

Dynamic Portfolio Choice with Parameter Uncertainty and Model Uncertainty

Hening Liu

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Abstract

My dissertation examines dynamic portfolio and consumption decisions taking into account both parameter uncertainty and model uncertainty. Parameter uncertainty arises from the estimation error when the investor estimates the unknown parameters in investment opportunities. Model uncertainty is incorporated to capture the investor's lack of confidence about the estimated model. These two types of uncertainty are expected to generate different implications on the optimal stock allocation and consumption.

1 Introduction

Dynamic portfolio and consumption decisions are usually analyzed in a setting where investment opportunities contain a stock, whose returns are stochastic, and a bond, whose returns are deterministic. An investor chooses optimal stock allocation and consumption instantaneously to maximize her lifetime utility. In reality, an investor may not fully observe the investment opportunities but has to estimate the unknown parameters that govern the investment opportunities. When the investor performs the estimation, the estimation error causes parameter uncertainty. For example, the mean return of a stock has to be estimated instead of assuming to be known precisely. The estimated mean return is random and follows a stochastic process. The investor is willing to induce hedging demand to hedge against the stochastic variations of the estimates. Several literature (to name a few, Brennan [1998], Xia [2001], Honda [2003]) has been done along this dimension.

The novel feature of my dissertation is that the dynamic portfolio and consumption decisions are to be examined in the presence of not only parameter uncertainty but also model uncertainty. In my dissertation, parameter uncertainty, similar to Guidolin and Timmermann (2007), is caused by an investor's dynamically learning regime switching investment opportunities. The regime switching model is employed to capture the cyclical behavior of the stock market. To be specific, suppose that the mean return of a stock is subject to two regimes. Each regime's mean return and the transition probabilities across the two regimes can be estimated through certain econometric methods. The investor cannot observe the current regime but has to update her beliefs of the probabilities of the underlying regimes by

observing relevant signals including market returns and dividend yields. In Guidolin and Timmermann (2007), the investor's perceived probabilities of the underlying regimes can be obtained by Hamilton (1989)'s filtering method applied to discrete data. However, my dissertation is to develop new mechanism to obtain the filtered probabilities in a continuous time setting.

Rational expectation theory assumes complete confidence of an investor about the model that describes the investment opportunities and is to be estimated by econometricians. Thus the investor does not share the same concern with econometricians about the validity of the model itself while econometricians often conduct model comparisons in terms of likelihood-based tests. In reality, however, the investor often lacks confidence about the estimated model and worries the model is misspecified. Moreover, because estimating mean returns is difficult and there are unknown factors that cannot be learned by the investor but still affect the investment opportunities, it is reasonable to assume the investor takes a less confident position than an investor with rational expectation. To incorporate this consideration, my dissertation introduces model uncertainty in addition to parameter uncertainty in analyzing dynamic portfolio and consumption choices. For instance, although a lot of evidences found that returns are predictable, the predictability is subject to uncertainty. Even the predictability is different among the regimes of the investment opportunities, the predictive relation that is estimated may be misspecified in each regime. The investor tends to surround the predictive model in each regime with a set of possible models. These models are difficult to distinguish statistically from the benchmark model. This type of model uncertainty is expected to generate different impacts on the optimal stock allocation and consumption from parameter uncertainty, and risk as well.

2 Research Questions and Design

My dissertation has two closely related research projects. In the first project, I assume no return predictability and the unobserved mean return of a stock follows a regime switching process with a high regime and a low regime. An investor needs to update her beliefs of the likelihoods of the underlying regimes by observing market returns. Then the stochastic dynamic optimization problem can be formulated to maximize an utility function that takes into account model uncertainty and the investor's aversion to such uncertainty. It has been known that when an investor learns the unknown parameters throughout investment horizon, the optimal stock allocation has two components, the myopic component and the intertemporal hedging component. The myopic component relies on the current estimates of the parameters and is instantaneously mean-variance efficient while the intertemporal hedging component is induced to hedge against future fluctuations of the estimates. By incorporating model uncertainty, it is possible to examine the effects of model uncertainty on both myopic portfolio and intertemporal hedging portfolio. It

has been found (Chen and Epstein [2002], Maenhout [2004]) that model uncertainty reduces the myopic portfolio. However, its impact on the hedging portfolio is yet to be examined for an investor dynamically learning her investment opportunities. I expect another hedging component to appear to hedge against model uncertainty. This component acts to hedge for the reduction in the myopic component caused by model uncertainty. The hedging component for parameter uncertainty and the component for model uncertainty are anticipated to be opposite in sign and tend to offset each other. Then it is interesting to ask whether the importance of the intertemporal hedging demand increases or decreases as a result of model uncertainty. In other words, is the investor more likely to be myopic and ignore future changes in investment opportunities when model uncertainty persists? Moreover, the horizon dependence of optimal portfolio can also be altered by model uncertainty. Brennan [1998] and Honda [2003] found as horizon increases, the investor would invest less in stock. Model uncertainty may change this scenario because the hedging demands for model uncertainty and parameter uncertainty interact with each other to affect the optimal stock allocation. Thus the question that whether younger investor (with long horizon) should invest more or less will be analyzed taking into account model uncertainty. The result may generate new insights on the conventional wisdom that younger investor should invest more in equity. Moreover, it has been already found that an investor ignoring learning investment opportunities can incur large welfare loss¹. Then what is the effect of model uncertainty on this welfare gain from dynamic learning? Will an investor benefit more or less from learning more uncertain investment opportunities? I expect that the greater the model uncertainty, the investor benefits less from learning because she perceives her investment opportunities to be fuzzier and is less willing to learn. If model uncertainty can make the welfare gain to be negative, then the simple constant investment strategy for i.i.d. returns even outperforms the optimal strategy taking into account learning.

The second project extends the first one by introducing return predictability. An additional hedging demand is called for to hedge variations of the predictive variables. Market timing, in the presence of return predictability, refers to the dependence of optimal stock allocation on the current value of the predictive variable. After introducing model uncertainty, the intensity of market timing depends crucially on model uncertainty through its impact on the hedging components for both the predictive variable and parameter uncertainty.

3 Backgrounds on Literature

The seminal works of Merton [1969] and Samuelson [1969] derives constant optimal portfolio shares in a continuous-time setting when stock returns follow a simple i.i.d process and the investor has an iso-

¹ See Xia [2001] and Guidolin and Timmermann [2007] for example.

elastic utility function. When the investment opportunity set is stochastic, the intertemporal hedging demand emphasized by Merton [1971] becomes important to the dynamic portfolio choice problem. A non-myopic long-horizon investor optimally chooses allocation strategy that deviates from the mean-variance efficient portfolio. Several literature have analyzed dynamic portfolio choice either when the risk premium follows a mean-reverting process or when returns are predictable. By solving for the analytical solutions (Kim and Omberg [1996], Wachter [2002]) and through numerical computations (Campbell and Viceira [1999], Barberis [2000], Campbell et al. [2004]), they found that the stochastic variations of the investment opportunities greatly increases the stock allocation that also increases for a long-horizon investor. Optimal consumption and portfolio choice in stochastic investment opportunity set has also been studied by other literature such as Schroder and Skiadas [1999] and Liu [2007]. Detemple et al. [2003] uses the Monte Carlo method to compute optimal portfolios for a wide class of stochastic mean return and interest rate.

The effect of parameter uncertainty on the optimal portfolio has been analyzed by Bawa and Klein [1976] in a single-period setting. A few of more recent literature (for example, Brennan [1998], and Honda[2003]) shows in a continuous time setting, parameter uncertainty reduces the optimal stock allocation. The result also implies that younger investors tend to invest less in stock. Xia [2001] examines dynamic portfolio allocation for an investor who is confronted with both return predictability and parameter uncertainty. The result indicates the non-monotonic dependence of the optimal stock allocation on the investment horizon. My dissertation differs from Xia [2001] in two respects. First, Xia [2001] considers the conditional Gaussian predictability while my dissertation examines the regime switching predictability. Second, in my dissertation, the predictive relation is subject to model uncertainty in each regime, a feature that is ignored by Xia [2001].

Recently, a growing body of literature (for example, Anderson et al. [2003], Maenhout [2004], Chen and Epstein [2002], Hansen and Sargent [2006], Hansen et al. [2006], Epstein and Schneider [2007]) emphasizes the distinction between model uncertainty (Knightian uncertainty, ambiguity) and risk, and also examines its importance in finance and macroeconomics. The Ellsberg Paradox and other related evidences illustrate that this distinction is behaviorally important. Chen and Epstein[2002] and Maenhout [2004] have obtained closed-form optimal portfolio rules taking into account model uncertainty in a continuous time setting with i.i.d returns. Maenhout [2006] combines time-varying investment opportunities and model uncertainty together to solve for optimal portfolio rules. Epstein and Schneider [2007] studies learning under ambiguity and its implications on portfolio choice. But the model assumes the investor learns an unobserved but constant parameter. My dissertation, by considering regime switching parameters, can exploit richer dynamics of investment opportunities.

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