

A New Information Share Measure
and Its Application to Futures and Spot Markets

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Abstract

In this paper, we modify the information share (IS) originally proposed by Hasbrouck (1995) as a way of measuring price discovery. The proposed modified information share (MIS) leads to a unique measure of price discovery instead of the upper and lower bounds associated with the original IS. The performance of MIS is compared with the alternate price discovery method based on the permanent-temporary decomposition proposed by Gonzalo and Granger (1995) (PT/GG) using simulations with 1,000 replications applied to the same three examples considered by Hasbrouck (2002). The MIS is found to perform much better than the PT/GG method. The empirical application of the MIS to 9 different commodities indicates that the price discovery takes place mostly in the futures market. The evidence supports the transaction cost hypothesis, which states that the price discovery is expected to take place in the market with lower transaction cost.

I. Introduction

Whether price reflects the fundamental value of a security traded in a financial market is one of the fundamental questions in finance. This question is of interest to academicians, policy makers as well as practitioners. The answer to this question would be positive in well-functioning, efficient financial markets. This is because, in these markets, any new information, that affects the fundamental value of the security, should be quickly reflected in the price. However, the new information would not be instantaneously impounded into the price due to the existence of market imperfections like transaction costs, information asymmetry and regulations etc. Furthermore, the gradualness in the impounding of information has also to do with the possibility that an investor with private information may trade gradually in order to profit before his/her trade fully reveals the information.¹

The process in which the information gets reflected in the price becomes even more interesting if there are more than one market where the same security or very similar securities trade. For example, the same stock may be listed in multiple markets in the same country or multiple countries. In this case, we would be interested to know the price in which market reflects the new information first. In other words, we would like to know the market where the price discovery takes place first. This gives rise to the concept of *dominant* and *satellite* markets (Garbade and Silber, 1983). For example, in the case of multiple listing in multiple countries, we would like to know if the price discovery takes place in the domestic or foreign markets.

¹ See Kyle (1985), Admati and Pfleiderer (1988), Barclay and Warner (1993) and Chakravarty (2001) for discussions on this issue of stealth trading strategy followed by investors with private information.

Besides multiple listing, similar situation is encountered when dealing with securities which have derivatives. One can take a long position in an underlying security by buying underlying security itself or by taking long positions in derivative securities like futures and options. In this case, we would like to know whether the price discovery takes place in the security (cash) market or the derivatives (futures) market. If the price discovery depends on transaction costs, as suggested by Fleming, Osdiek and Whaley (1996), then we expect the price discovery to take place in the futures market instead of cash market due to low transaction cost associated with futures trading.

This brings us to the measurement of price discovery. In a pioneering work, Hasbrouck (1995) suggests a measure of price discovery commonly known as information share (IS). This is one of the most commonly used methods. One of the attractive features of this method has to do with the fact that it incorporates both the system dynamics as well as the characteristics of the innovations driving the dynamic system. This method is also consistent with the conventional argument that the security prices are martingale.

However, due to dependence of the IS measure on ordering of the series, we end up with upper and lower bounds for the IS measure instead of a single figure. This may not be problematic in situations where the bounds are close to each other. However, specific conclusions may be hard to reach if the bounds are far apart. In this paper, we suggest a modified version of information share (MIS), which leads to a unique measure of price discovery. Simulations of the three examples considered by Hasbrouck (2002) indicate that the MIS performs better than the original IS as well as the measure based on a model suggested by Gonzalo and Granger (1995).

In the paper, we also empirically analyze the price discovery mechanism in spot and futures markets. The research analyzing the relationship between the spot price and futures price dates back to Keynes (1930), Hicks (1946), Telser (1958) and Cootner (1960). 'Price discovery' and 'risk transfer' have been recognized as two important contributions of futures markets since late 1940's (Working, 1948, 1962; Silber, 1981). Even though the meaning of risk transfer is clear, the meaning of 'price discovery' has changed over time. Initially, price discovery referred to the use of futures prices for pricing cash market transaction (Working, 1948). Then, the term has also been used to refer to the unbiasedness hypothesis where the futures price is supposed to be the forecast of the spot price in the future.² Price discovery has also been interpreted to mean the Granger causality which is associated with the lead-lag relationship between the futures price and the spot price (Garbade and Silver, 1983; Chan, 1992).

The current use of the term refers to the impounding of new information in prices. Therefore, in this paper, we use the term to refer to the process in which the new information would get reflected in price. Price discovery in the futures and spot markets has been the interest of many researchers. This is evident from a large number of papers which have been published in this area. The papers include, e.g., Garbade and Silber (1983), Kawaller, Koch and Koch (1987), Stoll and Whaley (1990), Chan (1992), Wahab and Lashgari (1993), Grunbichler, Longstaff and Schwartz (1994), Fleming, Ostdiek and Whaley (1996), Tse (1999), and Roope and Zurbruegg (2002).

Using a version of vector autoregressive (VAR) model with one-period lag, Garbade and Silber (1983) find that about 75 percent of the new information is first

² See Telser (1958), Stoll (1979), Turnovsky (1983) and French (1986) for some theoretical models which implies that the futures price is equal to expected future spot price under certain conditions. See also Chakravarty (2001) for some empirical results.

incorporated in futures markets for wheat, corn and orange juice whereas this figure is only about 54 percent for Oats and Copper. Kawaller, Koch and Koch (1987), using a general VAR model, analyze S&P 500 index and index future and conclude that “futures price movements consistently lead index movements by twenty to forty-five minutes while movements in the index rarely affect futures beyond one minute.” Instead of VAR model, which uses only the lagged variables, Stoll and Whaley (1990) employ the lead-lag model suggested by Sims (1972) in analyzing S&P 500 and major market index (MMI) returns. They find that futures returns lead stock index returns by about five minutes on average, and occasionally by as long as 10 minutes or more, but there is a weak feedback from the cash market into the futures market. Using a similar model, Chan (1992) also finds similar results for MMI including the MMI components stocks, i.e., the asymmetric relationship holds between MMI futures and all components stocks. Grunbichler, Longstaff and Schwartz (1994) also find similar results where DAX index futures is found to lead the DAX index by 15 to 20 minutes. Fleming, Ostdiek and Whaley (1996) extend the Sims’ lead-lag model by including an error-correction term and analyze the S&P 500 index, S&P 500 futures and S&P 100 options. They also find that futures lead the index. Wahab and Lashgari (1993) use a vector error-correction (VEC) model to analyze the S&P 500 and FTSE 100 indices. Based on the significant adjustment coefficients for both futures and spot, they conclude that the feedback exists between the two markets. However, the adjustment term for futures is found to be much higher than for the spot, therefore, they make further conclusion that the lead from spot to futures is stronger (more pronounced) than the lead from futures to spot.

Instead of using the VAR, VEC and Sims' lead-lag models, recent researchers have used Hasbrouck's IS and Gonzalo-Granger permanent-temporary (PT/GG) decomposition methods. For example, Tse (1999) use the information share method suggested by Hasbrouck (1995) when analyzing the DJIA futures and cash markets. He finds that the IS bounds for futures to be (97.98, 78.60) and the bounds for spot to be (21.40, 2.02). Based on the IS bounds and size of adjustment coefficients, Tse (1999) concludes that "DJIA futures (market) is more informationally efficient than the underlying stock market". Finally, Roope and Zurbruegg (2002) use both IS and PT/GG methods to analyze MSCI Taiwan futures (TiMSCI futures) with MSCI Taiwan (TiMSCI) being the underlying instrument, and TAIEX futures with Taiwan stock index (TAIEX) being the underlying instrument. TiMSCI futures is traded in Singapore and TAIEX futures is traded in Taiwan. Based on the PT/GG method, they conclude that futures have higher information share than spot in each country. The same conclusion is reached using Hasbrouck IS measure for TiMSCI. However, for TAIEX futures, the IS bounds are (85.13, 6.27) and the IS bounds for TAIEX are (93.73, 14.87). This seems to imply that the spot has more IS than futures. However, since the bounds are wide and overlap each other, definite conclusions cannot be made. Based on the evidence presented above, it can be concluded that, in general, price discovery mostly takes place in the futures market.

In this paper, we use the IS methods as well as the PT/GG method to empirically estimate the price discovery for 9 different sets of spot and futures markets. According to PT/GG method, price discovery takes place mostly either only in the spot market or in both markets. Only in one case (corn), the price discovery is found to take place in

futures market. However, the results based on modified IS (MIS) method are quite different. According to the MIS method, the price discovery takes place mostly in the futures markets. As explained later, the PT/GG method completely ignores the innovations, which is expected to characterize the information arriving in different markets. Furthermore, the simulation results (presented in the paper) also indicate that the MIS performs better compared to the PT/GG method. Therefore, we suggest that the MIS method be relied upon in drawing conclusions. Based on the MIS, we conclude that the price discovery takes place mostly in the futures market. The evidence supports the transaction cost hypothesis, which states that the price discovery is expected to take place in the market with lower transaction cost. This evidence also supports the evidence found by other researchers in the area.

The remainder of the paper is divided into three sections. In Section II, the modified information share and other methods of measuring price discovery are discussed. The simulation results are also presented in this section. The empirical results are discussed in Section III. We conclude the paper in Section IV.

II. Measurement of Price Discovery

Hasbrouck (1995), in a pioneering work, suggests a measure of price discovery, known as information share (IS), defined for a situation where the series involved have unit roots and are cointegrated with a single stochastic trend (i.e., the number of cointegrating vectors is equal to one minus the number of series considered). One of the attractive features of the information share is the fact that it incorporates both the system dynamics as well as the characteristics of the innovations driving the dynamic system.

However, since the information share depends on the ordering of the series, this leads to the upper and lower bounds for the information share instead of the unique information share. In this paper, we modify the Hasbrouck's information share in such a way that it leads to a unique information share which will be referred to as modified information share (MIS).

In this section, we will describe the Hasbrouck's IS as well as the modified IS (MIS). In addition to these two price discovery measures, we will also describe other alternative methods currently being used. These include methods based on the permanent-temporary decomposition of non-stationary series suggested by Gonzalo and Granger (1995) as well as the model proposed by King et al. (1991). However, first we would describe the general structure of cointegrated system with single common stochastic trend.

Let Y_t be an $n \times 1$ vector of unit-root processes where it is assumed that there exist $n - 1$ cointegrating vectors, which implies an existence of single common stochastic trend (Stock and Watson (1988)). Based on Engle and Granger (1987), the series have the following vector error-correction (VEC) representation:

$$\Delta Y_t = \Pi Y_{t-1} + \sum_{i=1}^k A_i \Delta Y_{t-i} + \varepsilon_t, \quad \Pi = \alpha \beta^T \quad (1)$$

where β and α are $n \times (n - 1)$ matrices of rank $(n - 1)$. The columns of β consist of the $(n - 1)$ cointegrating vectors and each column of α consists of adjustment coefficients. The matrix Π is decomposed in such a way that $\beta^T Y_t$ consists of $(n - 1)$ vector of stationary series. The covariance matrix of the error term is given by $E[\varepsilon_t \varepsilon_t^T] = \Omega$. Now

we will describe the alternative methods of measuring the price discovery starting with the information share method proposed by Hasbrouck (1995).

A. Hasbrouck Information Share (IS):

Following Stock and Watson (1988), Hasbrouck (1995) transforms equation (1) into the following vector moving average (VMA) representation:

$$\Delta Y_t = \Psi(L)\varepsilon_t \quad (2)$$

or, alternatively,

$$Y_t = Y_0 + \Psi(1)\sum_{i=1}^t \varepsilon_i + \Psi^*(L)\varepsilon_t \quad (3).$$

Since the two series are cointegrated, the Engle-Granger representation theorem (Engle and Granger (1987)) implies the following (de Jong (2002)):

$$\beta^T \Psi(1) = 0 \quad \text{and} \quad \Psi(1)\alpha = 0 \quad (4)$$

Therefore, we have $\Psi(1) = \beta_{\perp} \alpha_{\perp}^T$ where α_{\perp} and β_{\perp} are orthogonal vectors to α and β respectively. Equation (3) can be written as

$$Y_t = Y_0 + \beta_{\perp} \alpha_{\perp}^T \sum_{i=1}^t \varepsilon_i + \Psi^*(L)\varepsilon_t \quad (5)$$

Note that $\alpha_{\perp}^T \sum_{i=1}^t \varepsilon_i$ represents the common stochastic trend component, which follows a random walk process. Also note that $\Psi(1)\varepsilon_t$ represents the long-run impact of innovation on price. It is also clear that existence of $(n-1)$ cointegrating vectors implies that the

impact matrix $\Psi(1)$, which is the sum of the moving average coefficients, has rank 1. Furthermore, in the case considered by Hasbrouck (1995), the rows of $\Psi(1)$ are identical.³ Let ψ represent the identical row of $\Psi(1)$. Hasbrouck (1995) mentions that $\psi\varepsilon_t$ constitutes the long-run impact of the innovations on each of the prices and suggests the following measure of information share (IS) of market j for the case where the covariance matrix Ω is diagonal (i.e., the innovations are independent):

$$S_j = \frac{\psi_j^2 \Omega_{jj}}{\psi \Omega \psi^T}. \quad (6)$$

Note that ψ_j is the j -th element of the identical row of the impact matrix $\Psi(1)$. The information share measure when the covariance matrix is not diagonal is given by

$$S_j = \frac{([\psi F]_j)^2}{\psi \Omega \psi^T} \quad (7)$$

where F is the Cholesky factorization of Ω and $[\psi F]_j$ represents the j -th element of the row vector ψF ⁴. Since the Cholesky factorization depends on the ordering, equation (7) will provides us with an information share for a particular ordering. By considering all

³ In Hasbrouck (1995), different series corresponds to the prices of the same security being traded in multiple markets. Therefore, in equilibrium, all the prices must be equal.

⁴ When $n = 2$, it can be shown that $F = \begin{bmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sigma_2(1-\rho^2)^{\frac{1}{2}} \end{bmatrix}$ and $\Omega = FF^T$.

possible orderings we can compute the upper and lower bounds on IS (see Hasbrouck (1995) for detail).

B. Modified Information Share (MIS):

In this paper, we propose a new measure of information share that is independent of the ordering and, therefore, leads to a unique measure of information share. Hasbrouck (1995) assumes the following factor structure for the innovations (i.e., equation (15) in Hasbrouck (1995)):

$$\varepsilon_t = Fz_t, \quad E[z_t] = 0, \quad E[z_t z_t^T] = I \quad (8)$$

where F is taken as the Cholesky factorization of Ω , i.e. $\Omega = FF^T$. However, as mentioned above, this leads to the upper and lower bounds for the information share, instead of a unique information share, when the innovations are not independent. This has to do with the fact that the information share, based on Cholesky factorization, depends on the ordering of the series.

We can make the information share independent of the ordering by assuming a different factor structure to be discussed next. Let Φ represent the correlation matrix of the innovations. Let Λ represent the diagonal matrix with diagonal elements being the eigen values of the correlation matrix Φ , where the corresponding eigen vectors are given by the columns of matrix G . Finally, let V be a diagonal matrix containing the innovation standard deviations on the diagonal. Then the following transformed

innovation z_t^* can be shown to have zero mean and identity covariance matrix, i.e.,

$$E[z_t^*] = 0 \text{ and } E[z_t^* (z_t^*)^T] = I :$$

$$z_t^* = G\Lambda^{-1/2}G^T V^{-1}\varepsilon_t \quad (9)$$

Then, we have the following factor structure for the innovations:

$$\varepsilon_t = F^* z_t^* \quad (10)$$

where $F^* = [G\Lambda^{-1/2}G^T V^{-1}]^{-1}$. Note that $\Omega = F^* (F^*)^T$. Then, under this factor structure, the modified information share (MIS) is given by

$$S_j^* = \frac{([\psi F^*]_j)^2}{\psi \Omega \psi^T} \quad (11)$$

It is important to note that under this new factor structure, the resulting information share measures are independent of ordering. Therefore, this leads to a unique measurement of information share instead of upper and lower bounds.

If the number of series considered is 2, we have the following:

$$G = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \Lambda = \begin{bmatrix} (1+\rho) & 0 \\ 0 & (1-\rho) \end{bmatrix}, \text{ and}$$

$$F^* = \begin{bmatrix} 0.5(\sqrt{1+\rho} + \sqrt{1-\rho})\sigma_1 & 0.5(\sqrt{1+\rho} - \sqrt{1-\rho})\sigma_1 \\ 0.5(\sqrt{1+\rho} - \sqrt{1-\rho})\sigma_2 & 0.5(\sqrt{1+\rho} + \sqrt{1-\rho})\sigma_2 \end{bmatrix}.$$

C. King-Plosser- Stock-Watson Method:

King et al. (1991) consider the model represented by equation (2) to be a reduced form model where a constant vector is included in the model. Therefore, the reduced form model is given by

$$\Delta Y_t = \mu + \Psi(L)\varepsilon_t \quad (12)$$

Corresponding structural model is considered as follows:

$$\Delta Y_t = \mu + \Gamma(L)\eta_t \quad (13).$$

One can get the reduced form model (12) from the structural model (13) with $\varepsilon_t = \Gamma_0\eta_t$ and $\Psi(L) = \Gamma(L)\Gamma_0^{-1}$. The identification of the structural model from the reduced form model requires imposition of some restrictions. In the case where there is only one common stochastic trend, like the one considered here, King et al. (1991) suggests the following restriction on the cumulative impulse response matrix:

$$\Gamma(1) = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \quad (14).$$

This restriction is obtained by assuming that the innovation to the permanent component is uncorrelated with the innovations to the transitory component. It is interesting to note that the normalization imposed on the parameter (equation (14)) implies that the cumulative impulse responses to the permanent innovation of all series are unity.

Tse (1998), Booth, So and Tse (1999) and Tse (2000) use the above model to analyze the price discovery mechanism where they compute the forecast error variance decomposition (VDC) as well as cumulative impulse response function (CIR) for different lags. The market, for which the largest part of the forecast error variance is attributed to common trend innovation for a given lag, is considered informationally efficient. Similarly, the price discovery is supposed to take place in the market with the greatest CIR for a given horizon (lag).

However, this technique is associated with some problems. Firstly, empirically it is possible that the variance decomposition may change with lag with one market being informationally efficient for certain set of lags but not for other lags. The analysis based on CIR also suffers from this problem in addition to the fact that as lag increases they all tend to 1 by construction (see equation (14)).

D. Gonzalo-Granger Permanent-Temporary Measure (PT/GG):

Gonzalo and Granger (1995) suggests a way of decomposing the vector of non-stationary series Y_t into permanent (common factor) component f_t (which is non-stationary or I(1) series) and transitory (stationary) component \tilde{Y}_t . The identification of these components is achieved by assuming that (i) the permanent component is a linear

function of the original series and that (ii) the transitory component does not Granger-cause the permanent component in the long run. Under these identification conditions, the series Y_t can be written as follows:

$$Y_t = Af_t + \tilde{Y}_t \quad (15)$$

This method of decomposing the original series into permanent and transitory components will henceforth be referred to as “PT/GG” method. The permanent component f_t (under linearity condition) can be written as

$$f_t = \theta^T Y_t \quad (16)$$

where θ is the permanent component coefficient vector. The dimension of the permanent component (number of permanent components) is equal to the number of common stochastic trends in the system, which is assumed to be one. Therefore, in the case under consideration, f_t is a one dimensional series.

The argument for using PT/GG method is based on the assumption that the permanent component represents the information. As each of the original non-stationary series potentially contributes to the permanent component (equation (16)), one can use θ_i (the i^{th} component of the coefficient vector θ) to measure the contribution of market i (or, the i^{th} component of Y_t) to the price discovery process. This is the approach taken by Booth et al. (1999, 2002) and Harris, McInish and Wood (2002). For example, if $\theta_1 = 0$,

this implies that the first market (or, Y_{1t}) has no contribution to the price discovery. Similarly, if $\theta_2 = 0$, then the second market has no contribution to the price discovery. Specifically, Harris et al. (2002) suggests the use of the elements of θ as measures of price discovery after the normalization so that the sum of the elements is equal to 1.

Gonzalo and Granger (1995) has shown that $\theta = \alpha_{\perp}$, where α_{\perp} is a column vector orthogonal to the adjustment coefficient vector α , i.e., $\alpha_{\perp}^T \alpha = 0$. Therefore, the permanent component f_t can be written as follows:

$$f_t = \alpha_{\perp}^T Y_t \quad (17)$$

In order to see the relationship between this representation and Stock-Watson common stochastic trend representation, we can substitute equation (5) into equation (17) to get the following (see de Jong (2002)):

$$f_t = \alpha_{\perp}^T Y_t = \alpha_{\perp}^T Y_0 + \alpha_{\perp}^T \beta_{\perp} \alpha_{\perp}^T \sum_{i=1}^t \varepsilon_i + \alpha_{\perp}^T \Psi^*(L) \varepsilon_t \quad (18)$$

Or, alternatively,

$$f_t = \alpha_{\perp}^T Y_t = \alpha_{\perp}^T Y_0 + \delta \alpha_{\perp}^T \sum_{i=1}^t \varepsilon_i + \alpha_{\perp}^T \Psi^*(L) \varepsilon_t \quad (19)$$

where the scalar parameter δ is given by $\delta = \alpha_{\perp}^T \beta_{\perp}$. Therefore, it is clear that Gonzalo-Granger permanent component (or common factor) consists of the Stock-Watson common stochastic trend plus a stationary series. Therefore, Gonzalo-Granger permanent component is non-stationary (I(1)) but not necessarily martingale or pure random walk

(Hasbrouck (2002)). Harbrouck (2002) considers this property to be a disadvantage of PT/GG by saying that “... disadvantage of the PT/GG procedure is that the common factor is generally not martingale, and is therefore of questionable economic relevance.”

E. Performance Comparisons using Simulated Market Data:

In order to compare the performance of alternative methods, we apply these methods to simulated market data for the three examples considered by Hasbrouck (2002). We normalize the permanent component coefficient vector θ so that the sum of the coefficient is equal to unity. Due to some serious problems associated with King et al. (1991) method, as described earlier, we exclude this method. In the simulation, we use the sample size of 100,000 observations as in Hasbrouck (2002).⁵ However, rather than one run, we repeat the estimation 1,000 times with the same sample size. This allows us to create the confidence intervals for comparison. The simulation is performed for the same three examples considered by Hasbrouck (2002) which are briefly presented below.

Let m_t denote the unobservable efficient price. The trade direction for market $i = 1, 2$ is denoted by q_{it} which takes values of either 1 or -1 with equal probability with q_{1t} being independent of q_{2t} . Finally, p_{it} denotes the price in market i . The three examples are given below:

Example 1: (A two-market Roll model)

$$m_t = m_{t-1} + u_t, \quad u_t \sim N(0,1) \quad (20)$$

⁵ The random number generator used is based on Matsumoto and Nishimura’s (1998) Mersenne Twister which has a super astronomical period of $2^{19937} - 1$.

$$p_{it} = m_t + q_{it} \quad (21)$$

Example 2: (Two markets with private information)

$$m_t = m_{t-1} + q_{1t} \quad (22)$$

$$p_{1t} = m_t + q_{1t} \quad (21)$$

$$p_{2t} = m_{t-1} + q_{2t} \quad (23)$$

Example 3: (Two markets with public and private information)

$$m_t = m_{t-1} + q_{1t} + u_t, \quad u_t \sim N(0,1) \quad (24)$$

$$p_{1t} = m_t + q_{1t} \quad (21)$$

$$p_{2t} = m_{t-1} + q_{2t} \quad (23)$$

The result of the simulation is reported Table 1. For example 1, the PT/GG provides estimate for price discovery which is, on the average, very close the theoretical value of 0.5. Furthermore, 0.5 lies in the 90 percent confidence interval (between 5th and 95th percentile values). The PT/GG leads to auto-correlated change in efficient price (Δm_t) with variance which is almost double its theoretical value. However, the autocorrelations are significant only up to lag 1.⁶ The average modified IS estimate is also very close to the theoretical value. Furthermore, the 90 percent confidence interval is narrower for this measure compared to the PT/GG measure. This is also evident from the fact that the PT/GG measure has higher standard deviation (1.402) compared to the

⁶ Even though we only report the autocorrelation up to lag 2, the autocorrelations for higher order lags were small and insignificant.

standard deviation of modified IS measure (0.828).⁷ Therefore, it seems that for example 1, modified Hasbrouck measure performs better compared to PT/GG measure.

We get similar results for example 2, except that the variance of change in efficient price associated with PT/GG method is much higher than the theoretical value with average of 5.0265. As for the example 3, PT/GG measure performs (under estimates) poorly with the average price discovery which is about 40 percent away from the theoretical value and the associated variance of efficient price change twice the theoretical value. Furthermore, the 90 percent confidence interval does not include the theoretical value. Whereas the modified IS measure performs quite well. Even though it is biased downward, the average is much closer to the theoretical value. Therefore, it can be said that once we modify the Hasbrouck's IS measure to take care of bounds, it performs well compared to PT/GG measure in all three examples.

[Please insert Table 1 here]

III. Empirical Analysis

In this paper, we empirically test and estimate the price discovery mechanism in the spot and futures markets using different methods discussed in the previous section. In total, 9 commodities are included in the analysis covering stock index, currency, metal etc.⁸ We use the daily data on these series. Table 2 presents the names of the

⁷ Here the standard deviation represents the standard deviation of the 1,000 estimates of the market shares generated during the simulation.

⁸ Actually, we started out with 23 different commodities. However, the rests were dropped due to the fact that the series were found to be either stationary or not cointegrated. Note that the methods discussed in the paper applies to cases where the series are non-stationary and cointegrated.

commodities, sample periods and sample sizes considered. The unit-root tests, reported in Table 3, indicate that all the series are found to be unit root.

In this case, we have two series ($n = 2$) and the logarithm of spot price represents the first series and the logarithm of futures price represents the second series. The lambda max and trace test statistics based on Johansen (1991) are reported in Table 4. Both lambda max and trace tests indicate the existence of a single cointegrating vector for each of the 9 commodities considered.

[Please insert Table 2, Table 3 and Table 4 here]

Having established the existence of a single cointegrating relationship, we perform the test on price discovery based on PT/GG method. One advantage of the PT/GG method is that we can perform the tests on the elements of the permanent component coefficient vector θ . Due to the orthogonality of the permanent component coefficient vector and adjustment coefficient vector (α), i.e., $\theta = \alpha_{\perp}$, the null hypothesis that the price discovery does not take place in the first market (spot market), i.e., $H_0 : \theta_1 = 0$, is equivalent to $H_0 : \alpha_2 = 0$. Similarly, the null hypothesis that the price discovery does not take place in the second market (futures market) can be expressed as $H_0 : \theta_2 = 0$ or, equivalently, as $H_0 : \alpha_1 = 0$. These tests can be performed using the Chi-squared test based on Johansen (1991) approach.

The normalized cointegrating vectors, the adjustment coefficient vectors as well as the chi-squared tests are reported in Table 5. The Chi-squared tests indicate that for TOPIX, FTSE 100, CAC 40 and British Pound, the price discovery take place in the spot

market. For Corn, the price discovery takes place in the futures market. However, for All Ordinary, Coffee and Silver the price discovery seem to take place both in the spot as well as in the futures markets. As for Nikkei 225, even though at least one of the adjustment coefficients is expected to be significant (due to the existence of cointegrating vector based on trace and lambda max statistics reported in Table 4), both came out to be significant only at 15 percent. Therefore, based on this 15 percent level of significance, the price discovery seems to take place in both markets for Nikkei 225. Even though lower transaction cost implies that the price discovery should take place mostly in the futures market, our empirical results, based on PT/GG method, indicate that the price discovery take place mostly either in the spot market or both markets.

[Please insert Table 5 here]

As mentioned earlier, PT/GG method completely ignores the innovations driving the price system. Therefore, we estimate the Hasbrouck's (1995) information shares (IS) as well as the modified IS measure (MIS), which incorporate the information on innovation variances in addition to the characteristic of the dynamic system. The results are summarized in Table 6. For many commodities like Nikkei 225, TOPIX, FTSE 100, CAC 40 and British Pound, the upper and lower bounds for IS are far apart and those bounds for spot markets are close to the bounds for the futures markets. As shown in the Appendix, a large innovation correlation coefficient leads to such results. Indeed, large sample innovation correlations, reported in Table 6, for these commodities can explain such results.

However, the modified information share (MIS) provides a much clearer picture. The MIS measures are also reported in Table 6. Surprisingly, based upon MIS, we find that the price discovery takes place mostly in the futures market instead of the spot market for all commodities except for TOPIX where the price discovery seem to take place in both markets with the modified information share of the spot and futures markets being equal to 47.1 and 52.9 percent respectively. This result is different from the one obtained using PT/GG method. This difference can be explained by the difference in innovation standard deviation. For example, the standard deviations of the futures innovations are found to be higher than that of the spot innovations with the exception of Corn and Nikkei 225 in which case the innovation standard deviations are almost equal (see the last columns of Table 6).

[Please insert Table 6 here]

Since the results based on PT/GG method and modified IS measure are fairly different in nature, we need to decide on which method should be relied upon. As explained above, PT/GG method completely ignores the innovations, which is expected to characterize the information arriving in different markets. Furthermore, the simulation results also indicate that, for the examples considered, the modified IS measure performs better compared to PT/GG method. Therefore, we feel that modified IS method must be relied upon in drawing conclusions. Based on modified IS measure, we conclude that the price discovery takes place mostly in the futures market. The evidence supports the transaction cost hypothesis, which states that price discovery is expected to take place in the market with lower transaction cost.

IV. Conclusion

In a pioneering work, Hasbrouck (1995) suggests a measure of price discovery commonly known as the information share (IS). The attractive feature of IS has to do with the fact that it incorporates both the system dynamics as well as the characteristics of the innovations driving the dynamic system. In addition, it is consistent with the conventional argument that the security prices are martingale. However, the Hasbrouck's method leads to the upper and lower bounds for IS instead of a unique measure. In this paper, we propose a modified information share (MIS) that is unique.

In this paper, we also compare the performance of the MIS with another price discovery method based on the permanent-temporary decomposition method suggested by Gonzalo and Granger (1995) (PT/GG) using simulated data. Specifically, we perform simulations with 1,000 replications (with individual series of length 100,000 observations) using the same three examples considered by Hasbrouck (2002). In terms of bias and size of the confidence intervals, we find the MIS method to perform much better compared to the PT/GG methods.

As an empirical application, we estimated the price discovery using MIS and PT/GG methods for 9 sets of spot and futures markets. The PT/GG method indicates that the price discovery mostly takes place either in the spot market or both spot and futures markets. However, the MIS indicates that the price discovery takes place mostly in the futures markets. As explained in the main text, PT/GG method completely ignores the innovations, which is expected to characterize the information arriving in different markets. Furthermore, the simulation results also indicate that, for the examples

considered, the MIS performs better compared to PT/GG method. Therefore, we suggest that the MIS method must be relied upon in drawing conclusions. Based on the MIS, we conclude that the price discovery take place mostly in the futures market. The evidence supports the transaction cost hypothesis, which states that the price discovery is expected to take place in the market with lower transaction cost. Our conclusion agrees with the evidence found by other researchers in the area.

APPENDIX

In this appendix, we show the dependence of the IS bounds on the correlation between the spot and futures innovations. Note that the Cholesky factorization matrix is given by

$$F = \begin{bmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sigma_2\sqrt{1-\rho^2} \end{bmatrix}.$$

This leads to the following bounds on information shares

$$\begin{aligned} S_1^{\max} &= \frac{\psi_1\sigma_1 + \psi_2\rho\sigma_2}{\psi\Omega\psi'} & \text{and} & & S_2^{\min} &= \frac{\psi_2\sigma_2\sqrt{1-\rho^2}}{\psi\Omega\psi'}, \\ S_2^{\max} &= \frac{\psi_2\sigma_2 + \psi_1\rho\sigma_1}{\psi\Omega\psi'} & \text{and} & & S_1^{\min} &= \frac{\psi_1\sigma_1\sqrt{1-\rho^2}}{\psi\Omega\psi'}. \end{aligned}$$

Therefore, when the correlation approaches one, the maximum IS of the first market approaches the maximum IS of the second where the minimum for both approaches zero.

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Table 1: Market 1's Price Discovery Share (%)

0	Gonzalo-Granger Permanent Temporary (PT/GG)				Hasbrouck Information Share			Modified IS
	PT/GG	$Var(\Delta m_t)$	$\rho(1)$	$\rho(2)$	Lower IS	Upper IS	$Var(\Delta m_t)$	
Panel A: Example 1								
Structural Model	50.000	1.0000	0.0000	0.000				
Average	49.974	2.0005	-0.2500	-8.2E-05	21.951	78.105	1.0003	50.034
5 th Percentile	47.612	1.9865	-0.2550	-6.0E-03	20.774	76.933	0.9562	48.632
95 th Percentile	52.176	2.0139	-0.2450	5.0E-03	23.063	79.175	1.0454	51.330
Std. Deviation	1.402	0.0085	0.0029	3.4E-03	0.694	0.690	0.0276	0.828
Panel B: Example 2								
Structural Model	100.000	1.0000	0.0000					
Average	100.165	5.0265	-0.4000	-9.0E-06	99.713	99.716	0.9968	99.715
5 th Percentile	95.782	4.5139	-0.4120	-6.0E-03	99.562	99.563	0.9538	99.567
95 th Percentile	104.804	5.5990	-0.3880	6.0E-03	99.849	99.848	1.0438	99.849
Std. Deviation	2.772	0.3353	0.0072	3.6E-03	0.090	0.084	0.0282	0.085
Panel C: Example 3								
Structural Model	100.000	2.0000	0.0000					
Average	60.054	2.0113	0.0010	-1.2E-04	90.124	98.253	2.0004	94.975
5 th Percentile	55.624	1.7535	-0.0670	-6.0E-03	89.278	97.905	1.9020	94.393
95 th Percentile	64.809	2.3124	0.0700	5.0E-03	90.965	98.577	2.0992	95.538
Std. Deviation	2.803	0.1693	0.0420	3.2E-03	0.514	0.211	0.0591	0.358

Note: $\rho(i)$, $i = 1, 2$ denotes the autocorrelation of change in efficient price (Δm_t) at lag i . Modified IS refers to the unique information share measure obtained by modifying the Hasbrouck's IS as suggested in the paper. The 95th percentile for PT/GG measure is above 100 percent. This could be due to the fact that the estimate of θ_2 came out to be small but negative even though theoretically it is supposed to be zero.

Table 2: Summary of Futures Contracts

	Commodity	Sample Period	Sample Size
1	Nikkei 225	September 5, 1988 – December 31, 1997	2432
2	TOPIX	September 5, 1988 – December 31, 1997	2432
3	FTSE100	May 3, 1984 – December 31, 1997	3564
4	CAC40	March 1, 1989 – December 31, 1997	2305
5	All Ordinary	January 3, 1984 – December 31, 1997	3651
6	Corn	January 2, 1979 – December 31, 1997	4956
7	Coffee	January 2, 1979 – December 31, 1997	4956
8	Silver	January 2, 1979 – December 31, 1997	4956
9	British Pound	January 2, 1986 – December 31, 1997	3129

Note. This table lists the commodities, sample periods, and sample sizes for the 25 different futures contracts used for empirical analyses in this study.

Table 3: Results of the Unit-Root Tests on Futures and Spot Prices

		Logarithm of Spot Price		Logarithm of Futures Price	
		ADF Test	PP Test	ADF Test	PP Test
1	Nikkei 225	-1.177	-1.075	-1.136	-1.076
2	TOPIX	-1.008	-1.022	-1.142	-1.108
3	FTSE 100	-0.657	-0.645	-0.723	-0.723
4	CAC40	-1.077	-1.066	-1.392	-1.243
5	All Ordinary	-1.545	-1.538	-1.647	-1.550
6	Corn	-2.236	-2.530	-2.467	-2.412
7	Coffee	-1.524	-1.856	-2.257	-2.429
8	Silver	-1.890	-1.971	-1.806	-1.866
9	British Pound	-2.556	-2.561	-2.701*	-2.701*

Note: This table lists the results of the Augmented Dickey-Fuller and Phillips-Perron Unit-Root Tests on the logarithm of Spot and Futures prices. The critical values are – 2.57, –2.87 and –3.43 at 10%, 5% and 1% level of significance respectively.

*** significant at 1%

** significant at 5%

* significant at 10%

Table 4: Johansen-Juselius Tests on Number of Cointegrating Vectors

		No. of Cointegrating Vectors = 0 $r = 0$			No. of Cointegrating Vectors = 1 $r = 1$		
		Eig. Val.	λ_{\max}	Trace	Eig. Val.	λ_{ma}	Trace
1	Nikkei 225	0.0346	85.449***	87.45 ***	0.0008	2.008	2.008
2	TOPIX	0.0288	70.873***	72.98 ***	0.0009	2.115	2.115
3	FTSE 100	0.0249	89.685***	96.90 ***	0.0020	7.224	7.224
4	CAC40	0.0260	60.696***	63.05 ***	0.0010	2.355	2.355
5	All Ordinary	0.0359	133.205***	137.78 ***	0.0013	4.582	4.582
6	Corn	0.0150	75.030***	81.78 ***	0.0014	6.754	6.754
7	Coffee	0.0096	47.861***	50.98 ***	0.0006	3.125	3.125
8	Silver	0.0552	280.933***	285.11 ***	0.0008	4.178	4.178
9	British Pound	0.0256	81.169***	87.94 ***	0.0022	6.775	6.775

Note: The critical values for λ_{\max} statistic for $r = 0$ are 13.75, 15.67, and 20.20 at 10%, 5% and 1% respectively and for $r = 1$ are 7.52, 9.24 and 12.97 at 10%, 5% and 1% respectively. The critical values for Trace statistic for $r = 0$ are 17.85, 19.96 and 24.60 at 10%, 5% and 1% respectively and for $r = 1$ they are the same as for λ_{\max} statistic.

Table 5: The Estimation of the Normalized Cointegrating Vectors ($\beta_1 S_t + \beta_2 F_t + \beta_3 = 0$) and Loading Vectors (α)

		Cointegrating Vector			Adjustment Vector		Chi-Square Test	
		β_1	β_2	β_3	α_1	α_2	$\alpha_1 = 0$ $\theta_2 = 0$	$\alpha_2 = 0$ $\theta_1 = 0$
1	Nikkei 225	1	-0.985	-0.145	-0.072	0.069	2.444	2.265
2	TOPIX	1	-0.985	-0.103	-0.028	0.123	0.491	7.472 ***
3	FTSE 100	1	-1.001	0.017	-0.025	0.046	1.733	4.271 **
4	CAC40	1	-1.018	0.141	0.008	0.083	0.063	5.850 **
5	All Ordinary	1	-0.990	-0.073	-0.097	0.043	77.451 ***	4.181 **
6	Corn	1	-1.109	0.643	-0.027	-0.001	31.918 ***	0.021
7	Coffee	1	-1.269	1.328	-0.009	0.011	8.735 ***	9.755 ***
8	Silver	1	-0.994	-0.032	-0.152	0.216	37.288 ***	71.080 ***
9	British Pound	1	-1.020	0.004	-0.008	0.085	0.066	7.669 ***

Note: The chi-square statistic has degrees of freedom equal to 1. The critical values are 2.706, 3.841 and 6.635 at 10%, 5% and 1% respectively. Note that the null hypothesis $\alpha_1 = 0$ is equivalent to the null hypothesis $\theta_2 = 0$ and vice versa.

Table 6: Estimation of Information Share

	ρ	Information Share of Spot Market			Information Share of the Futures Market			Information Share based on PT/GG		Innovation Standard Dev.	
		Modified IS	Upper Bound	Lower Bound	Modified IS	Upper Bound	Lower Bound	Spot (θ_1)	Futures (θ_2)	Spot (%)	Futures (%)
Nikkei 225	94.26	0.403	0.930	0.005	0.597	0.995	0.070	0.490	0.510	1.378	1.375
TOPIX	91.17	0.471	0.943	0.033	0.529	0.967	0.057	0.814	0.186	1.145	1.285
FTSE 100	92.97	0.344	0.884	0.001	0.656	0.999	0.116	0.649	0.351	0.901	1.058
CAC40	95.79	0.414	0.947	0.003	0.586	0.997	0.053	1.106	-0.106	1.105	1.196
All Ordinary	47.53	0.150	0.362	0.022	0.850	0.978	0.638	0.308	0.692	0.758	1.446
Corn	74.04	0.211	0.608	0.004	0.789	0.996	0.392	-0.025	1.025	1.411	1.327
Coffee	58.54	0.086	0.329	0.000	0.914	1.000	0.671	0.562	0.438	1.833	2.223
Silver	60.73	0.308	0.628	0.067	0.692	0.933	0.372	0.587	0.413	2.088	2.152
British Pound	93.63	0.389	0.917	0.005	0.611	0.995	0.083	0.919	0.081	0.669	0.701

* ρ denotes the sample correlation between the spot and futures innovations. The information shares of spot and futures markets θ_1 and θ_2 are normalized so that $\theta_1 + \theta_2 = 1$..