1976, Skewness preference and the valuation of risk assets, *Journal of Finance* 31, 1085-1100.

[8] Kraus, A., and R. Litzenberger, 1983, On the distributional conditions for a consumption-oriented three moment CAPM, *Journal of Finance* 38, 1381-1391.

[9] Lim, K.-G., 1989, A new test of the three-moment capital asset pricing model, *Journal of Financial and Quantitative Analysis* 24, 205-216.

[10] Smith, D.R., 2006, Conditional coskewness and asset pricing, working paper, Simon Fraser University.