Nominal Mortgage Contracts and the Effects of Inflation on Portfolio Allocation

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Abstract

Under a standard mortgage contract the household's mortgage payment is fixed in nominal terms. In order to transfer assets from home equity to the financial portfolio, the household must refinance the mortgage and reset the nominal payment to the current price level. The higher the rate of inflation, the faster the real mortgage payment declines over the life of the mortgage, and the household faces higher refinancing costs, and re-balance their portfolio between home equity and financial assets less frequently. As the rate of inflation increases, households hold larger positions in home equity earlier in the life-cycle, and smaller positions later in the life-cycle. The costs associated with resetting the nominal mortgage payment when refinancing also helps explains why retired households hold such a significant portion of their wealth in housing.

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1 Introduction.

When attempting to discern the avenue through which inflation can affect the real macro-economy, a common assumption is that prices are “sticky”. These “sticky” prices are usually taken to represent long-term wage contracts, or menu costs associated with changing prices. These examples overlook perhaps one of the “stickiest” of prices, the nominal mortgage contract. While wage contracts often are re-negotiated every year or two, a standard mortgage results in the fixed nominal mortgage payments for up to 30 years. Despite the fact that mortgage interest rates are nominal and reflect both current and anticipated levels of inflation, the nature of the fixed nominal payment does impact investment behavior over the life-cycle. The real mortgage payment declines over the life of the contract, while the payment on a new mortgage increases in pace with inflation. This friction discourages households from more frequently re-balancing their total portfolio and shifting assets from home equity into financial assets.

Housing wealth is a vitally important but understudied component of household wealth. The single most significant asset for many households in the United States is the equity held in their home. Flavin and Yamashita (2002) used the Panel Study of Income Dynamics to show that among homeowner households with a head between 18 and 30 years old, 67.8% of their portfolio is in their home. In the same article the authors documented how a household’s exposure to risk through their housing wealth could impact the portfolio allocation of their financial wealth. Fernández-Villaverde and Krueger (2001) document how housing could be used as collateral to relax lending constraints. These papers, among others, demonstrate that to understand the accumulation and composition of household wealth, one must first understand housing wealth.

One approach to modeling housing wealth's role in the life-cycle is to develop an abstract model that captures only a few of the most important aspects of housing as an investment good. Papers such as Martin (2001) and Fernández-Villaverde and Krueger (2001) follow this approach. This type of model's advantages are that many can be solved analytically, or embedded in a general equilibrium framework and solved numerically. The primary disadvantage is the relatively narrow scope of such a model. The second approach is to sacrifice simplicity for a more complicated partial equilibrium model that can be solved numerically using stochastic dynamic programming. Examples of this approach include Li and Yao (2004) and Hu (2002). Its advantage is that as a more complex model it presents a more realistic picture of the role of housing wealth over the life-cycle. The downside is an upper limit on the model's complexity level, beyond which the solution times are no longer tractable. Parallel processing can extend this upper limit in a grid-cluster or super-computer environment. The greater complexity of the model requires great care in presenting the results and currently precludes the option of embedding the model in a general equilibrium framework.

Both of the approaches described above are important and legitimate. Many of the questions the more detailed partial equilibrium models can address are outside the scope of the general equilibrium models. By explicitly including so many different aspects of housing wealth simultaneously, the partial equilibrium model is extraordinarily flexible. The detailed partial equilibrium model in this paper provides important insight into how to develop an more abstract general equilibrium model that can still capture the complexities associated with modeling housing wealth.

The dynamic stochastic optimizing framework adopted for the household for this paper is based on Rust and Phelan (1996). Rust and Phelan set up and solve a dynamic programming problem of labor supply with incomplete markets, Social Security, and Medicare. The dynamic programming problem in their paper is solved by discretizing the continuous state spaces and then using backward recursion to solve for the optimal value of the continuous choice variable at each point on the state space grid. The detailed rules governing the Social Security and Medicare application processes and benefits are embedded in the income transition matrix. The model in this paper has a similar structure, but instead embeds the detailed characteristics of the mortgage market contract in the income transition matrix.

The model's design allows households to choose their current consumption, their savings, their savings allocated to risky assets, which type of housing to occupy, and whether to refinance their mortgages. The housing tenure choice includes a rental unit, a small home, and a large home. Households may increase the sizes of their mortgage balances through the use of a cash-out refinance. Renters choose the size of the rental unit so that the intra-period marginal utility of housing is equal to the marginal utility of non-durable consumption. The size of the large and small homes are fixed in terms of the number of housing units they represent. Households face uncertainty in the returns on risky assets and housing, the probability of survival, and a transitory shock to income; which is otherwise a deterministic function of age. The model includes moving, maintenance, and transaction costs. Both the option to and the costs of defaulting on a mortgage are also included in the model. The model is solved given the terms of a traditional 30-year fixed rate mortgage contract. The values of non-structural parameters, such as returns on different types of assets, the survival probability, mortgage terms, and income process, are taken from historical data.

The model's solution is then used to demonstrate how the rate of inflation, even when it is perfectly anticipated and unchanging over the life-cycle, can distort the household's portfolio allocation. Several additional versions of the model are solved, one version where the mortgage contract is in real and not nominal terms and another version with a historically high rate of inflation and a nominal mortgage contract. The levels of housing and financial wealth as well as the allocation of the financial portfolio are compared across the different inflation regimes.

2 A Partial Equilibrium Model of Housing Wealth with Mortgage Contracts.

This section describes the structure of the finite-horizon life-cycle model of a household's savings, investment, and housing decisions. The structure of the model described here was chosen to highlight the effects of mortgage contracts on the evolution of housing wealth. In order to keep the model focused and tractable aspects such as endogenous labor supply an margin account for risky asset are excluded. The section concludes with a discussion of the method used to solve the household's optimization problem. The structure of the model is actually quite straight forward, with most of the complexities embedded in the wealth transition rules. Households receive utility from the consumption of both a non-durable good and the stock of housing that they own. Each period in the model represents a single year. Table **??** in Appendix B provides a listing of the model parameters and their definitions.

Their optimization problem is to maximize their lifetime utility, defined as:

$$E\sum_{t=20}^{80}\beta^t\rho_t U(c_t,h(i_t))+\beta^t(1-\rho_t)(\theta_A U_B(A_t)+\theta_H U_B(H_t)-\theta_D U_B(D_t)) \quad c_t>0,\forall t \tag{2.1}$$

$$U(c_t,i_t)=\frac{(c_t^{1-\phi}h(i_t)^{\phi})^{1-\lambda}}{1-\lambda} \tag{2.2}$$

$$U_B(b)=\frac{b^{1-\lambda}}{1-\lambda} \tag{2.3}$$

where,

- c_t represents the consumption of non-durables;
- $h(i_t)$ represents the number of units of housing services consumed, given the housing tenure choice in period t (note that while the number of units of housing services consumed varies with tenure choice the utility gained

from a unit of services does not vary);

- A_t, H_t, and D_t are respectively the value of the financial assets, home, and mortgage debt left as bequests;
- β represents the discount rate;
- ρ_t is the survival probability;
- ϕ represents the measure of preference between of housing and consumption;
- λ represents a measure of risk aversion; and,
- θ_A, θ_H, and θ_M represent bequest parameters.

A household lives at most 80 years. It faces uncertainty about its survival, temporary income shocks, and the rate of return on both housing and risky assets. In addition to the stochastic elements for income and the rate of return on risky assets, the households may experience an additional shock. A small probability exists that the household will experience unemployment in one period, reducing income to zero. Also, a small independent probability exists of a stock market crash where the household will lose 100% of its investment in the risky financial asset. The probability of a stock market crash is in addition to the regular standard deviation associated with the stochastic rate of return on risky assets. Households also are not allowed to consume negative amounts of non-durable goods. The price of the consumption good is set equal to unity and the rental price of housing is set equal to a constant ratio of the underlying price of the housing unit. The inflation rate is constant and known.

2.1 Consumption of Housing

While consumption of the non-durable good in the model is continuous, the choices for housing consumption are partially discrete. The model has three different alternatives for housing: a rental unit, a small home, and a large home, represented by the corresponding symbols i_r, i_s, and i_l. The number of housing units available to rent is continuous while the number of housing units provided by a small or large home is fixed. Renter households are able to choose the number of housing units that equalizes their intra-period marginal utility from housing to their intra-period marginal utility from non-durable consumption.

$$\frac{\delta U(c_t, h(i_t))}{\delta c_t} = \frac{\delta U(c_t, h(i_t))}{\delta h(i_t)} \tag{2.4}$$

Optimal rental units may now be defined as a function of consumption,

$$h(i_r) = (\phi/(1-\phi))c_t \tag{2.5}$$

Many other factors in the model are conditional on current housing tenure, including rent or mortgage payments, maintenance costs, level of utility derived from housing, and the rate of appreciation in home value. The size of a small home is set equal to that of a median priced home, while the size of a large home is set to be twice that of a median priced home.

2.2 Accumulation of Financial Wealth and the Income Process

A household is “born” at age 20 with zero financial and housing wealth. It starts off as a renter with no savings. In each period it receives a draw from an age-dependent income process. The model contains no permanent income shock, only transitory shocks. In retirement, pension income is set to 60% of the deterministic portion of age 65 income. Pension income is still subject to transitory shocks, representing uncertainty regarding medical costs. Households can store their wealth in two different classes of assets, financial and real. The household’s financial assets are held in a portfolio of risk free and risky assets. The household can, at no cost, rebalance its financial portfolio between risk free and risky assets every period. Households with zero wealth face a binding liquidity constraint for financial assets in that they cannot borrow against their future income. Households also cannot purchase leveraged portfolios, where they borrow at the risk free rate to invest more in the risky asset. In addition to moving to one of the three types of housing, $\{i_r, i_s, i_l\}$, the household can also decide to stay in its current home, $\{i_{t+1} = i_t\}$. Households may also either add to their mortgage balance through a cash-out refinance or reduce their mortgage balance through a pre-payment refinance.

The transition rule for the level of financial wealth is defined as:

$$
\begin{aligned}
(2.6) \qquad A_{t+1} = {} & (1 + (1-\gamma)(\alpha_t \widetilde{r_{s_t}} + (1-\alpha_t)r))(A_t - c_t - X_t(i_t, \kappa_t) + \\
& G_t(i_t, i_{t+1}, \kappa_t) + Z_t(\kappa_t, \kappa_{t+1})) + (1-\gamma)\widetilde{e_{t+1}} + \gamma I_t(i_t, \kappa_t) \\
& s.t. \quad A_{t+1} \geq 0 \ \& \ 0 \leq \alpha_t \leq 1
\end{aligned}
$$

where,

- A_t is the level of financial assets in period t;
- A_{t+1} is a random variable that depends on the stochastic rate of return on risky assets ($\widetilde{r_{s_t}}$) in period t and the realizations of earning ($\widetilde{e_{t+1}}$) in period $t+1$;
- α_t is the share invested in risky assets in time t;
- r is the deterministic rate of return on risk-free assets;
- $X_t(i_t, \kappa_t)$ (equation (2.13)) is the housing costs incurred in period t for a household currently choosing tenure type i_t with a mortgage κ_t years old;
- $I_t(i_t, \kappa_t)$ (equation (2.15)) is the mortgage interest paid;
- $G_t(i_t, i_{t+1}, \kappa_t)$ (equation (2.16)) is the net gain for a household choosing i_t this period and i_{t+1} next period;
- $Z_t(\kappa_t, \kappa_{t+1})$ (equation (2.17)) is the net gain from cash-out refinancing; and

- γ is the tax rate on income and capital gains (note that both income and capital gains have the same tax rate and taxes on capital gains are paid immediately).

The net gain from a home sale is tax-free and the mortgage interest paid is deducted from taxable income. Both the housing expenses and the amount of the mortgage interest deduction are functions of the current housing choice and age of mortgage. Refinancing is modeled as a choice to lengthen the remaining number of years on the mortgage, or inversely, to shorten the current age of the mortgage. The model only allows cash-out refinancing and does not allow prepayments. The age of a mortgage for a rental unit or a mortgage that has been paid off is zero. Households receive their wages at the same time they realize the returns on their investment from the previous period. As a result, the state variable A_t represents all available cash on hand, consisting of previous financial wealth and current income.

The income process is defined as a deterministic function of age plus a transitory shock, as shown below in log form:

$$\log(e_t) = \psi_0 + \psi_1 t + \psi_2 t^2 + \varepsilon_e \tag{2.7}$$

$$\varepsilon_e \sim N(0, \sigma_e)$$

The real rate of return on risky assets is a random variable with the distribution:

$$r_{s_t} \sim N(\eta_s, \sigma_s^2) \tag{2.8}$$

where η_s is the expected real rate of return on the risky asset and σ_s^2 is the variance.

2.3 Price of Housing

In addition to the portfolio of financial assets, households can also store their wealth in real assets by purchasing a house. It is only through the purchase of a house, and the acquisition of a mortgage loan, that households can borrow against their future income. The use of durable goods as collateral is in the same spirit as Fernández-Villaverde and Krueger (2001). The only mortgage contract available to the household in this model requires a 20% down payment; has a term of 30 years; and requires mortgage payments based on a fixed interest rate and the size of the original mortgage. The mortgage balance and the mortgage payment are both in nominal terms while the rest of the model is in real terms. Households selling their home are also required to pay a transaction cost equal to 10% of the value of the home that they are purchasing. This represents realtors' fees, credit checks, and other expenses associated with the purchase.

The real price of housing has a positive trend over time. The purchase price of either a small or large home increases non-stochastically by the average market price increase in each period. The value of homes that have already been purchased changes according to a stochastic process, with the expected increase equal to the non-stochastic market price increase. A household that has had a series of excellent draws in home price appreciation will own a home worth relatively more than a comparable home on the market. A household that has had a series of poor draws in home price appreciation will own a home worth relatively less than a comparable home on the market.

The price per housing unit is the same across all types of housing. Large homes cost more than small homes because they provide more units of housing for the homeowner to consume. Renters may choose as small or as large a home to rent as they wish. Their rent is proportional to the current market value of their chosen home. As the value of housing units change, so do their rental rates. The value of owner-occupied units evolves stochastically while the value of newly purchased and rental units are set equal to the current deterministic market price. The market price of a housing unit is the number of housing units, $h(i_t)$, multiplied by the current market price of a housing unit, $(1+\eta_h)^t P_0$. The value of an owner-occupied unit is the value of the unit from the previous period, H_t, multiplied by the realized rate of appreciation for that unit in that period, $(1+\widetilde{r_h})$. The price of owner-occupied housing is allowed to evolve differently from the market price of housing in order to capture the idiosyncratic aspect of housing returns. The formulas for the market price of home type i_t ($P_t(i_t)$) and the housing wealth (H_{t+1}) transition rule are:

$$P_t(i_t) = (1+\eta_h)^t P_0 h(i_t) \tag{2.9}$$

$$H_{t+1} = \begin{cases} H_t(1+\widetilde{r_h}), & i_{t+1} = i_t \\ P_t(i_t), & i_{t+1} \in i_s, i_l \\ 0, & i_{t+1} = i_r \end{cases} \tag{2.10}$$

$$r_h \sim N(\eta_h, \sigma_h^2) \tag{2.11}$$

where P_0 is the price of a single unit of housing in period 0; $\widetilde{r_h}$ is the realized rate of appreciation on housing in period t; η_h is the expected rate of appreciation on housing; and σ_h^2 is the variance of the house price growth. Note that home prices are in real terms, the increase in the market price of housing is note due to general inflation, but a real increase in the value of the house with time.

2.4 The Mortgage

A significant source of the complexity in the model is the need to include the age of the mortgage in the state space. In the model this adds a discrete state variable with thirty-one discrete values, resulting in over 1.7 million points in the final state space. The computational techniques used to solve a problem of this scope are discussed briefly at the end of this section. The reason for including the age of the mortgage in the state space is the nature of the 30-year self amortizing mortgage. First, the actual equity households hold in their home is the difference between the value of the home minus the remaining balance on the outstanding mortgage. To accurately track the value of the household's home equity, it is necessary to track both the value of the home and the mortgage balance independently. The nature of the mortgage contract further complicates what would be a logical solution, the addition of a third continuous state variable for mortgage debt. The principal paid on a self amortizing mortgage is not constant over the life of the mortgage. Initial payments are almost completely composed of interest, with very little principal being paid. The final payments on a 30-year mortgage on the other hand are almost completely principal, with very little interest being paid. Therefore, the transition rule for mortgage debt is a function of the age of the mortgage. The fact that the mortgage balance and mortgage payment are in nominal terms provides an additional motivation for including the age of the mortgage in the state space. The real values of the mortgage balance and payment decline steadily over the life of the mortgage due to inflation.

The mortgage payment is based on the home price when purchased, and only changes when the household refinances the mortgage or sells the house. A cash-out refinance increases the number of years left on the mortgage. The formula for the real value of a mortgage payment at time t after κ_t years on a house of type i_t is:

$$M_t(i_t,\kappa_t) = \pi(1-\mu)P_{t-\kappa_t}[(1-(1+\pi)^{-\kappa_t})(1+\nu)^{\kappa_t}]^{-1} \tag{2.12}$$

where π is the nominal mortgage interest rate; ν is the inflation rate; and μ is the required down payment.

The cost of housing services also reflects the maintenance costs paid by homeowners. As a result, the formula for the real cost of housing services is:

$$X_t(i_t,\kappa_t) = \begin{cases} M_t(i_t,\kappa_t) + \delta H_t, & i_t \in i_s, i_l \\ 0.06P_t(i_r), & i_t = i_r \end{cases} \tag{2.13}$$

where δ is the percent of current home value required in maintenance costs. Rent is equal to 6% of the current market value of the unit being rented and renters pay none of the maintenance costs for the property.

The present value of the household's home equity is the current value of the house minus the amount of the outstanding mortgage balance. While the value of the house increases or decreases according to the stochastic return on housing, the outstanding mortgage balance is a monotonically declining function of the age of the mortgage. The formula for the real value of the mortgage balance at time t after κ_t years on a house of type i_t is:

(2.14)

$$D_t(i_t, \kappa_t) = \begin{cases} M_t(i_t, \kappa_t)\frac{1-(1-\pi)^{\kappa_t - 30}}{\pi}, & i_t \in i_s, i_l \ \ \& \ \ \kappa_t \leq 30 \\ 0, & (i_t \in i_s, i_l \ \ \& \ \ \kappa_t > 30) \quad \text{or} \quad (i_t = i_r) \end{cases}$$

The formulas for the mortgage payment is used to calculate the amount of mortgage interest paid for tax purposes. The values must be adjusted back from the real terms since this deduction is in nominal terms. The formula for the mortgage interest deduction is:

$$I_t(i_t, \kappa_t) = \pi M_t(i_t, \kappa_t)(1 - (1+\pi)^{\kappa_t - 30})(1+\nu)^{\kappa_t} \tag{2.15}$$

2.5 Gains from Sale or Refinancing

The net gain after paying transaction costs and down payments for a household choosing next period's tenure $i_{t+1} \in \{i_r, i_s, i_l\}$ is given by:

$$G_t(i_t, i_{t+1}, \kappa_t) = \begin{cases} H_t - D_t(i_t, \kappa_t) - \mu P_t(i_{t+1}) - \tau H_t - \chi, & i_{t+1} \neq i_t \\ 0, & i_{t+1} = i_t \end{cases} \tag{2.16}$$

where τ is the transaction cost; μ is the downpayment rate; and χ is a fixed moving cost paid regardless of which type of housing is being purchased. When the household chooses not to move, $i_{t+1} = i_t$, it has zero net gain.

The net gain after choosing to refinance a mortgage is defined as the sum of the difference between the mortgage balances before and after the refinance and a fee for the transaction. Interest rates are constant in this model, so there is never any incentive to refinance at a lower interest rate. The only benefit of refinancing is to extract home equity in order to invest in financial assets or smooth consumption. When no refinance occurs $\kappa_{t+1} = \kappa_t + 1$ and the net gain is zero.

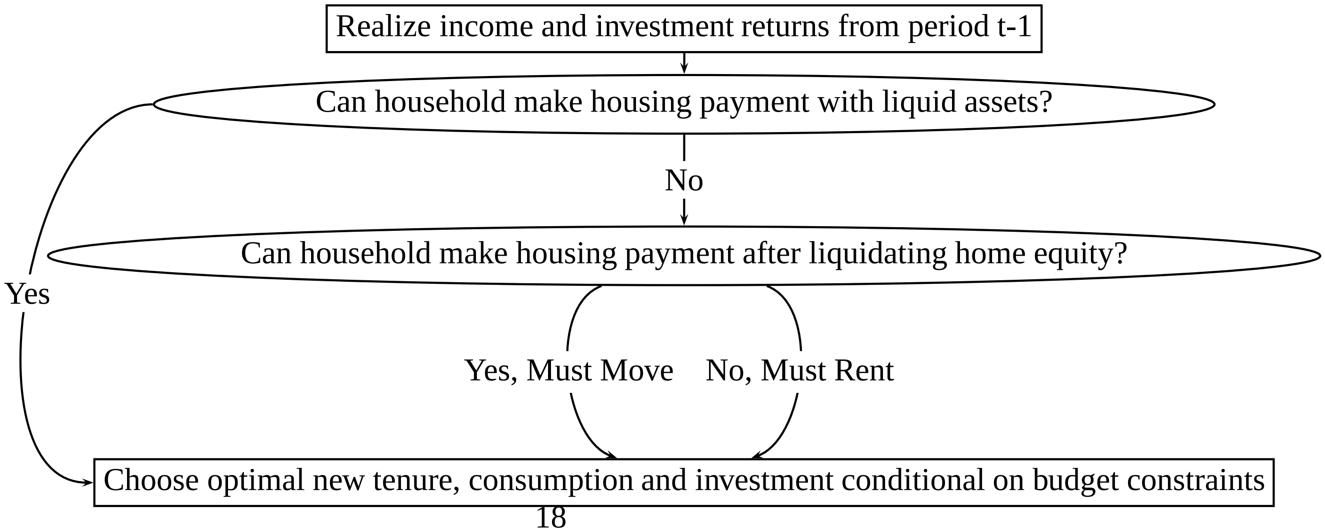
$$Z_t(\kappa_t, \kappa_{t+1}) = \begin{cases} (1-\zeta) D_t(i_t, \kappa_{t+1}) - D_t(i_t, \kappa_t), & \kappa_{t+1} \neq \kappa_t + 1 \\ 0, & \kappa_{t+1} = \kappa_t + 1 \end{cases} \tag{2.17}$$

where ζ represent the transaction costs associated with refinancing. When there is a cash-out refinance the household is increasing the number of years left on the mortgage, $\kappa_{t+1} < \kappa_t + 1$ and $Z_t(\kappa_t, \kappa_{t+1}) > 0$. Only households who choose not to move in a given period may choose to refinance.

The effect of steadily increasing home prices provides another argument for the inclusion of the age of the mortgage as a state variable. Due to the steady increase in home prices, the initial mortgage on a given home today would be significantly greater than the mortgage on a similar home twenty years ago. The current mortgage payments on these two similar homes would reflect this, with the mortgage payment for the home with the twenty-year old mortgage being significantly less than the payment for the home with the new mortgage. The implication is that there might be some economic value to the ability to lock-in the recurring housing expense at a fixed level while the market price of housing fluctuates. This allows the model to capture the role of housing as a hedge against variability in rents, as argued by Sinai and Souleles (2003).

2.6 Default Penalties

FIGURE 2.1: Timing of Decisions



The model also contains a default penalty. In any period the household must be able to cover its housing expenses, including the rent or mortgage and maintenance costs. If it fails to do so, it must move the next period into rental housing, forfeiting all its home equity and all its financial equity above some small nominal amount. Households that can cover their expenses by selling their current house and extracting their home equity are allowed to do so. Households that can afford the associated transaction costs may also avoid defaulting through a cash out refinance. The advantage of this for the household is the ability to keep its housing equity. Current consumption is also constrained to equal that same small nominal amount. The first constraint, shown in equation (2.18), affects those households that are forced to move but can avoid defaulting and the second constraint affects those households that default. The restriction that A_{t+1} may not be negative, combined with the definitions of $X_t(i_t, \kappa_t)$, $Z_t(\kappa_t, \kappa_{t+1})$, and $G_t(i_t, i_{t+1}, \kappa_t)$, along with the budget constraint, create an upper bound on possible levels of non-durable consumption, and also rule out some possible choices of housing tenure. If the household cannot afford the down payment for a large home without incurring negative wealth, it is not allowed to move to such a home. The flow chart above shows how the default penalties affect the household's decisions.

2.7 Optimization Problem and Value Functions

The household's optimization problem is to choose variables $c_t, \alpha_t, i_{t+1}, \kappa_{t+1}$ given a series of state variables $t, \kappa_t, i_t, A_t, H_t$ to optimize equation (2.1) given equations (2.2) (2.16). The household only has one choice of mortgage contract, with a fixed downpayment rate. The choice variable κ_{t+1} capture the ability of a household to cash-out home equity by refinancing, and therefore reduce the effective age of the mortgage as described above.

The value function of the household is the maximum utility, subject to the default constraints of the value functions for the households that choose next period tenure type $i_{t+1} \in \{i_r, i_s, i_l, i_t\}$:

(2.18)

$$(A_t - X_t(i_t, \kappa_t) < 0) \;\; \& \;\; (A_t - X_t(i_t, \kappa_t) + \max_{i_{t+1}, \kappa_{t+1}} (G_t(i_t, i_{t+1}) + Z_t(\kappa_t, \kappa_{t+1}))) > 0) \Rightarrow$$

$$V_t(i_t, A_t, H_t, \kappa_t) = \max_{i_{t+1} \neq i_t or \kappa_{t+1} \neq \kappa_t + 1, c_t, \alpha_t} V_t^{i_{t+1}}(i_{t+1}, A_t, H_t, \kappa_t)$$

(2.19)

$$(A_t - X_t(i_t, \kappa_t) < 0) \;\; \& \;\; (A_t - X_t(i_t, \kappa_t) + \max_{i_{t+1}, \kappa_{t+1}} (G_t(i_t, i_{t+1}) + Z_t(\kappa_t, \kappa_{t+1}))) > 0) \Rightarrow$$

$$V_t(i_t, A_t, H_t, \kappa_t) = U(\omega, h(i_t)) + \beta \rho_t V_t(i_r, \omega, 0, 0) + \beta(1 - \rho_t)\theta_A U_B(\omega)$$

(2.20)

$$(A_t - X_t(i_t, \kappa_t) > 0) \Rightarrow$$

$$V_t(i_t, A_t, H_t, \kappa_t) = \max_{i_{t+1} \in \{i_r, i_s, i_l\}, c_t, \alpha_t, \kappa_{t+1}} V_t^{i_{t+1}}(i_m, A_t, H_t, \kappa_t)$$

where ω is the amount of consumption and wealth protected in default from creditors. Equation (2.18) is the value function when the households recurring housing expenses, $X_t(i_t, \kappa_t)$, are greater than their available liquid assets, A_t, but if their net equity after selling or refinancing their home is positive, $(A_t - X_t(i_t, \kappa_t) + \max_{i_{t+1},\kappa_{t+1}}(G_t(i_t, i_{t+1}) + Z_t(\kappa_t, \kappa_{t+1})))$. Faced with this constraint, the household must either more, $i_{t+1} \neq i_t$, or refinance, $\kappa_{t+1} \neq \kappa_t + 1$. Equation (2.19) is the value function when the household cannot cover their recurring housing expenses out of their liquid assets and their net equity after selling or refinancing their home is negative. These households must move to a rental unit ,$i_{t+1} = i_r$, and have both their consumption and remaining wealth limited to ω. Equation (2.20) is the value function when the households can cover their recurring housing expenses out of their liquid assets. The only limits to their choices are those embedded in the constraints in equation (2.6).

The value function conditional on next period's tenure choice i_{t+1} is:

$$V_t^{i_{t+1}}(i_t, A_t, H_t, \kappa_t) = \begin{cases} \max\limits_{c_t,\alpha_t} U(c_t, h(i_t)) + \beta\rho_t V_t(i_{t+1}, A_{t+1}, H_{t+1}, 1) + & \\ \beta(1-\rho_t)(\theta_A U_B(A_t) + \theta_H U_B(H_t) - \theta_D U_B(D_t)), & i_{t+1} \in \{i_r, i_s, i_l\} \\ \max\limits_{c_t,\alpha_t,\kappa_{t+1}} U(c_t, h(i_t)) + \beta\rho_t V_t(i_{t+1}, A_{t+1}, H_{t+1}, 1) + & \\ \beta(1-\rho_t)(\theta_A U_B(A_t) + \theta_H U_B(H_t) - \theta_D U_B(D_t)), & i_{t+1} = i_t \end{cases} \tag{2.21}$$

such that equations (2.2) to (2.20) hold.

The structure of this problem contains several significant sources of non-continuity. The first is the discrete nature of housing tenure, which functions as both a choice and a state variable. The second main source of the non-continuity is the structure of the value function, which is defined as the maximum of over sixty-six different value functions, one for each possible combination of four tenure choices or two refinance options and eleven portfolio allocations. This non-continuity of the model prevents the use of analytical methods to derive a solution. It also prevents the derivation of Euler equations. The model is instead solved using computational methods based on the methods used in Rust and Phelan (1997).

The code used to solve this problem is in C. One solution of the problem initially took roughly two weeks on a dual processor Pentium Xeon 1.8GHz with 512K L2 cache and 1GB of RAM running Linux. In order to improve the runtime, the code was re-written to take advantage of parallel processing, using the Message Passing Interface (MPI) standard. In this version of the code one processor is designated the master while a pool of other processors are designated slaves. As the model is solved recursively by year, the master distributes the current value function for all previous years to the slaves. Each slave then solves for the optimal value function for a sub-set of state spaces for the given year. The slaves then return the new value function values to the master. The master then combines the new values with the value function for the previous year, completing the recursion for one year. The problem was solved using 61 high-performance Digital Alpha 64-bit microprocessors running at 450MHz each on a scalable parallel Cray T3E at the Pittsburgh Supercomputing Center. One solution involved roughly 1.3 billion evaluations of the value function and took roughly eight and a half hours.

3 Baseline Model Results.

The parameter values for the model calibration are chosen to be consistent with other models in the relevant literature. The parameter values for the size of small and large homes are set so that they represent, respectively, a home 80% and 120% the size of a median priced home. The ϕ value of 0.2 reflects the share of total household expenditures allocated to housing expenditures in the 2001 Consumer Expenditure Survey from the U.S. Department of Labor. This paper does not represent a serious attempt to calibrate a model of housing wealth or to estimate the maximum likelihood parameters of such a model. The goal is to see how closely the model can match certain stylized facts while using fairly standard and common parameter values. Appendix A contains more information on the values of the market and preference parameters chosen. A series of graphs of the policy functions, from one of the calibrated models, for households receiving different series of shocks are then presented, to illuminate the factors driving the economic decisions of the household. Finally, some results from simulations based on the baseline model are given. The baseline model matches several patterns seen in the empirical data.

To better explore the implications of the model, 1,000 simulations are generated using the calibrated model. The table and figures below contain the results from these simulations. Households begin at age 20 as renters with no assets. Households retire at age 65 and live to at most 80 years of age. The simulations track their accumulation of housing and financial wealth over their lifetime. Figures 3.1 and 3.3 present the simulation results across the life cycle. These figures show the role of housing over the life cycle, and how consumption and investment decisions are linked to housing decisions.

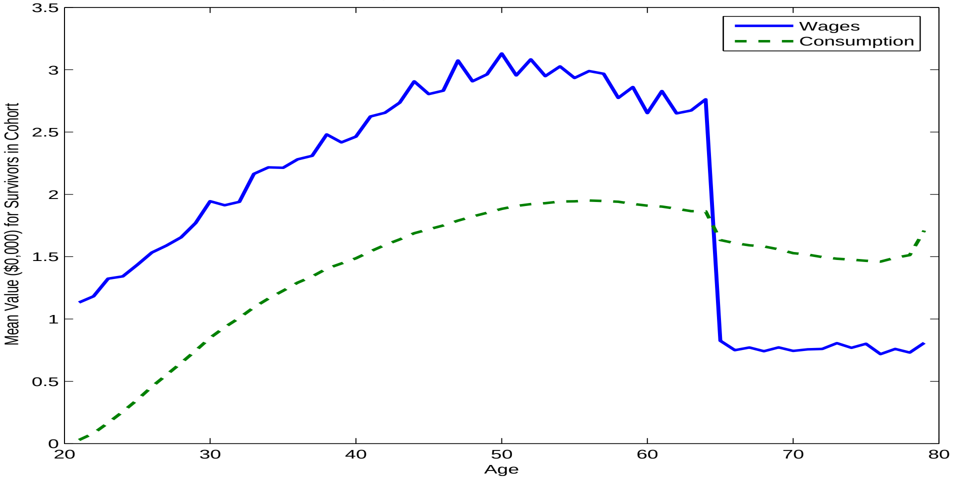
Figure 3.1 shows the consumption and income paths over the life-cycle. The sharp drop in income in retirement can be seen in panel (a), while consumption is much smoother. Panel (b) shows the path of consumption as a share of total wealth. Younger households who are aggressively saving for a downpayment consume the smallest share of their wealth. Once households become homeowners, their consumption as a share of total wealth climbs, peaking near 16% around the age of 30. As households approach retirement, they start to accumulate more wealth, and consumption as a share of total wealth starts to fall reaching a low point of 9% at age 65. In retirement households draw down their savings and consumption as a share of total wealth climbs again. At retirement the average household has roughly forty-five times their annual income saved in both housing and financial wealth.

The importance of housing wealth in retirement is emphasized by the next set of figures. Figure 3.2 (a) shows that housing wealth has a hump over the life cycle, reaching a peak at 60 and starting to decline as households approach retirement. The brief plateau in the growth of housing wealth at age 50 is caused by many households either trading down to smaller homes or refinancing their existing mortgage in order to lock in nominal mortgage payments for the rest of their expected life. Financial wealth, shown in Figure 3.2 (b), is more sharply humped and peaks at age 65.

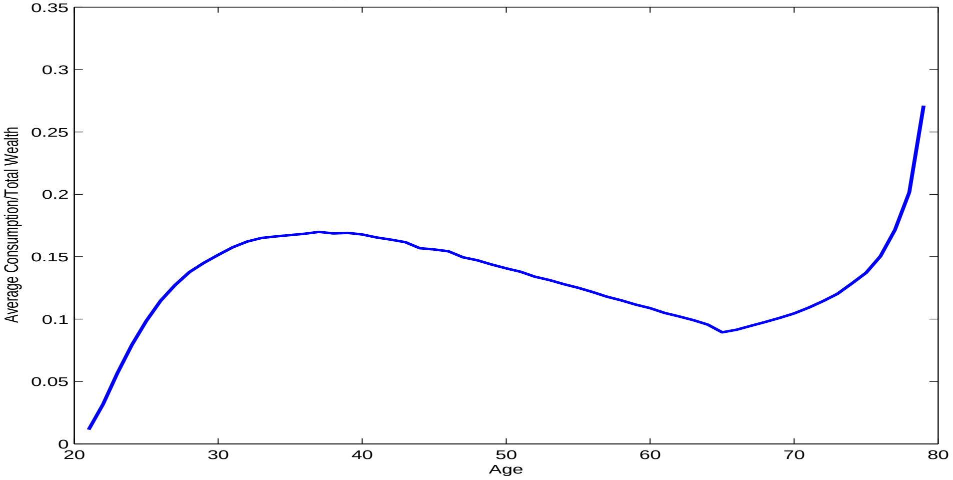
One implication of the model is that accumulated home equity is used to fi-

FIGURE 3.1: Consumption and Income

(a) Wages and Consumption



(b) Consumption as Share of Total Wealth



(c) Ratio of Total Wealth to Income

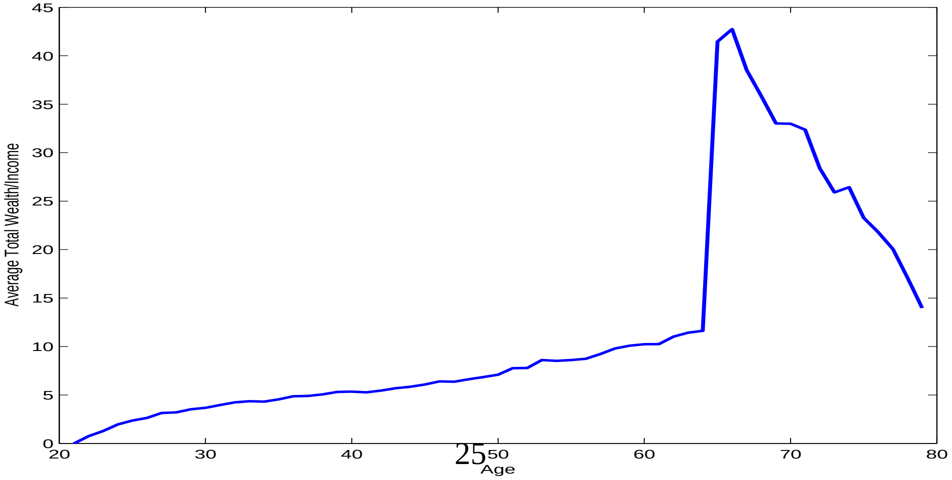
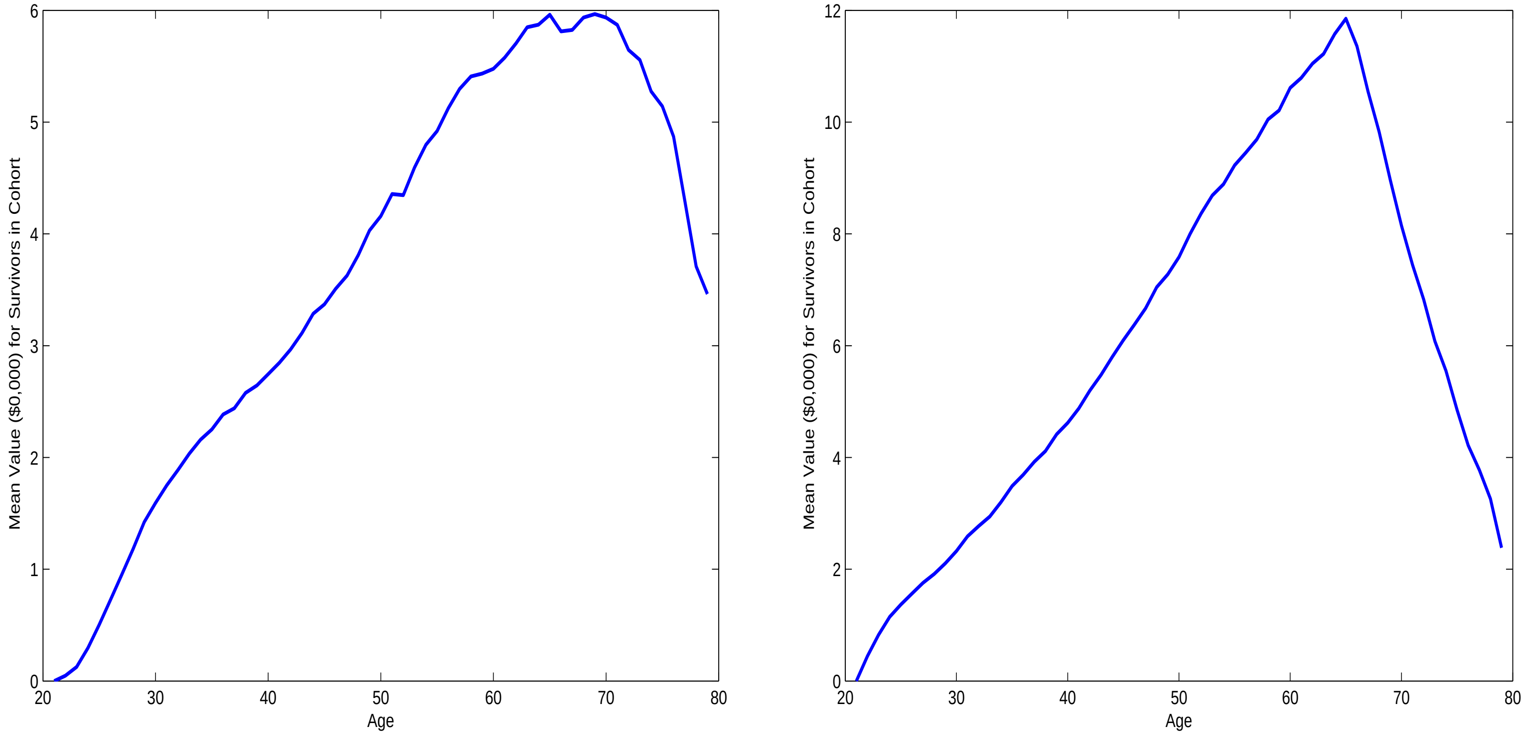
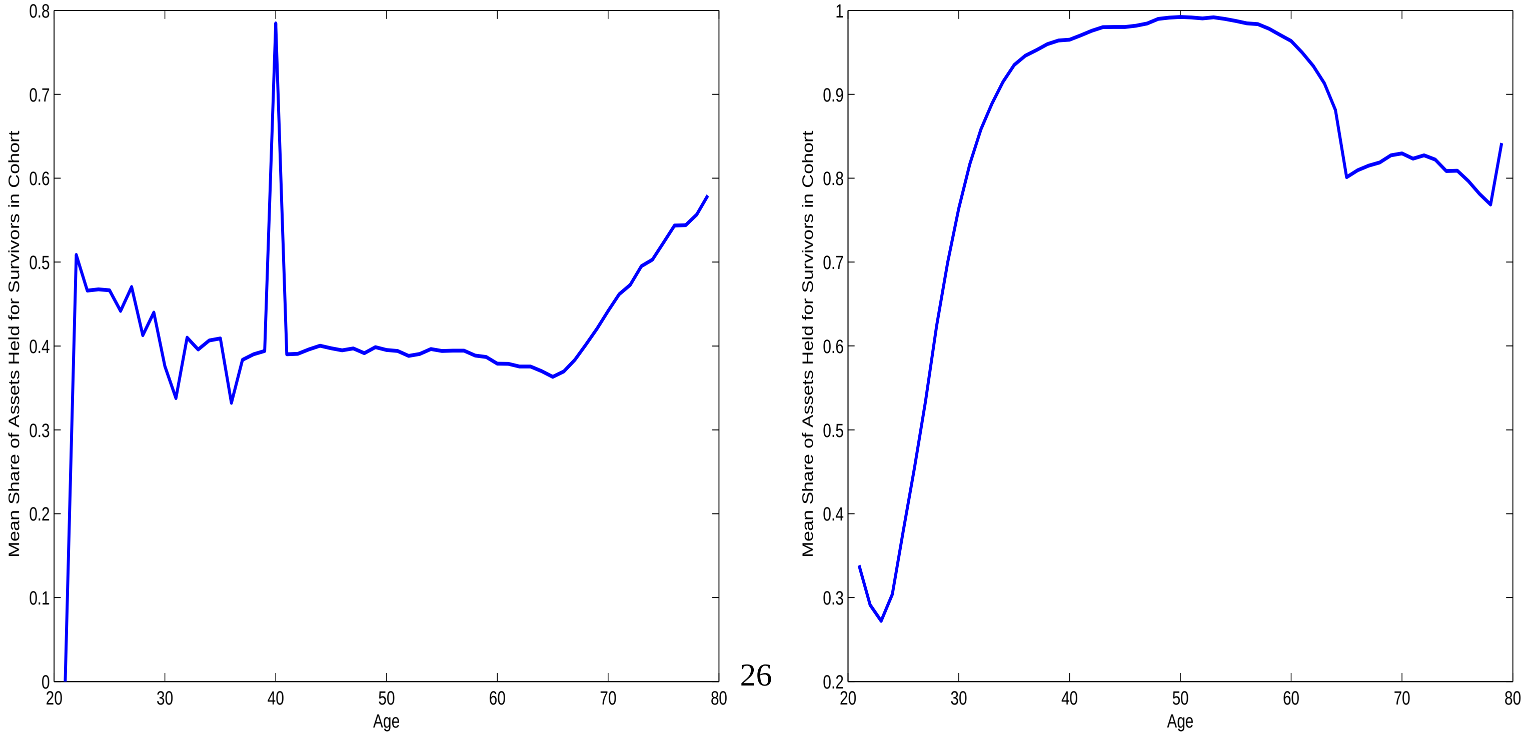


FIGURE 3.2: Wealth and Portfolio Choice



(a) Simulated Housing Wealth

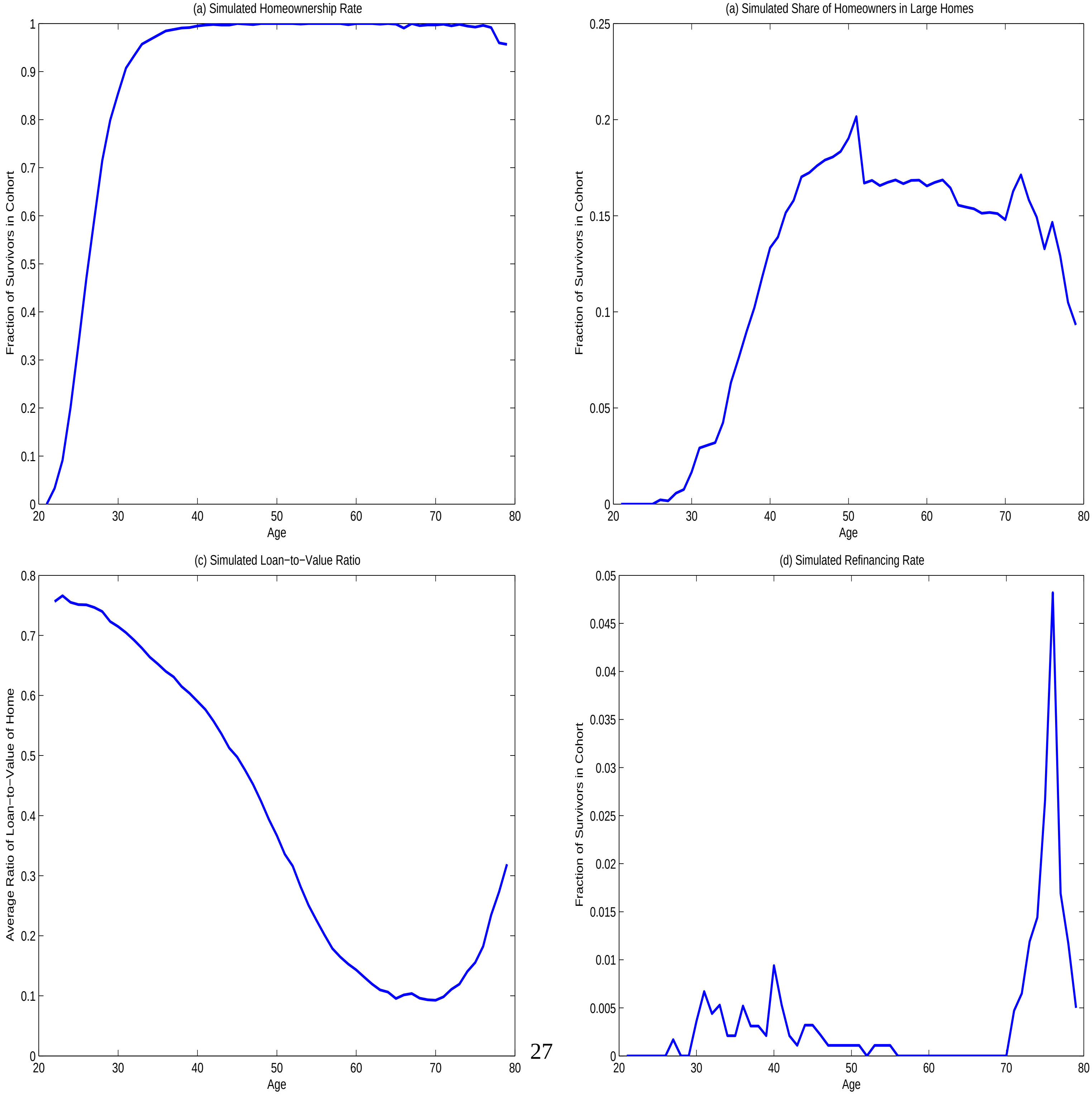
(b) Simulated Financial Wealth



(c) Simulated Share of Housing Assets for Owners

(d) Simulated Risky Share of Financial Assets

FIGURE 3.3: Housing Tenure Choice



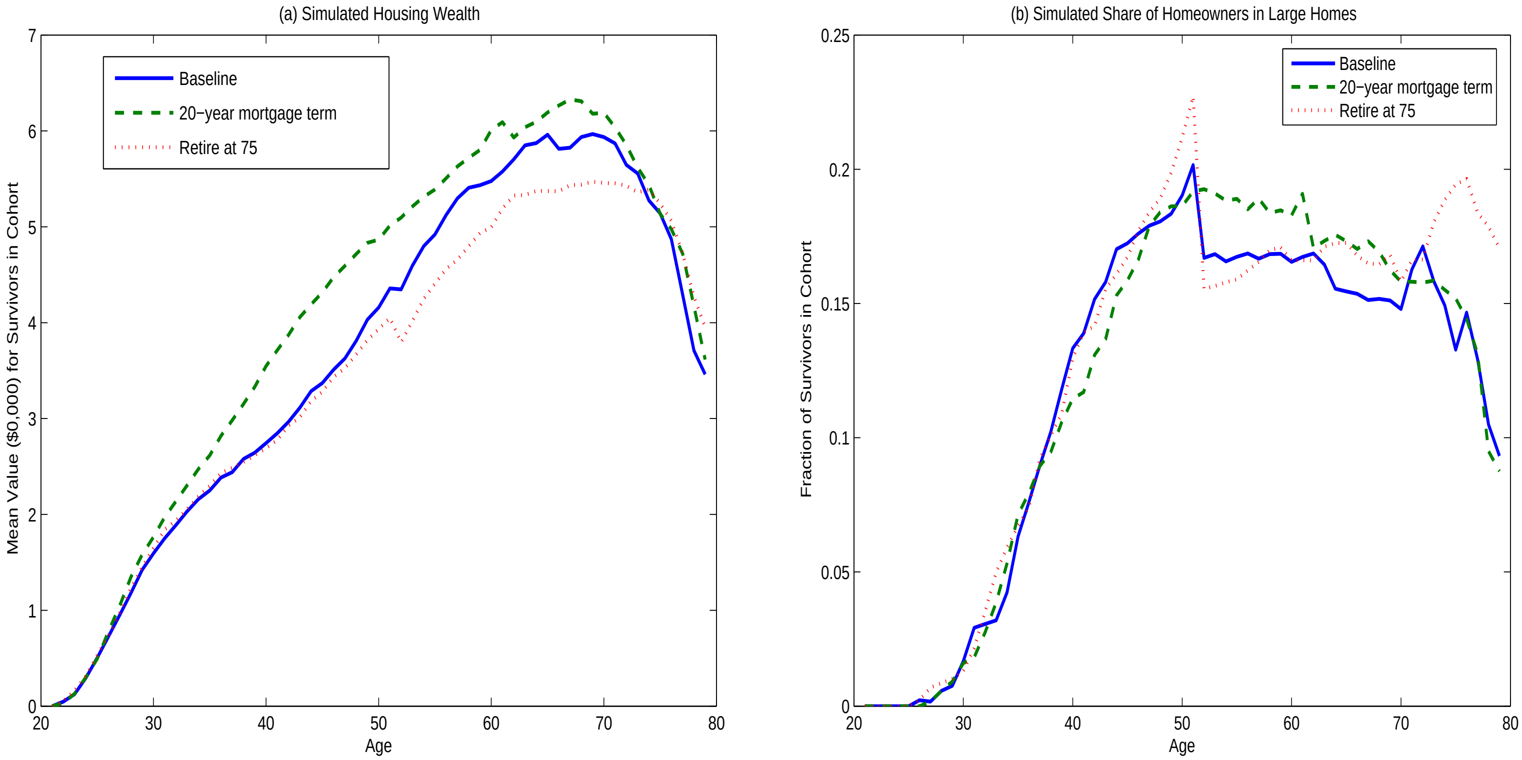
nance the consumption of non-durables in only late in retirement. The actual role of housing wealth among the elderly is a bit more complicated. Venti and Wise (2000) found that housing wealth was not in fact used to support non-housing consumption. They find that households resort to their home equity only when faced by a significant shock such as the death of a spouse or a serious illness. This is similar to the finding in Sheiner and Weiss (1992) that anticipation of death and illness significantly increases the probability that households reduce their home equity. These conclusions find additional support in the results of this model, in that households do not tap into housing wealth in retirement until their reserves of financial wealth have been depleted. However the model does result in more rapid decline in housing wealth than seen in the data. The lack of health status as a state variable and the connection between health status and retiree tenure choice might explain this failure of the model.

Figures 3.2 (c) and 3.2 (d) provide the most significant results of the model. As Figure 3.2 (c) shows, the simulated share of assets held in housing is consistently near 40%, a bit below the empirical average of 67%. The housing share is high among young households who must invest a large portion of their savings in a downpayment. As financial wealth grows faster than housing wealth this share falls initially. The jagged nature of the curve reflects a combination of refinancing and trading up as younger households try to keep their portfolios balanced while taking advantage of their greater financial resources to purchase larger homes. The rate of increase in the share climbs in retirement, as households draw down financial wealth prior to extracting home equity. Household’s face significant transaction costs, due in part to the nature of the mortgage contract, to access their home equity. As a result, households turn to financial equity initially to fund consumption in retirement. This partially matches the ”over-investment” in housing seen in the empirical data, as reported by Flavin and Yamashita, using a model of rational, forward looking agents. The implication is that while some degree ”over-investment” in housing is the result of something innate in the nature of the housing good or the mortgage contract used to purchase it and not the result of sub-optimal behavior by non-rational consumers, the actual level of ”over-investment” in housing seen empirically cannot be fully explained with this model.

Figure 3.2 (d) shows the pattern of allocation in the financial portfolio over the life cycle. Young households who are aggressively saving for or already have large shares of their wealth tied up in downpayments invest less in the risky asset, as do older households who have drawn down their financial wealth relative to their housing wealth. The risky portfolio share peaks around age 50, just when the households start to actively shift their total portfolio away from home equity.

The final set of figures from the simulations document the role of housing over the life-cycle. Figure 3.3 (a) shows home-ownership increasing rapidly for younger households and declining very slightly in retirement. The share of homeowners living in larger homes has a similar hump, as seen in Figure 3.3 (b), with a sharp drop at age 50. Both of these charts document the strategy of households trading down in retirement to access housing wealth to finance consumption. Figure 3.3 (c) documents an interesting pattern. Households who have recently purchased their homes are required to have an initial loan-to-value ratio of 80%. They are then able to pay down their mortgage through the regular amortization schedule and the average loan-to-value ratio falls. The average loan-to-value ratio seems to stabilize at 10% before climbing late in retirement in response to a surge in cash-out refinancing. Figure 3.3 (d) reports the level of refinancing activity over the life-cycle. Younger households and those who have just purchased their homes take advantage of refinancing to re-balance their portfolios and smooth their income. Older households start to use cash-out refinances to access their equity.

FIGURE 3.4: Why Trade Down at 50?



In Figure 3.3 (b) there was a sharp drop in the share of households living in large homes at age 50, with the share falling from a high of 20% to 16%. The timing of this sudden shift into smaller homes is a result of the 30-year mortgage combined with a maximum age of 80 imposed by the model specifications. Households take advantage of the 30-year mortgage term to lock in their nominal mortgage payments for the rest of their natural lives. Figuree 3.4 provides additional support for this hypothesis. In addition to the baseline simulations this figure also reports the simulations with the a 20-year mortgage and when retirement is delayed until 75. The goal is to demonstrate that the shift into smaller homes is driven by the length of the mortgage term and not the proximity to retirement. When the retirement age is 75 and the mortgage term is 30 years, the shift to smalle homes still happens at age 50. When the mortgage term is shortened to 20 years and the retirement age remains at 65, the shift to smaller homes occurs at age 60. These alternate scenarios show that the shift to smaller homes is driven by the household's desire to lock-in their nominal mortage payment in retirement. The benefit of this strategy is that while they will continue to receive a constant stream of utility from their home, the real value of the mortgage payments will fall due to inflation. In effect, households are purchasing an annuity where the stream of real payments, the difference between the implicit rent and the real mortgage cost, will increase with time and be at its highest during retirement when income is at its lowest.

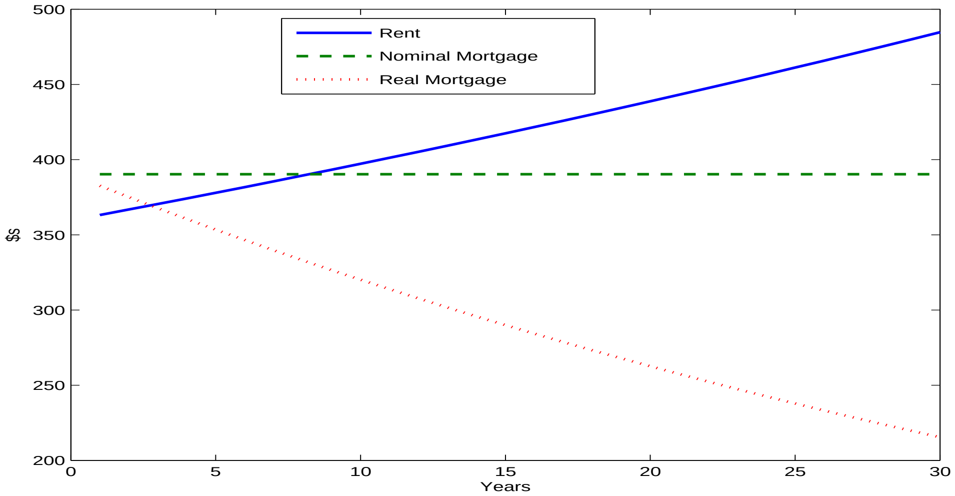
This section has established the most significant accomplishment of the model the ability to partially match the ”over-investment” in housing seen in the data within a framework of rational, forward-looking agents. This section has also argued that the optimal share of risky assets in the financial portfolio is effected by both the level of investment in housing and the endogenous tenure choices of the household. The next section will build on these results with a detailed examination of how the demand for housing and the allocation of household portfolios differs under different levels of inflation.

4 The Effects of Inflation on Portfolio Allocation.

The previous section provided evidence that the baseline model can match certain stylized facts about housing wealth over the life-cycle. This section determines exactly how portfolio allocation across the life-cycle changes under different levels of inflation. Each alternative model is then re-solved and the corresponding sets simulations generated. The levels of wealth accumulation, housing demand, refinance activity, and portfolio allocation under each alternative assumption are then compared to the base case.

All that is needed in order to simulate the effects of high inflation, or of mortgage contracts that are real and not nominal is to set the inflation parameter respectively to a higher level or to zero. The presence of nominal mortgage contracts effectively shifts the costs of home-ownership forward over the term of the mortgage. Figure 4.1 documents how the real value of the mortgage payment declines over the life of the mortgage. This is of course factored into the rate of the original mortgage and partially explains the gap between the mortgage and

FIGURE 4.1: Rent and Mortgage Payments



risk-free rate. Figures 4.2 and 4.3 show how housing demand and portfolio allocation differs under different inflation rates. The main impact of inflation on the households portfolio allocation is to increase a current homeowner's costs associated with taking out a new mortgage, either to refinance an existing loan or to move to a different sized house. As a result, the allocation between home equity and financial assets are rebalanced less frequently, with younger households in particular holding larger shares of their wealth in home equity under higher rates of inflation.

High inflation increases the rate at which the nominal mortgage payments are discounted over time. As a result, there is a much more pronounced move from large to small homes at age 50 under the high inflation scenario. Households are eager to purchase small homes at age 50 and lock in their nominal mortgage payments for the rest of their life. In fact they almost never move or refinance

FIGURE 4.2: Wealth and Portfolio Choice

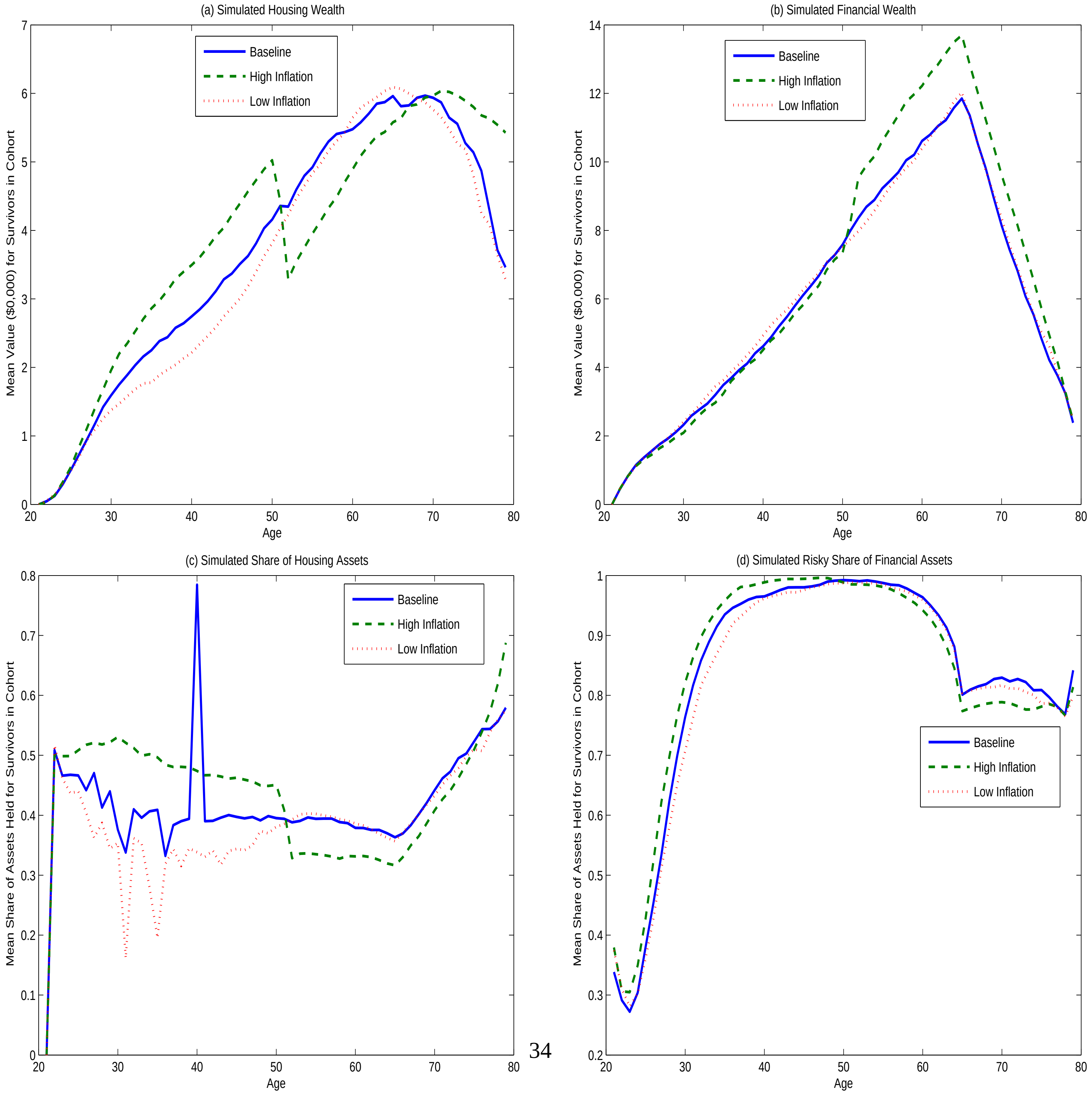
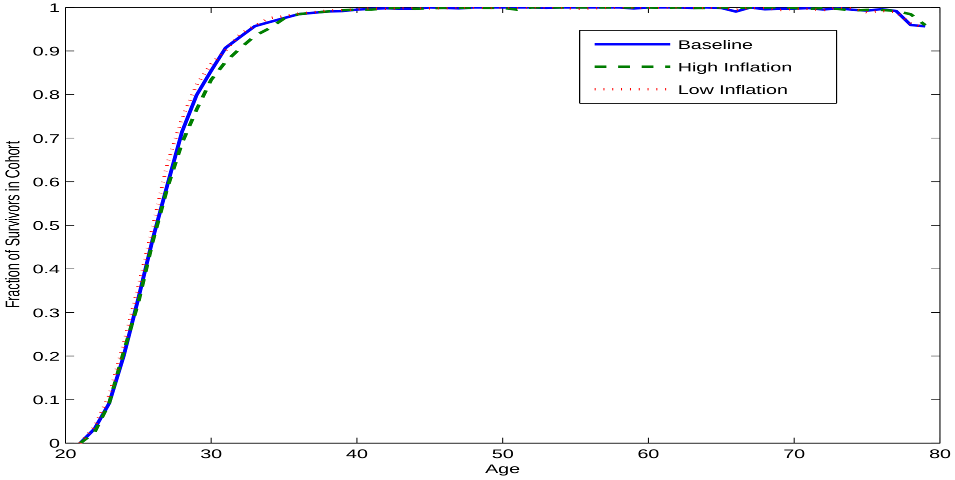
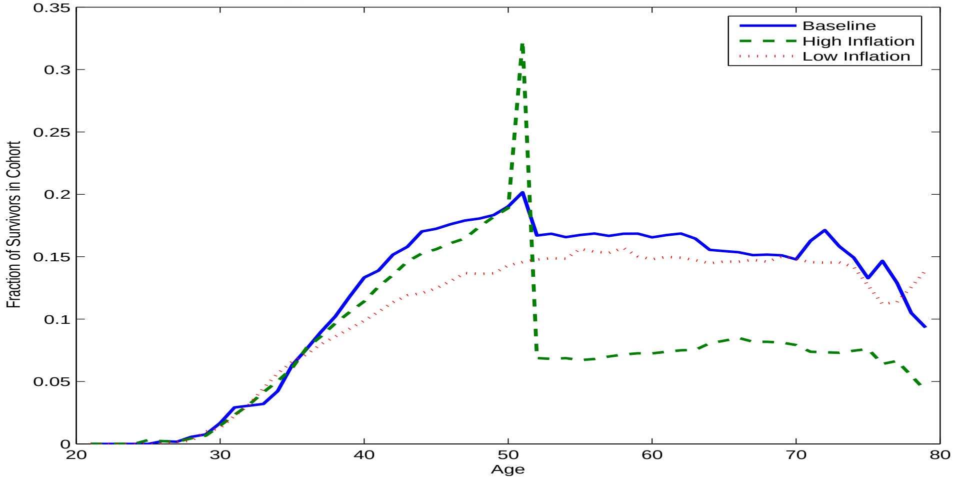


FIGURE 4.3: Housing Tenure Choice

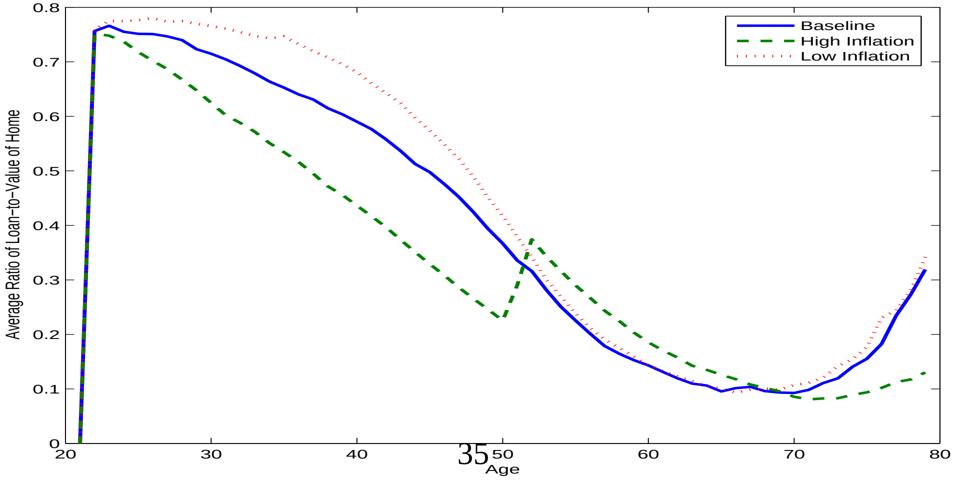
(a) Simulated Homeownership Rate



(a) Simulated Share of Homeowners in Large Homes



(a) Simulated Loan−To−Value Ratio



after age 50, due to the increased value of the inflation discounting of the nominal mortgage payment. The main effect of zero inflation is to reduce the demand for large homes and remove the tendency to lock in nominal mortgage payments at age 50 as the real mortgage payments no longer decline with the age of the loan. The large number of households buying smaller homes at age 50 result in a large shift of wealth from home equity to financial equity, significantly reducing the share of wealth held in home equity. Once they move into the smaller homes, they prefer to never move or refinance again in an effort to minimize the size of their recurring mortgage payment in real terms during retirement. The higher the rate of inflation, the faster the share of wealth held in housing grows in retirement as households refinance and re-balance less frequently. As was shown previously, the pattern of trading down at age 50 is driven by the nature of the nominal mortgage contract and not retirement decisions.

Households anticipate this large influx of home equity into their financial portfolios at age 50 under the high inflation regime. In response, they feel comfortable keeping a slightly larger share of their financial portfolio in risky assets during this period. Under the high inflation regime, the gap between the current mortgage payment for current homeowners and the payment on a new mortgage or rent is significantly greater, especially for those with long tenures. This increases the costs associated with the default risk, which forces the household to break their current mortgage contract with it's low real mortgage payment. This increased cost of default under the high inflation regime encourages households to hold larger amounts in financial wealth and to hold smaller shares of this wealth in risky assets.

Homeowners face two different expenses in purchasing their homes. The first expense is the upfront costs, or the downpayment. The second expense is the mortgage payments made on a monthly basis over the life of the loan. These recurring expenses are significant, in that if the household cannot make these payments they will default on the mortgage and loose their existing home equity. Households in this model trade down precisely because they fear they will not be able to make the mortgage payments once their income falls in retirement. Nominal mortgage payments under high inflation are especially attractive to households in their fifties. The real mortgage payment is initially high, during the prime of their earnings potential. The real mortgage payment then falls rapidly during retirement, just when their income also falls. Nominal mortgage payments allow households to shift the expense of housing forward while insuring the consumption of housing later. In this way housing is a form of annuity which provides housing services instead of cash payments. The ability to shift the cost of housing forward is due to the nominal mortgage contract. The higher the rate of inflation, the greater the shift in costs.

5 Conclusion.

The baseline model developed in this paper succeeds in partially replicating home equity's large position in household level portfolios. The implication is that while some degree ”over-investment” in housing is the result of something innate in the nature of the housing good or the mortgage contract used to purchase it and not the result of sub-optimal behavior by non-rational consumers, the actual level of ”over-investment” in housing seen empirically cannot be fully explained. The model shows how the allocation of the financial portfolio varies in response to the position in housing wealth and tenure decisions. The initial introduction of risky housing can increase the demand for risky financial assets, as household gain the ability to diversify across uncorrelated risky assets. As the exposure to risky housing assets grows however, household respond to the increased background risk by reducing their demand for risky assets.

The model is then used to demonstrate how perfectly anticipated inflation, even when reflected in the nominal interest rate on mortgages, can distort the households portfolio allocation over the life-cycle. The nominal mortgage contract results in declining real mortgage payments. The longer a mortgage is held, the great the difference in their mortgage payment and the payment for a mortgage refinanced at the current price. This friction discourages households from shifting assets from home equity to financial portfolios more frequently. As the rate of inflation increases, households hold larger positions in home equity earlier in the life-cycle, and smaller positions later in the life-cycle. The costs associated with resetting the nominal mortgage payment when refinancing also helps explains why retired households hold such a significant portion of their wealth in housing.

Appendix A - Baseline Model Parameter Values

The parameter values for the baseline model are chosen to be consistent with other models in the relevant literature. As was discussed in the Section 3, the income process consists of a deterministic and a transitory factor. The income process is based on the results of regressions of Social Security earnings on age and age-squared. The dependent variable is the log of the wage income in constant 1990 dollars. The transitory factor of wage is reflected in the estimated standard error of the regression. The wage is converted from log to level terms in the model. At age 65 the level of the deterministic wage falls to a flat level equal to 60% of the last period's income before any transitory shocks, representing a system of forced retirement and a defined benefit pension plan. The coefficients and standard deviation used in this version of the model are shown in Table A-1 below.

TABLE A-1: Log Income Regression Results

Constant	ψ_0	7.28626
Coefficient Age	ψ_1	0.10278
Coefficient of Age^2	ψ_2	-0.00098
Std. Dev.	σ_w	0.80778
R^2		15.5%
Probability of Unemployment	υ	1%

The market price of a housing unit is the result of setting the deterministic home price at age 60 with the National Association of Realtors' 1990 median home price. It is assumed that a median home consists of 10 housing units. The home prices are converted to constant 1990 dollars and the deterministic home price series are calculated using the historical average return. The average and standard deviation of the return on housing are at taken from Li and Yao (2004) and are consistent with Campbell Cocco (2003). The mortgage interest rate used is the average rate on loans with 80% loan-to-value ratios as reported by Freddie Mac from 1969 to 2001, adjusting for the inflation rate. The percent required for downpayment represents the minimum needed to avoid paying mortgage insurance. The transaction, maintenance, and moving costs are based on survey data provided by the National Association of Realtors. The values chosen for the current version of the model are presented in Table B-4 below. The risk and return on risky assets follows Yao and Zhang (2004).

TABLE A-2: Values of Market Parameters

Parameter Name and Definition	Symbol	Value
Real risk free rate of return	r	2%
Price of 1 housing unit, at age 60	$P_{60}(1)$	1.003
Size of small homes	$h(i_s)$	8
Size of large homes	$h(i_l)$	12
Mean of real return on housing	η_h	1%
Standard deviation of housing return	σ_h	11.5%
Mean of real return on risky asset	η_s	6%
Standard deviation of risky asset return	σ_s	15.7%
Probability of 100% loss on risky asset	ς	1%
Mortgage interest rate	π	5%
Percent required as downpayment	μ	20%
Percent of home price lost to transaction costs	τ	10%
Maintenance costs	δ	0.7%
Moving costs	χ	0.3
Tax Rate	γ	30%
Refinancing Costs	ζ	3%
Inflation	ν	2%

Note: Units are in $10,000s or percent.

The values for the preference parameters shown in Table A-3 below were chosen to replicate certain stylized facts about the role of owner-occupied housing in portfolios, specifically the large share of total wealth held in home equity. An λ value of 2 represents a relatively low, but realistic, level of risk aversion. An β value of 0.96 is a commonly used discount rate. The ϕ value of 0.2 reflects the share of total household expenditures allocated to housing expenditures in the 2001 Consumer Expenditure Survey from the U.S. Department of Labor. The discount rate for bequests are 0.8 for θ_A, 0.8 for θ_H, and 0.8 for θ_M. They are chosen to imply that households would rather consume one additional dollar than leave an additional dollar as a bequest and that households place a premium on leaving their homes as bequests relative to other assets.

TABLE A-3: Values of Structural Parameters in Calibrated Model

λ	β	ϕ	θ_A	θ_H	θ_M
2	0.96	0.2	0.8	0.8	0.8

Appendix B - Model Parameter Definitions

TABLE B-4: Model Parameter Definitions

Parameter Name and Definition	Symbol
Consumption	c_t
Tenure Choice, next period	i_{t+1}
Share of Financial Assets held in risky assets	α
Age of Mortgage (Refinancing=Change Age of Mortgage)	κ_{t+1}
Tenure Choice, this period	i_t
Current Age of Mortgage	κ_t
Value of Financial Assets	A_t
Value of Home	H_t
Tenure Choice, rent	i_r
Tenure Choice, own small house	i_s
Tenure Choice, own large house	i_l
Number of housing service units for tenure choice i_t	$h(i_t)$
Realized Earnings	$\widetilde{e}_t$
Remaining Mortgage Balance	D_t
Recurring Housing Costs	$X_t(i_t, \kappa_t)$
Mortgage Interest paid	$I_t(i_t, \kappa_t)$
Net Gain/Loss from Home Sale/Purchase	$G_t(i_t, i_{t+1}, \kappa_t)$
Net Gain from Cash-Out Refinancing	$Z_t(\kappa_t, \kappa_{t+1})$
Mortgage Payment	$M_t(i_t, \kappa_t)$
Risk Aversion	λ
Discount rate	β
Housing Utility Coefficient	ϕ
Bequest Parameter - Financial Assets	θ_A
Bequest Parameter - Housing	θ_H
Bequest Parameter - Mortgage Debt	θ_M
Survival Probability	ρ_t

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