The Role of the Leverage Effect in the Price Discovery Process of Credit Markets*

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Abstract

Starting from the analysis of a levered firm's capital structure, I show that corporate default risk becomes measurable through the leverage effect, that is, the negative correlation observed between stock returns and changes in stock volatility. In this model, the firm's debt-to-asset ratio governs the elasticity of default probabilities relative to equity prices. A large dataset of S&P 500 firms and an extended timeframe (2008-2018) is used to examine the model's empirical implications. The impact of the corporate leverage in the transmission mechanism between stock and credit default swap (CDS) markets is uniform across firms and robust to market conditions. Equity and credit markets are more likely to be co-integrated when firms employ a higher debt-to-asset ratio. Although the stock market generally dominates the price discovery process, a small cluster of highly-leveraged firms exhibits a dominant CDS market share. Under the effect of corporate leverage, the credit market attracts informed trading and arbitrage resources.

JEL classification: G12; G13; G14

Keywords: Capital structure arbitrage; Credit markets; Leverage effect; Co-integration; Price discovery.

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1. Introduction

Merton's (1974) structural model of credit risk implies that individual stock and corporate debt securities are reasonably close substitutes. The integration of credit and equity markets suggested by the standard financial theory incites rational investors to bet on the convergence of these two markets by arbitraging stocks against credit derivatives (Kapadia and Pu, 2012). The risk limits to capital structure arbitrage strategies being low,[1] neither the equity market nor the credit market should dominate the price discovery of credit risk. However, recent studies have shown that insider trading may occur in the credit derivatives market and impound the price discovery process (e.g., Acharya and Johnson, 2007; Kryzanowski, Perrakis, and Zhong, 2017). Such breaches to market efficiency raise the question as to which market attracts informed trading and arbitrage resources.

In this paper, I put forward a structural model of the leverage effect to interpret the dominance of credit markets in the price discovery process. In this model, the corporate leverage governs the transmission of price information between the credit and the stock market. When the firm's financial leverage is low, the informational content of the credit market is low and produces low-intensity signals. As a result, credit traders are mostly noise or liquidity traders, and the bulk of the price discovery process primarily occurs in the stock market, in line with multiple empirical studies (e.g., Hilscher, Pollet, and Wilson, 2015). Conversely, when the firm's financial leverage gradually increases, rational and sophisticated credit investors acquire a gradual advantage in the gathering and the processing of information related to the firm's credit quality. As credit traders tend to monopolize the incorporation of private information into prices, the transmission to the stock market intensifies due to the effect of the corporate leverage. The shift in intensity then pushes stock traders to chase the trend and morph into noise traders without their knowing. Everything happens as if the corporate leverage made informed trading migrate to the credit market.

My structural model for price transmission draws on the following insight. Relying on the economic concept of elasticity,[2] I focus on the elasticity of default probabilities relative to stock

[1]For the limits to arbitrage, see, for example, Shleifer and Summers (1990) and Shleifer and Vishny (1997).

[2]The term *elasticity* refers to situations where a change of $\delta\%$ in a dimensionless financial quantity x generates a

prices. The intuition behind the credit-equity elasticity is that it conveys the joint correlation between changes in the firm's market value and changes in the firm's credit quality. Additionally, it delivers the optimal hedge ratio sought by capital structure arbitrageurs.[3] The paper's key result is then as follows. The structural framework proves to be rich enough to link the credit-equity elasticity to the elasticity of equity variance relative to stock prices. Simultaneously, the latter captures the so-called "leverage effect"[4] as a linear function of the debt-to-asset ratio, one of the critical indicators of the firm's financial health. Consequently, a simple function of the firm's financial leverage turns out to encapsulate the signal for informed trading of the capital structure.

The model put forward offers far-reaching empirical implications. First, the credit-equity elasticity hypothesis brings support to the presence of a long-run equilibrium relationship between a firm's credit spreads and equity prices. The intuition is that CDS and stock time series cannot drift too far apart from the equilibrium because capital structure arbitrageurs will act to restore the equilibrium relationship. By capturing the non-linear effect of the firm's leverage, this co-integrating vector is distinct from the linear combination of the credit spread and the stock price already investigated in the literature (e.g., Narayan, Sharma, and Thuraisamy, 2014).

Second, the model provides new testable hypotheses concerning the price discovery process at work in credit markets. If equity and credit prices are co-integrated, the permanent-transitory decomposition of Gonzalo and Granger (1995) ensures that they must track a common long-memory component, or efficient price (Hasbrouck, 1995). Meanwhile, an error-correction mechanism must absorb transitory shocks to reflect arbitrage across equity and credit markets. By specifying the co-integrated credit-equity system, our model allows computing the implicit efficient price of credit. Each market's contribution to the price discovery process then becomes accessible.

Third, as already underscored in the literature (Kapadia and Pu, 2012; Choi and Kim, 2018), exogenous barriers to arbitrage such as funding constraints, liquidity risks, or short-sale impediments interfere with the co-movements in the equity and credit markets. This paper hypothesizes

change of $e\delta\%$ in quantity x^e for a δ close to 0.

[3]See Schaefer and Strebulaev (2008) for a study of the hedge ratios produced by structural models of credit risk.

[4]Coined by Black (1976), the term conventionally designates the negative correlation empirically observed between stock price returns and changes in volatility.

that the non-linear impact of the leverage effect may be one of the endogenous sources for the lack of integration between the credit and equity markets.

This paper uses a large dataset of S&P 500 firms and an extended timeframe (2008-2018) to examine the transmission of pricing information from the stock market to the credit default swap (CDS) market. By identifying the genuine price innovations arising in the stock market, I offer an empirical methodology to identify the non-linear impact of the financial leverage on the information flow transiting to the credit market. This leveraged transmission mechanism to the CDS market appears (i) more intense than a linear direct transmission channel, (ii) uniform across firms, (iii) robust to market conditions.

Most firms in the sample reject the null hypothesis of no (leveraged-)co-integration between their equity and CDS markets. The empirical analysis shows that entities are more likely to be co-integrated when (i) they belong to a business sector perceived as more leveraged; (ii) they employ a higher debt-to-asset ratio; (iii) their CDS price is more volatile. These findings provide evidence that an increase in financial leverage ramps up market activity in capital structure arbitrage. One of the indirect market effects of corporate leverage is thus to intensify the integration between credit and equity markets.

For those firms which are significantly co-integrated, this study draws on the vector error-correction (VECM) approach of Gonzalo and Granger (1995) to identify the respective contributions of each market to the price discovery process. The CDS market share appears to be low and below 30% for the vast majority of firms, consistent with the CDS "sideshow" hypothesis (Hilscher, Pollet, and Wilson, 2015). However, a small cluster of highly-leveraged firms exhibits an extremely dominant CDS market share close to 100%. This new finding provides reliable evidence for the role of the leverage effect in the price discovery process.

This paper relates to the vast empirical literature that investigates the price discovery process in credit markets. The conventional view states that credit pricing information primarily flows from stock markets to credit markets due to lower transaction costs (e.g., Hilscher, Pollet, and Wilson, 2015). The alternative view underscores the role of private information in the flow of pricing information from credit markets to stock markets (e.g., Acharya and Johnson, 2007). The most recent literature suggests that both credit and equity markets should potentially lead and lag the other market (Forte and Peña, 2009; Marsh and Wagner, 2016; Lee, Naranjo, and Velioglu, 2018). By studying the endogenous, non-linear impact of the firm's capital structure, this paper departs from a work of literature mainly focused on exogenous and linear transmission effects.

The article proceeds as follows. Section 2 contains the main theoretical contribution, while Section 3 discusses the economic implications of the theory. Section 4 describes the data used in the empirical analysis developed in Section 5. Finally, Section 6 concludes the article.

2. The Model

I now introduce a simple structural framework to build a new approach to the leverage effect.

2.1 Structural framework

We start with a basic structural model of the firm in the vein of Black and Cox (1976), Leland (1994, 1998), Leland and Toft (1996). The firm's unlevered asset value $V = (V_t)_{t \geqslant 0}$ evolves according to a geometric Brownian motion which is defined on a complete probability space $(\Omega, \mathscr{F}, \mathrm{P})$:

$$\mathrm{d}V_t = \mu_V V_t \mathrm{d}t + \sigma_V V_t \mathrm{d}B_t, \tag{1}$$

where μ_V is the asset growth rate, $\delta > 0$ is the net cash outflow rate from the firm paid to stockholders, σ_V^2 is the instantaneous variance of the return on the firm, and B_t is a Brownian motion. Fully-informed managers operate the firm and have access to the complete information filtration $\mathscr{F}_t := \sigma\{V_s : s \leqslant t\}$.

The firm has issued two types of financial claims: equity and an amount of debt interest and principal. Debt is issued to benefit from the tax shield offered at the constant tax rate $\theta \in [0;1]$. We model it as a consol bond, which is paying interests indefinitely at some constant coupon rate, $c > 0$. The optimal amount of debt and its coupon rate may be chosen at time 0 by the structural planners, depending on the initial firm valuation. The considered company is subject to default risk, and the stopping time of the firm's default is $\tau_B := \inf\{t > 0 : V_t \leqslant V_B\}$, where $V_B > 0$ is a default boundary to be endogenously determined later by having the shareholders optimally liquidate the firm. The rule of absolute priority governs the distribution of assets to bondholders in case of liquidation. Liquidation costs are assumed to be a proportion of the remaining asset value V_B.

Fully-informed managers of the firm are the agents of equity shareholders. They are entrusted with the strategic choice of optimally liquidating the firm for the benefit of stockholders. As we do not consider agency costs of equity, the managers choose a liquidation policy modeled as an $(\mathscr{F}_t)$-stopping time τ_B to maximize the residual asset value of the firm at time t:

$$S_t = \sup_{\tau_B \in \mathscr{T}} \mathrm{E}\left[\int_t^{\tau_B} e^{-r(s-t)}(\delta V_s + (\theta - 1)c)\mathrm{d}s \middle| \mathscr{F}_t\right], \tag{2}$$

where $\mathscr{T}$ is the set of $(\mathscr{F}_t)$-stopping times. Under the technical assumption that the expected asset growth rate μ_V is below the risk-free rate, Duffie and Lando (2001) solve the optimal control program (2) to find explicit functions for the endogenous default boundary, the optimal equity value, and the value of debt.

To obtain a first passage default model consistent with a reduced-form representation, I follow Duffie and Lando (2001) by assuming noisy accounting information. I assume that debtholders, contrary to the firm's managers, have only access to incomplete accounting information on the state of the firm value V. More precisely, the investors in the secondary debt market have access to the following pieces of information:

- *Noisy accounting information.* At selected reporting dates t_1, t_2, ..., the bondholders have access to a noisy accounting report of assets, $\hat{V}_t$.
- *Default state of the firm.* At each time t, the debtholders know whether the firm managers have liquidated the firm and bankruptcy has taken place. They observe and use rationally the default indicator process $1_{\{\tau_B \leqslant t\}}$.

At each time t, the information filtration available to the bond market is thus given by the σ-

algebra $\mathscr{G}_t := \sigma\left\{\left(\hat{V}_{t_1}, \hat{V}_{t_2}, \cdots, \hat{V}_{t_n}\right), 1_{\{\tau_B < s\}} \middle| 0 \leqslant s \leqslant t\right\} \subset \mathscr{F}_t$, where t_n is the latest noisy reporting date before t. As shown by Giesecke (2006) in a more general setting, the imperfect observation of the firm's assets naturally yields a $\mathscr{G}_t$-intensity process λ of the default stopping time τ_B.

I now turn to the structural analysis of the leverage effect. The next formulation of the leverage effect is a simple consequence of our structural framework. It provides a preliminary link between the logarithmic slope of the equity local volatility surface and an adjusted value of the firm's corporate leverage.

Lemma 1. *The logarithmic slope of the equity local volatility surface, $\sigma(S_t,t)$, is linked to the firm's financial leverage by the relationship:*

$$\frac{\partial \sigma}{\partial \ln(S)} = -\sigma \cdot (\ell - \varepsilon_\ell), \tag{3}$$

where ℓ is the debt-to-asset ratio,[5] *and the adjustment to the financial leverage is given by:*

$$\varepsilon_\ell := \frac{S S_{VV}}{S_V^2}, \tag{4}$$

where subscripts denote partial derivatives with respect to the firm's asset value V. Moreover:

(a) $\lim_{V\to\infty} \varepsilon_\ell = 0$;

(b) $\lim_{V\to V_B} \varepsilon_\ell = \frac{1}{2}$;

(c) ε_ℓ *is continuous and bounded over* $[V_B;\infty)$;

(d) Except for states of the firm close to default ($V \approx V_B$), numerical simulations indicate that $\varepsilon_\ell \ll \ell$. In these conditions, the slope of the equity instantaneous volatility surface is determined by ℓ at first order in the firm's leverage.

Proof. See Appendix A. □

[5] The debt-to-asset ratio is specified with market values instead of book values. This modeling choice reflects not only the firm's tangible assets and working capital but also its intangible assets and growth opportunities.

2.2 Structural approach to the "leverage effect"

To substantiate the role of the firm's financial leverage in the so-called leverage effect, limiting oneself to Equation (3) presents serious shortcomings. Indeed, as it depends on the model of the stock price dynamics, the local volatility surface is not observable in the market. It is thus preferable to rely on a model-free formulation of the volatility surface. Let introduce the Black-Scholes implied volatility $\widehat{\sigma}_T(K)$ for a given strike price K and given time to maturity T.[6] The local volatility slope appearing in Equation (3) is known to be a good predictor of the asymmetry of the implied volatility surface observed in options markets. More precisely, the local volatility skew is twice as steep as the implied volatility skew for short times to expiration (see, for example, Gatheral, 2006). However, as the following technical result derived by Hagan and Woodward (1999) via singular perturbation theory shows, the assumption of short times to expiration may safely be relaxed at a small technical cost.

Lemma 2 (Hagan and Woodward, 1999). *Assume a time-separable instantaneous volatility surface: $\sigma(S_t,t) \equiv \alpha(t)\sigma(S_t)$. The instantaneous volatility surface $\sigma(\cdot)$ may be inferred from the volatility surface $\widehat{\sigma}(\cdot)$ implied by the options market through the following affine transformation:*

$$\widehat{\sigma}_T(K) = \sigma\left(\frac{S+K}{2}\right)\sqrt{\frac{1}{T}\int_0^T \alpha^2(s)\mathrm{d}s}, \qquad \text{for all} \quad K,S>0. \tag{5}$$

Proof. See Appendix B. □

Let define the slope of the implied volatility surface in the log-strike space as:

$$\widehat{\Sigma}_T := \frac{\partial \widehat{\sigma}_T(K)}{\partial \ln(K)}. \tag{6}$$

An immediate consequence of Lemma 2 is to provide a structural interpretation based on leverage of the implied volatility "skew" $\widehat{\Sigma}$, that is, the negative relationship between implied volatility and

[6] $\widehat{\sigma}_T(K)$ is the volatility number to be input in the Black-Scholes-Merton model (Black and Scholes, 1973) in order to match the European-style call price $C(K,T)$ observed in the options market.

strike price.

Proposition 1 (Structural leverage effect). *Assume a time-separable local volatility surface: $\sigma(S_t, t) \equiv \alpha(t)\sigma(S_t)$. The slope of the implied volatility surface is linked to the firm's debt-to-asset ratio through:*

$$\widehat{\Sigma}_T = -\frac{\sigma}{2}(\ell - \varepsilon_\ell)\,\overline{\alpha}_T, \tag{7}$$

where ℓ is the debt-to-asset ratio, ε_ℓ is given by Equation (4), and $\overline{\alpha}_T := \sqrt{\frac{1}{T}\int_0^T \alpha^2(s)\mathrm{d}s}$. The variance-equity elasticity is then given by:

$$e_v := \frac{\mathrm{d}\sigma^2/\sigma^2}{\mathrm{d}S/S} = -2(\ell - \varepsilon_\ell). \tag{8}$$

Proof. See Appendix C. □

The alternate hypothesis traditionally advanced for the asymmetry of the implied volatility surface (e.g., Bekaert and Wu, 2000; Wu, 2001) is the volatility feedback effect.[7] Notice how Equation (7) captures the volatility feedback effect (increases in volatility imply increases in $|\widehat{\Sigma}|$) on top of the leverage effect (increases in leverage imply increases in $|\widehat{\Sigma}|$). In our current structural framework, it is thus the role of the firm's financial leverage to dampen or magnify a possible volatility feedback effect.

In view of Equation (8), it comes as no surprise that some authors call the variance-equity elasticity a "leverage coefficient" (Das and Sundaram, 2007), although the authors recognize that "there is no direct interpretation of this parameter within the Merton framework." The academic literature devoted to the constant-elasticity-variance (CEV) stock price process has rarely addressed the practical problem of parameter estimation.[8] To the best of my knowledge, Equation (8) is

[7]The economic mechanism goes as follows. As an increase in stock market volatility raises expected stock returns (Campbell and Hentschel, 1992), current stock prices then decline to adjust to these revised expectations. As a result, an increase in volatility ($\sigma \uparrow$) is correlated with negative stock returns, thus raising the value of out-of-the-money stock options and the implied volatility skew ($|\widehat{\Sigma}| \uparrow$).

[8]Beckers (1980) initiates an econometric approach to the variance-equity elasticity estimation. Schroder (1989) is the first to outline the influence of the firm's debt-equity ratio. More recently, De Spiegeleer, Schoutens, and Van Hulle (2014) further elaborate upon it in the context of the modeling of hybrid securities.

the first theoretical result to provide an unambiguous structural estimate for the variance-equity elasticity.

2.3 The credit-equity elasticity

To exploit the approach to the variance-equity elasticity developed in Section 2.2, I now introduce a simple economic model to link default probabilities with equity volatilities. We reinforce the assumption of financial market completeness by assume that equity options are continuously tradeable within a significant range of exercise prices before the default event. A stock option trading continuum is needed to exhibit optional equity structures liable to replicate the main features of a conventional CDS instrument. The ultimate purpose is to match the higher moments of the implied volatility surface, such as the volatility "skew."

If the positive correlation between CDS spreads and the levels and slopes of the implied volatility surface is well known from empiricists (e.g., Cremers et al., 2008), theoretical models that account for this close empirical relationship are still lacking. It is possible, however, to rely on sensitivity-matching analysis to get a better understanding of the links between default probabilities and the dynamics of the implied volatility surface. Grounded in the replication of a CDS instrument by an equity option structure, the following result provides a workable relationship between the CDS spread and the implied volatility skew.

Lemma 3 (Zimmermann, 2015). *Under the assumption of an stock option trading continuum, the firm's default probability on its debt at a given maturity T is linked at first order to the at-the-money implied volatility, $\widehat{\sigma}_{ATM}$, and the implied volatility skew, $\widehat{\Sigma}$, as follows:*

$$\lambda_T = k \cdot \widehat{\sigma}_{ATM} \cdot |\widehat{\Sigma}_T|, \tag{9}$$

where the constant normalizing factor k is typically independent of the equity volatility and reflects the expected recovery rate on the debt.

Proof. See Appendix D. □

I now combine the insights from Proposition 1 and Lemma 3 to derive the main result of the paper. We introduce the credit-equity elasticity defined as:

$$e_\lambda := \frac{\mathrm{d}\lambda/\lambda}{\mathrm{d}S/S}. \tag{10}$$

The next result provides a structural estimate for the credit-equity based on the firm's financial leverage.

Proposition 2 (Credit-equity elasticity). *Under the assumptions of a separable local volatility surface and a stock option trading continuum, the credit-equity elasticity is equal to the variance-equity elasticity and amounts to twice the firm's adjusted financial leverage:*

$$e_\lambda = -2(\ell - \varepsilon_\ell), \tag{11}$$

where ℓ is the debt-to-asset ratio and ε_ℓ is given by Equation (4). The case of an un-levered firm ($\ell \equiv 0$) is consistent with the Black-Scholes paradigm ($e_\sigma \equiv 0$) in which the perfect de-correlation between credit spreads and stock prices ($e_\lambda \equiv 0$) means that the stock price process cannot reach zero and that the default probability reduces to zero.

Proof. See Appendix E. □

3. Model Implications

In this section, I show that the model's main result, Proposition 2, provides new refutable hypotheses for future empirical research.

3.1 The credit-equity power relationship

The main implication of Proposition 2 is that over a small period Δt, the firm's default probability follows a power relationship relative to the stock price. Taking CDS spreads quoted in the credit market as a natural proxy for default probabilities yields the relationship:

$$\mathrm{CDS}_{t+\Delta t} = \mathrm{CDS}_t \cdot \left(\frac{S_t}{S_{t+\Delta t}} \right)^{2(\ell - \varepsilon_\ell)}. \tag{12}$$

Notice how the power function captures the loose credit-equity de-correlation when stock prices increase, and the sharp credit-equity re-correlation when stock prices fall. In contrast with alternative parameterizations of credit spreads by stock prices based on logarithmic or exponential functions, the power function also ensures sound boundary conditions when stock prices fall close to zero or tend to infinity.[9] Another comparative advantage lies in the scalability of inputs, which can be multiplied by any factor without altering relevant empirical aspects.[10]

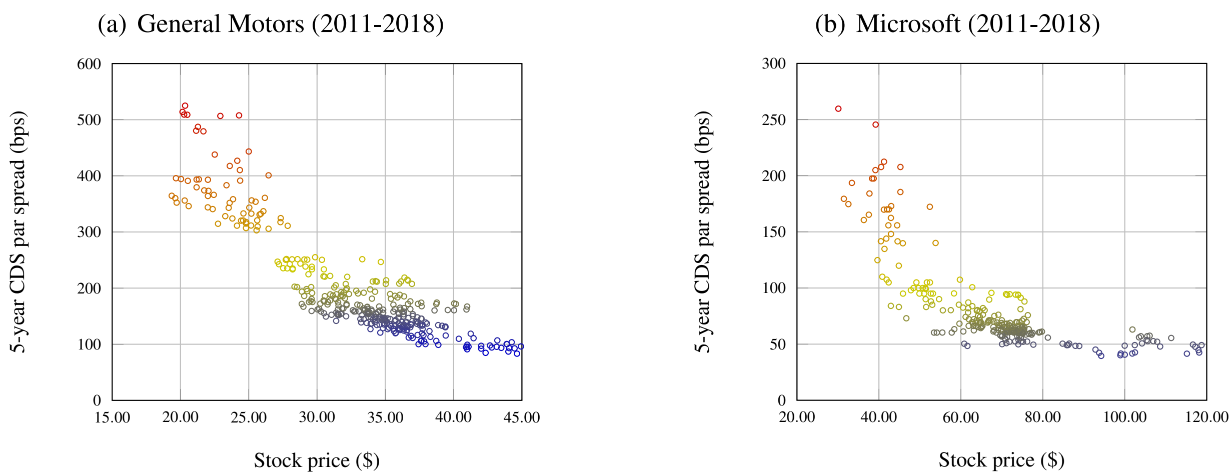
Figure 1 provides an empirical illustration of the credit-equity power relationship (12). The scatter plots display 5-year CDS par spreads against common stock prices for General Motors and Microsoft over the period 2011-2018. The monotonicity and the convexity predicted by Equation (12) are recognizable. When the equity market value rises significantly, the firm's improved financial health is expected to enhance its creditworthiness. As a result, default probabilities tend smoothly toward a floor, as illustrated in Figure 1.b (Microsoft). Conversely, a significant fall in the equity market value is expected to signal higher odds of financial distress. As a consequence, default probabilities sharply increase, as illustrated in Figure 1.a (General Motors).

[9]The superior capability of power parametric functions for data fitting is not an isolated case in the financial domain. Also known as the family of constant relative risk aversion (CRRA) in the economic literature, the power family is widely used in economics and other social sciences (e.g., Wakker, 2008; Gabaix, 2009).

[10]For example, credit risk should be an invariant across the different quoting currencies of the firm's common stock.

Figure 1. Credit-equity elasticity

This figure plots weekly CDS par spreads (5-year, senior unsecured contract) against weekly closing prices for the common equity. Time period: October, 2011 to September, 2018. Data source: Thomson Reuters.



3.2 Co-integration and price discovery

Letting the time interval Δt grow in Equation (12) and taking the natural logarithm of both sides suggests that the log-price of credit and the log-price of leveraged equity should be co-integrated. The long-run equilibrium is:

$$\ln(\text{CDS})_t = \beta_0 + \beta_1 \times (\ell - \varepsilon_\ell)_t \ln(\text{Stock})_t + \eta_t, \tag{13}$$

where β_0 captures the equilibrium's inception point, $(1, -\beta_1)'$ is the co-integrating vector, and η_t is a stationary random variable representing the dynamic behavior of the log-CDS-equity price ratio (disequilibrium error). The economic intuition behind Equation (13) is that the log-prices of CDS and leveraged equity should form a co-integrated system because both relate to the fundamental value of credit risk. If the series drift too far apart because of market frictions, capital structure arbitrageurs will act to restore the long-run equilibrium.

The co-integration of credit and equity markets has important implications for the price discovery process of credit risk. The permanent-transitory decomposition of Gonzalo and Granger (1995) suggests that credit and equity prices share a common long-memory component, also referred to as the *efficient price* (Hasbrouck, 1995). Additionally, if equity and credit prices converge in the long run, there must be an error-correction mechanism reflecting arbitrage across both markets. The error-correction mechanism should absorb the transitory shocks that have no permanent effect on CDS and equity prices. As a result, either the CDS price or the stock price must be the first to move permanently, that is, to reflect innovations in the efficient price of credit.

Two price discovery metrics have been proposed in the literature to assess the relative speed at which a price series is the first to impound new information. The first approach initiated by Hasbrouck (1995) focuses on the variance of the innovations in the efficient price. The information share (IS) measures each market's contribution to this variance. The IS metric attributes price discovery to the market that first reflects innovations in the common factor. By contrast, the second approach builds on Gonzalo and Granger's (1995) insight that the common stochastic trend must be a linear combination of the original price series. The component share (CS) measures each market's contribution to this implicit common factor. The CS metric attributes price discovery to the market with the most substantial weight in the common factor (e.g., Baillie, Booth, and Tse, 2002; Hasbrouck, 2002; Yan and Zivot, 2010; Putnins, 2013), i.e., the market that adjusts least to the other.

3.3 Information transmission between stock and credit markets

The credit-equity power relationship (12) provides new testable hypotheses concerning the transmission mechanisms between stock and credit markets. Taking the logarithm of both sides of Equation (12) and conditioning upon the information filtration $\mathscr{G}_t$ available at time t to the credit market participants yields the stock return expected by capital structure arbitrageurs:

$$\mathrm{E}\left[(\text{Stock return})_t \,\middle|\, \mathscr{G}_t\right] = -\frac{(\text{CDS return})_t}{2(\ell - \varepsilon_\ell)_t}. \tag{14}$$

Later in Section 5.1, this paper tests the following transmission mechanism from credit to stock markets suggested by Equation (14):

$$(\text{Stock return})_t = \alpha + \beta \times \frac{(\text{CDS return})_t}{2(\ell - \varepsilon_\ell)_t} + \epsilon_t, \tag{15}$$

where the intercept α captures stock return premia that cannot be explained by the integration of credit and equity markets, the coefficient β measures the rate of transmission between markets, and ϵ_t is the pure stock innovation arising in the stock market, independently from the credit market. In this mechanism, the interaction between the firm's leverage and the CDS returns constitutes the signal which primarily matters to informed traders when filtering stock returns.

4. The Data

For this study, I consider daily closing CDS quotes for the most widely traded, North American reference entities. To build as much as possible a large and representative CDS universe, I impose three requirements. The first constraint is for bid-ask CDS quotes to be available in Thomson Reuters over an extended 10-year sample period running from September 20, 2008, to November 1, 2018. In particular, the firm must not have undergone any major credit event (corporate default, merger, or acquisition) leading to an early exit from the dataset over the sample period. The second constraint is for the corresponding common stocks to continuously trade on the S&P 500 stock index over the full sampling period. Finally, we ask for the historical leverage ratio to be available in Thomson Reuters over the full sampling period. For all the reference entities satisfying the previous three requirements, all the CDS quotes, stock market data, and leverage data are then consistently retrieved from the Thomson Reuters database.[11]

The final single name CDS list comprises a total of 204 corporate credits from the S&P 500 index. For consistency, I consider only CDS par spreads corresponding to U.S.-dollar denominated contracts on the most liquid tenor (5 years), the lowest seniority (senior unsecured debt), and

[11] Mayordomo, Peña, and Schwartz (2010) offer an in-depth comparative study of the Thomson Reuters database and five other public sources of corporate CDS prices.

the same restructuring clause (Modified Restructuring). Thomson Reuters provides end-of-day prices by collecting daily single-name CDS quotes from over 30 contributors around the world and applying a rigorous screening procedure to eliminate outliers or doubtful data. Final CDS quotes are thus composite mid spreads calculated by Thomson Reuters and expressed in basis points. The timing for the end-of-day composite calculation is in T+1 (5:00 am GMT). As this last update takes place after the end of trading for U.S. stocks, there is no bias in detecting information flows from stock markets to credit markets.

To measure the firm's financial leverage, I use the ratio of total debt book value to enterprise value:

$$\frac{\text{Short-term Debt} + \text{Long-term Debt}}{\text{Market Capitalization} + \text{Total Debt} + \text{Minority Interest} + \text{Preferred Stock} - \text{Cash}}. \tag{16}$$

A conservative approach to the financial leverage of financial institutions is in order. For banks, customer deposits do not appear in total debt while cash on hand includes due from other banks. For insurance companies, policyholders liabilities do not appear in total debt. The sample thus comprises estimates of the debt-to-asset ratio over the period 2008-2018. Notice that the fluctuations of the firm's market capitalization on top of the changes in total debt book value entail daily variations in the leverage data set.

Table 1 provides summary statistics for the CDS levels, the leverage data, and the characteristics of the firms in the sample.

5. Empirical Analysis

In this section, I first provide empirical evidence for the effect of the firm's financial leverage in the transmission of price information from the stock market to the credit market.

Table 1. Descriptive statistics

The table reports summary statistics for firm characteristics (Panel A), overall equity and CDS returns (Panel B), and equity and CDS returns with opposite signs (Panel C). The sample consists of 204 U.S. firms over the period September 20, 2008, to November 1, 2018, including only trading days with available CDS spread observations and equity returns. Sample statistics are computed across all observations. Data source: Thomson Reuters.

	5^{th} perc.	25^{th} perc.	Median	Mean	75^{th} perc.	95^{th} perc.	SD	Observations
Panel A: firm-level statistics								
Firm CDS level (mid-price, bps)	25	45	70	113	120	315	188	467,330
Firm leverage (debt to assets)	0.07	0.15	0.23	0.29	0.37	0.69	0.20	467,330
Firm size (mkt. cap., $bn)	4.59	11.29	22.29	44.55	48.51	178.32	58.49	467,330
Firm debt (book value, $bn)	1.34	3.41	6.73	27.85	13.51	99.11	88.92	467,330
Daily observations	974	2,320	2,499	2,291	2,515	2,520	453	467,330
Panel B: equity and CDS returns								
Equity daily return (%)	−2.82	−0.78	0.05	0.03	0.88	2.76	2.06	467,330
CDS daily return (%)	−3.36	−0.10	0.00	−0.02	0.06	3.36	4.31	467,330
Panel C: opposite-sign equity and CDS returns								
Equity daily return (%)	−3.20	−0.91	0.03	−0.04	0.93	2.90	2.25	206,506
CDS daily return (%)	−4.26	−0.75	−0.01	−0.02	0.21	4.53	4.49	206,506

	♯ Firms	Mean CDS level (bps)	Mean leverage	Mean debt ($bn)	Mean size ($bn)	Observations
Panel D: business sector-level statistics						
Basic Materials	15	122	0.25	4.65	15.63	34,630
Consumer Cyclicals	32	160	0.27	9.49	27.72	79,126
Consumer Non-Cyclicals	23	63	0.21	11.13	52.07	56,099
Energy	18	108	0.23	10.82	62.68	40,033
Financials	36	136	0.44	107.49	47.96	74,178
Healthcare	21	61	0.19	11.17	62.96	48,218
Industrials	30	87	0.25	19.01	41.83	69,609
Technology	11	157	0.23	12.97	71.13	26,213
Telecommunications	3	147	0.41	65.74	120.96	5,884
Utilities	15	120	0.48	14.54	17.27	33,340

5.1 Identifying pure stock innovations

I first describe the methodology for identifying true innovations in the stock market due to information revelation. For each firm, I run a regression of stock percentage changes on a constant, four lags of CDS percentage changes to absorb any transmission of delayed information from the credit market, and four stock return lags to capture any autocorrelation in the stock market. To take the elasticity of CDS returns relative to stock returns into account as predicted by the model and Equation (15), the specification also includes interactions of the CDS returns (both contemporaneous and lagged) with the firm's leverage. This approach starts from the conventional view that credit pricing information primarily flows from stocks to CDS (e.g., Hilscher, Pollet, and Wilson, 2015).

Specifically, I estimate the following specification for each firm i:

$$\begin{aligned}(\text{Stock return})_{i,t} = \alpha_i + \sum_{k=0}^{4}\left[\beta_{i,k} + \frac{\beta^{\ell}_{i,k}}{(\text{Leverage})_{i,t}}\right](\text{CDS return})_{i,t-k} \\ + \sum_{k=1}^{4}\gamma_{i,k}(\text{Stock return})_{i,t-k} + \epsilon_{i,t}\ .\end{aligned} \tag{17}$$

I view the residuals $\epsilon_{i,t}$ from each of these regressions as independent innovations arriving in the stock market. These innovations are either not relevant or just not appreciated by the credit market at the time. By contrast, the coefficients $\beta_{i,0}$ and $\beta^{\ell}_{i,0}$ are akin to linear and "leveraged" measures of the feedback information flowing from the CDS market to the stock market. This approach is similar to the one by Acharya and Johnson (2007) who isolate CDS market innovations at time t by controlling for both stock and CDS returns between t and $t-k$.

The contemporaneous linear response $\beta_{i,0}$ is statistically significant at the 5% level for 22% of the firms. The contemporaneous leveraged response, $\beta^{\ell}_{i,0}$, is even more significant at 33%. For the sake of robustness, I then consider the following aggregated measures:

$$\beta_i := \sum_{k=0}^{4}\beta_{i,k}, \qquad \beta^{\ell}_i := \sum_{k=0}^{4}\beta^{\ell}_{i,k}, \tag{18}$$

Table 2. Feedback information from CDS to stock markets

In the first stage, we run for each firm i the time-series regression:

$$(\text{Stock return})_{i,t} = \alpha_i + \sum_{k=0}^{4}\left[\beta_{i,k} + \frac{\beta_{i,k}^{\ell}}{(\text{Leverage})_{i,t}}\right](\text{CDS return})_{i,t-k} + \sum_{k=1}^{4}\gamma_{i,k}(\text{Stock return})_{i,t-k} + \epsilon_{i,t} \qquad (17)$$

p-values are calculated via robust standard errors corrected for heteroscedasticity and serial correlation (Newey-West, 1987). In the second stage, firms are ranked into quintiles based on the first-stage estimates of $\beta_i = \sum_{k=0}^{4}\beta_{i,k}$ (resp. $\beta_i^{\ell} = \sum_{k=0}^{4}\beta_{i,k}^{\ell}$), Q1 being the quintile with the smallest (most negative) estimates and Q5 being the quintile with the largest estimates. The summary statistics reported for each quintile are the medians (across firms) of the time-series means of the characteristics for each firm. Within each quintile, p-values across firms are combined via Fisher's sum of logarithms method. ***, ** and * denote statistical significance at the 0.1%, 1%, and 5% levels, respectively. Data source: Thomson Reuters.

	Q1	Q2	Q3	Q4	Q5
Panel A: Properties of firms in different β-quintiles					
Median β_i	−0.335	−0.126	−0.022	0.136	0.463
Within-quintile p-value	0.000***	0.011**	1.000	0.851	0.000***
Median CDS level (bps)	102	77	66	69	81
Median firm leverage	0.29	0.19	0.20	0.23	0.36
Panel B: Properties of firms in different β^{ℓ}-quintiles					
Median β_i^{ℓ}	−1.687	−0.946	−0.523	−0.112	0.364
Within-quintile p-value	0.000***	0.000***	0.000***	0.999	0.014**
Median CDS level (bps)	68	82	96	76	80
Median firm leverage	0.23	0.27	0.26	0.25	0.21

These measures capture the overall feedback information flowing from the CDS market to the stock market at the firm level. The aggregated linear response β_i remains significant at the 5% level for only 18% of the firms. However, the level of statistical significance of the aggregated leveraged response β_i^{ℓ} now rises to at least 49% of the firms. This gap in the level of statistical significance stands a first hint as to the role of the leverage effect in the feedback price transmission from credit markets to stock markets.

Table 2 sorts the firms into quintiles based on their aggregated response and examines the average characteristics for firms within each quintile. Panel A of Table 2 shows that the linear aggregated response β_i can be positive, in stark contrast with structural models of default risk. Neither the CDS level nor the leverage appears to vary much across quintiles. When combining p-values within quintiles, only the lowest and the highest quintiles display statistical significance. By contrast, Panel B of Table 2 shows that the aggregated leveraged response β_i^{ℓ} is negative for most firms, in line with structural models of credit risk. Moreover, the high degree of combined statistical significance is almost uniform across quintiles. In this sense, β_i and β_i^ℓ appear as complementary measures of the feedback transmission channel existing from the CDS market to the stock market.

5.2 Evidence of leveraged information

I can now exploit the stock price innovations identified in the previous section to study the information flow from stock markets to credit markets. To bring to light the leverage effect predicted by the model and Equation (15), the specification of expected CDS returns includes interactions of the stock returns (both contemporaneous and lagged) with the firm's financial leverage. The specification also contains four lags of CDS percentage changes to purge the credit market of any residual autocorrelation. Finally, I allow the information flow to vary conditionally to specific market conditions.

More specifically, I estimate the following panel specification by pooled regression:

$$(\text{CDS return})_t = a + \sum_{k=0}^{4} b_k^\ell (\text{Leverage})_t (\text{Stock innovation})_{t-k} + \sum_{k=1}^{4} c_k (\text{CDS return})_{t-k} + e_t, \quad (19)$$

where the first-stage residuals $\hat{\epsilon}_{i,t}$ provide a proxy for the real stock innovations. The linear combination $\sum_{k=0}^{4} b_k^\ell$ offers a measure of the "leveraged" information flowing unconditionally from the stock market to the credit market. The stock market direction allows to condition specification (19) by using separate regression coefficients on the positive and negative part of each of the five lagged stock innovation terms. Similarly, conditioning upon stock innovation intensity enables to obtain more granular insights into the leveraged information flow.

Table 3 reports estimates for the specification (19). The main finding here is that the overall flow of leveraged information from stock to credit markets is highly significant at the 0.1% threshold. The measure has the awaited negative sign predicted by structural models of credit risk. Its value (-0.462) is significantly higher than the flow of direct, unleveraged information (-0.310).

Table 3. Leveraged information from stock to CDS markets

This table reports OLS panel estimates and t-statistics for the coefficients of the following pooled regression:

$$(\text{CDS return})_t = a + \sum_{k=0}^{4} \left[b_k + b_k^{\ell} (\text{Leverage})_t + b_k^{\ell,D} (\text{Leverage})_t (\text{Dummy})_t \right] (\text{Stock innovation})_{t-k} + \sum_{k=1}^{4} c_k (\text{CDS return})_{t-k} + e_t \tag{19}$$

The first column reports the baseline model (no leverage, no dummy). Column (A) reports unconditional estimates (no dummy). Column (B) reports estimates conditioned on positive stock innovations ($\hat{\epsilon} > 0$) and negative stock innovations ($\hat{\epsilon} < 0$). Column (C) reports estimates conditioned on positive stock innovations in the top decile and negative stock innovations in the lowest decile. Column (D) reports estimates conditioned on CDS levels in the lowest quintile Q_1 (< 40 bps), medium quintiles, and the top quintile Q_5 (> 142 bps), respectively. Column (E) reports estimates conditioned on the leverage in the lowest quintile Q_1 (< 0.14), medium quintiles, and the top quintile Q_5 (> 0.41), respectively. t-statistics in parentheses are calculated via firm-clustered standard errors corrected for heteroscedasticity and serial correlation. ***, ** and * denote statistical significance at the 0.1%, 1%, and 5% levels, respectively. Data source: Thomson Reuters.

	Baseline model	(A)	(B)	(C)	(D)	(E)
a	-0.0001	-0.0001^{*}	-0.0003^{***}	-0.0004^{***}	-0.0001^{*}	-0.0001
	(-1.61)	(-2.11)	(-4.11)	(-3.38)	(-2.04)	(-1.69)
$\sum_{k=0}^{4} b_k$	-0.310^{***}					
	(-50.22)					
$\sum_{k=0}^{4} b_k^{\ell}$		-0.462^{***}				
		(-34.86)				
$\sum_{k=0}^{4} b_k^{\ell,\hat{\epsilon}>0}$			-0.397^{***}	-0.437^{***}		
			(-24.73)	(-8.30)		
$\sum_{k=0}^{4} b_k^{\ell,\hat{\epsilon}>0,\text{ top}}$				-0.358^{***}		
				(-17.23)		
$\sum_{k=0}^{4} b_k^{\ell,\hat{\epsilon}<0}$			-0.511^{***}	-0.796^{***}		
			(-33.12)	(-15.40)		
$\sum_{k=0}^{4} b_k^{\ell,\hat{\epsilon}<0,\text{ lowest}}$				-0.498^{***}		
				(-24.95)		
$\sum_{k=0}^{4} b_k^{\ell,Q_1}$					2.424^{***}	-2.816^{***}
					(19.19)	(-14.39)
$\sum_{k=0}^{4} b_k^{\ell,Q_{2,3,4}}$					-0.444^{***}	-0.341^{***}
					(-30.48)	(-23.97)
$\sum_{k=0}^{4} b_k^{\ell,Q_5}$					-0.747^{***}	-1.215^{***}
					(-22.90)	(-32.74)
$\sum_{k=0}^{4} c_k$	0.039^{***}	0.045^{***}	0.044^{***}	0.044^{***}	0.044^{***}	0.043^{***}
	(9.48)	(16.45)	(16.25)	(16.26)	(16.15)	(15.67)
Obs.	455,690	455,690	455,690	455,690	455,690	455,690

Moreover, this finding is robust when conditioning upon the stock market direction. Column (B) shows that both the responses to positive and negative lagged innovations keep negative signs and the same magnitudes. The finding is also robust to the intensity of the stock information flow. The distribution of stock innovations being symmetrical, the lowest and highest deciles correspond to stock volatility above 67% per annum. When conditioning upon these extreme innovations, column (C) reveals that aggregated responses still keep negative signs and the same magnitudes.

To investigate the firm conditions in which leveraged information typically flows from stock to credit markets, I also estimate specification (19) conditionally upon different credit conditions:

$$
\begin{aligned}
(\text{CDS return})_t = a &+ \sum_{k=0}^{4} \left[b_k^{\ell} + b_k^{\ell,D} (\text{Dummy})_t \right] (\text{Leverage})_t \times (\text{Stock innovation})_{t-k} \\
&+ \sum_{k=1}^{4} c_k (\text{CDS return})_{t-k} + e_t,
\end{aligned}
\tag{20}
$$

where the first-stage residuals $\hat{\epsilon}_{i,t}$ proxy the real stock innovations. I interpret the linear combination $\sum_{k=0}^{4} b_k$ (resp. $\sum_{k=0}^{4} b_k^D$) as a measure of the unconditional (resp. conditional) leveraged information flow from the stock market to the credit market. To investigate the role of the firm's credit quality, I first condition by the credit spread level. I build three dummy variables to allocate the CDS level variation between the top quintile of the CDS distribution (above 142 basis points, corresponding approximately to a credit rating equal to or lower than A3/A-), the intermediary three quintiles (between 142 and 40 basis points), and the lowest quintile (below 40 basis points), respectively. Similarly, I probe the role of the firm's level of indebtedness by setting three dummy variables to allocate the leverage variation between the top quintile of the leverage distribution (above 0.41), the three intermediary quintiles (between 0.14 and 0.41), and the lowest quintile (below 0.14).

Table 3 reports estimates for the specification (20). Column (D) shows that the conditioned flow measure becomes positive when conditioning by low CDS levels. This unexplained positive response could signal either a low degree of informed trading in the CDS market or the absence of substantive information concerning credit risk. In other words, top CDS levels seem to impound a very substantial part of the leveraged price transmission. This finding could suggest a regime of informed revision of CDS quotes under conditions of financial stress.

Table 3 reveals a similar phenomenon when conditioning by top levels of indebtedness. The (unconditioned) response of column (A) increases from -0.462 to -1.215 in column (E), more than threefold an increase in intensity. Highly leveraged firms seem to produce even more informed revisions of CDS quotes instead of mechanical price transmission, once again suggesting the occurring of insider trading (Acharya and Johnson, 2007).

5.3 Leveraged information at the firm level

The pooled regression described above forces all firms to have the same dynamic properties, except as captured by market-conditioning dummy variables. I now estimate separate dynamics for each firm by allowing for firm fixed effects. To compare the intensity of the leveraged, non-linear information flow with the direct transmission of information, I also include five lags of unleveraged stock innovations. This alternative specification allows testing for differences among nested models at the firm level by running a likelihood ratio (LR) test. The LR test statistic then measures whether the inclusion of leveraged regressors significantly improves the goodness of fit of the regression model.

Specifically, I estimate the following specification for each firm i:

$$\begin{aligned}(\text{CDS return})_{i,t} = a_i + \sum_{k=0}^{4}\left[b_{i,k} + b^{\ell}_{i,k}(\text{Leverage})_{i,t}\right] \times (\text{Stock innovation})_{i,t-k} \\ + \sum_{k=1}^{4} c_{i,k}(\text{CDS return})_{i,t-k} + e_{i,t}\end{aligned} \tag{21}$$

where the first-stage residuals $\hat{\epsilon}_{i,t}$ provide a proxy for the real stock innovations.

Table 4. Leveraged information: two-stage cross-sectional estimation

In the first stage, I run for each firm i the time-series regression:

$$(\text{CDS return})_{i,t} = a_i + \sum_{k=0}^{4} \left[b_{i,k} + b^{\ell}_{i,k}(\text{Leverage})_{i,t} \right] (\text{Stock innovation})_{i,t-k} + \sum_{k=1}^{4} c_{i,k}(\text{CDS return})_{i,t-k} + e_{i,t}. \quad (21)$$

LR measures rejection of the null hypothesis $\text{H}_0 : \ b^{\ell}_{i,k} = 0 \ (0 \leqslant k \leqslant 4)$ by the likelihood ratio test. In the second stage, firms are ranked into quintiles based on the first-stage estimates of $b^{\ell}_i = \sum_{k=0}^{4} b^{\ell}_{i,k}$, Q1 being the quintile with the smallest (most negative) estimates and Q5 being the quintile with the largest estimates. The summary statistics reported for each quintile are the medians (across firms) of the time-series means of the characteristics for each firm. Within each quintile, p-values across firms are combined via Fisher's sum of logarithms method. ***, ** and * denote statistical significance at the 0.1%, 1%, and 5% levels, respectively. Data source: Thomson Reuters.

Panel A: Univariate properties of b and b^{ℓ}				
Average b_i	-0.017	Average b^{ℓ}_i	-1.303	
t-statistic	(-0.14)	t-statistic	(-1.74)	
		LR (p-value)	0.000^{***}	
Min	-3.277	Min	-142.903	
Max	18.000	Max	16.520	

	Q1	Q2	Q3	Q4	Q5
Panel B: Properties of firms in different b^{ℓ}-quintiles					
Median b^{ℓ}_i	-3.216	-0.930	-0.128	0.372	1.679
LR (p-value)	0.000^{***}	0.000^{***}	0.094	0.002^{***}	0.000^{***}
Median CDS level (bps)	59	77	129	97	74
Median firm leverage	0.22	0.22	0.32	0.29	0.20
Median firm size ($bn)	24.06	22.62	17.04	21.19	32.47
Median firm debt ($bn)	7.04	5.90	5.95	7.41	6.54

Table 4 reports estimates for the specification (21). Panel A shows the summary statistics for the estimated linear responses b_i. The mean is -0.017 and statistically insignificant, consistent with the findings of previous studies (Acharya and Johnson, 2007). By contrast, the mean of the leveraged response b^{ℓ}_i is -1.303 and significant, thereby validating the non-linear role of the leverage. As a robustness check, I also run for each firm an LR test of the null hypothesis $\text{H}_0 :$ $b^{\ell}_{i,k} = 0 \ (0 \leqslant k \leqslant 4)$. This procedure provides a collection of independent LR test statistics and p-values. I then use Fisher's combined probability test to fusion these p-values and to assess whether the inclusion of leveraged predictor variables improves the model's goodness of fit. Panel A reveals that this combined LR p-value is highly significant.

Panel B sorts the firms into quintiles based on their aggregated leveraged response and examine the median firm characteristics of each. The combined LR p-value turns out to be highly significant for four quintiles out of five. Moreover, there is a uniform distribution of firm characteristics across quintiles. These observations suggest that a specific category of firms does not impound the leveraged flow of information from stock to CDS markets.

5.4 Co-integration of credit and equity markets

In this section, I now investigate the role of the leverage effect in the co-integration of credit and equity markets. Equation (13) suggests that the log-CDS price and the leveraged log-stock price should form a co-integrated system. The intuition is that CDS spread and stock price time series cannot drift too far apart from the equilibrium because capital structure arbitrageurs will act to restore the long-run equilibrium relationship.

The testing procedure draws on the VECM approach to co-integration modeling suggested by Johansen (1988, 1991). Let introduce for each company i the credit-equity log-price process:

$$\mathbf{X}_{i,t} := \begin{bmatrix} \ln(\text{CDS})_{i,t} \\ \ln(\text{Stock})_{i,t} \end{bmatrix} \tag{22}$$

A preliminary step consists in checking that the two components of the log-price process are indeed integrated to the same order. For this purpose, I run systematic unit root tests for nonstationarity.

In the first stage, we then select each firm's optimal lag length by fitting a VAR model for the leveraged log-price process:

$$\mathbf{X}^{\ell}_{i,t} := \begin{bmatrix} \ln(\text{CDS})_{i,t} \\ (\text{Leverage})_{i,t} \cdot \ln(\text{Stock})_{i,t} \end{bmatrix}. \tag{23}$$

The order of the VAR, p_i, is selected by the Schwarz Bayesian information criterion. In the second stage, we estimate the following fully specified VECM by maximum likelihood:

$$\Delta\mathbf{X}^{\ell}_{i,t} = \Pi_i \mathbf{X}^{\ell}_{i,t-1} + \sum_{k=1}^{p_i} \Gamma_{i,k} \Delta\mathbf{X}^{\ell}_{i,t-k} + \boldsymbol{\mu}_i + \mathbf{u}_{i,t}, \tag{24}$$

where Π_i is the long-run impact matrix, $\Gamma_{i,k}$ are short-run impact matrices, $\boldsymbol{\mu}_i$ is a drift vector, and $\mathbf{u}_{i,t} = (u^1_{i,t}, u^2_{i,t})'$ are independent 2-dimensional Gaussian disturbances. Assuming $\mathbf{X}^\ell_{i,t}$ is co-integrated implies that the rank of Π_i can neither be null due to the error correction mechanism nor equal to 2 due to nonstationarity. As a result, Π_i must be of rank 1, and there must exist two vectors $\boldsymbol{\alpha}_i = (\alpha_{i1}, \alpha_{i2})'$ and $\boldsymbol{\beta}_i = (1, -\beta_i)'$ such that $\Pi_i = \boldsymbol{\alpha}_i \boldsymbol{\beta}'_i$. In this case, even though $\mathbf{X}^\ell_{i,t}$ is not stationary, the stochastic deviations from the long-run equilibrium must be stationary around a potential deterministic trend:

$$\eta_{i,t} := \boldsymbol{\beta}'_i \mathbf{X}^\ell_{i,t} + \rho_i t = \ln(\text{CDS})_{i,t} - \beta_i (\text{Leverage})_{i,t} \ln(\text{Stock})_{i,t} + \rho_i t \sim I(0). \tag{25}$$

We allow for a linear time trend in the co-integrating relationship to accommodate the trending nature of data like stock prices.

We use Johansen's (1988, 1991) likelihood ratio statistic to test for the rank of the long-run impact matrix. Rejecting the null hypothesis that $\text{rank}(\Pi_i) = 0$ validates the existence of a co-integrating vector $\boldsymbol{\beta}_i$. We repeat the testing procedure with the symmetric process:

$$\mathbf{Y}^\ell_{i,t} := \begin{bmatrix} \ln(\text{CDS})_{i,t} / (\text{Leverage})_{i,t} \\ \ln(\text{Stock})_{i,t} \end{bmatrix}. \tag{26}$$

Finally, we consider an entity i as co-integrated if either $\mathbf{X}^\ell_{i,t}$, or $\mathbf{Y}^\ell_{i,t}$, or both processes reject the null of non-co-integration at the statistical level of 5%.

Table 5 reports co-integration results. In a preliminary step, we run Augmented Dickey-Fuller tests to verify that the stock price and CDS spread time series share the same order of integration. To conserve space, we do not present the results of these tests, which provide unambiguous evidence that all the series are $I(1)$ and their first differences $I(0)$. These results are typical for asset price time series.

Panel A reports the number of significantly co-integrated firms that qualify for the VECM stage. We find support for co-integration at the 5% level for 106 entities out of 215, which represents 50%

Table 5. Co-integration tests results

This table reports co-integrated firms for which the Johansen trace statistic rejects the null hypothesis of non-co-integration $H_0 : \text{rank}(\Pi_i) = 0$ when estimating the VECM $\Delta\mathbf{X}^{\ell}_{i,t} = \Pi_i\mathbf{X}^{\ell}_{i,t-1} + \sum_{k=1}^{p_i}\Gamma_{i,k}\Delta\mathbf{X}^{\ell}_{i,t-k} + \boldsymbol{\mu}_i + \mathbf{u}_{i,t}$ (resp. $\Delta\mathbf{Y}^{\ell}_{i,t} = \Pi_i\mathbf{Y}^{\ell}_{i,t-1} + \sum_{k=1}^{p_i}\Gamma_{i,k}\Delta\mathbf{Y}^{\ell}_{i,t-k} + \boldsymbol{\mu}_i + \mathbf{u}_{i,t}$) by maximum likelihood. A time trend is included in the long-run relation. Panel A reports the number and proportion of co-integrated entities by threshold of co-integration. ***, ** and * denote statistical significance at the 0.1%, 1%, and 5% levels, respectively. Panel B reports firm-level sample statistics computed across all observations. Data source: Thomson Reuters.

	$p < 0.001^{***}$		$p < 0.01^{**}$		$p < 0.05^{*}$		
	♯ Firms	Prop. (%)	♯ Firms	Prop. (%)	♯ Firms	Prop. (%)	Total
Panel A: number of co-integrated firms							
Full sample	51	23.7	80	37.2	106	49.3	215
Basic Materials	4	26.7	4	26.7	5	33.3	15
Consumer Cyclicals	6	17.1	8	22.9	14	40.0	35
Consumer Non-Cyc.	3	13.0	6	26.1	9	39.1	23
Energy	3	15.8	7	36.8	8	42.1	19
Financials	19	52.8	23	63.9	26	72.2	36
Healthcare	2	8.7	9	39.1	13	56.5	23
Industrials	9	30.0	12	40.0	12	40.0	30
Technology	1	6.2	3	18.8	6	37.5	16
Telecommunications	0	0.0	1	33.3	3	100.0	3
Utilities	4	26.7	7	46.7	10	66.7	15
	5^{th} perc.	Median	Mean	75^{th} perc.	95^{th} perc.	SD	Obs.
Panel B: firm-level statistics of co-integrated firms							
Firm CDS level (bps)	26	75	117	130	309	204	241,054
Firm leverage	0.08	0.25	0.32	0.42	0.80	0.22	241,054
Firm size ($bn)	4.50	20.08	49.20	58.64	187.60	68.63	241,054
Firm debt ($bn)	1.24	6.91	42.05	16.86	358.51	117.00	241,054
Panel C: equity and CDS returns of co-integrated firms							
Equity daily return (%)	−2.81	0.05	0.02	0.86	2.73	2.14	241,054
CDS daily return (%)	−3.44	0.00	−0.02	0.06	3.41	5.47	241,054

of the firms. A third of the sample does not hint at co-integration at all, while 20 firms barely miss the 5% rejection threshold. The Financials, Telecommunications, and Utilities business sectors appear the most co-integrated, with co-integration ratios close or above 70%. Without surprise, Table 1 reveals that these three sectors are also the most dependent on external debt financing, with average debt-to-asset ratios well over 40%. This finding is the first hint of the role of the leverage effect in the integration of credit and equity markets.

Panel B reports summary statistics for co-integrated entities at the 5% threshold. The average debt-to-asset ratio (resp. CDS level) in this sub-sample is 3 points (resp. 4 basis points) higher. In the case of deeply co-integrated entities at the 0.1% level, unreported statistics show that this leverage differential is even more acute with an excess 4.5 points (resp. 14 basis points). With an average market capitalization of 44.5$bn versus 49.2$bn, the size of co-integrated firms seems on par with the overall sample. Their indebtedness, however, is much more pronounced with an average debt size of 42.1$bn versus 27.9$bn. Everything happens as if an increase in corporate leverage ramps up market activity in capital structure arbitrage. These additional findings provide reliable evidence that one of the leading market effects of corporate leverage is to intensify the integration between the credit and equity markets.

Panel C reports summary statistics for daily stock and CDS returns of co-integrated firms. While stock returns statistics are similar to the full sample, the standard deviation of CDS returns is much more pronounced, with 5.47% versus 4.53%. This finding is even more spectacular for highly co-integrated entities with an (untabulated) standard deviation at 6.85%. Without surprise, some excess volatility in CDS prices appears necessary to stimulate capital structure arbitrage activity.

5.5 Contributions to price discovery

I now study the contribution of each market to the price discovery process. The co-integrating feature suggests adopting the classical vector error-correction model (VECM) approach to price discovery formalized by Gonzalo and Granger (1995). The intuition is that error-correcting adjustments to transitory shocks must occur in either the stock market or the CDS market to maintain the long-run equilibrium relationship between both time series.

In the first stage, I test at the level of each firm i the co-integration of the CDS and stock price series following the methodology of Section 5.4. For those entities which are significantly co-integrated at the 5% threshold, I retrieve the co-integrating residuals $\widehat{\eta}_{i,t}$ and the lag order p_i providing the optimal fit for the underlying VAR process.

In the second stage, I measure the contribution of each market to the price discovery process by estimating the following VECM by OLS:

$$
\begin{aligned}
(\text{CDS return})_{i,t} &= \alpha_1 \widehat{\eta}_{i,t-1} + \sum_{k=1}^{p_i} b_{1,k} (\text{Leverage})_{i,t} (\text{Stock return})_{i,t-k} + \sum_{k=1}^{p_i} c_{1,k} (\text{CDS return})_{i,t-k} + u^1_{i,t}, \\
(\text{Stock return})_{i,t} &= \alpha_2 \widehat{\eta}_{i,t-1} + \sum_{k=1}^{p_i} b_{2,k} (\text{Stock return})_{i,t-k} + \sum_{k=1}^{p_i} c_{2,k} \frac{(\text{CDS return})_{i,t-k}}{(\text{Leverage})_{i,t}} + u^2_{i,t},
\end{aligned}
\tag{27}
$$

where the first-stage co-integrating residuals $\widehat{\eta}_{i,t}$ provide error-correcting terms, the lagged stock and CDS returns capture market imperfections, and $u^{1,2}_{i,t}$ are i.i.d. disturbances. If the stock market is contributing significantly to the price discovery process, then α_1 should be negative and statistically significant as the CDS market continuously adjusts to absorb transitory noise frictions. Conversely, if the CDS market dominates the price discovery process, α_2 should be positive and statistically significant as the stock market continuously responds to transitory shocks.

Comparing the adjustment coefficients $\hat{\alpha}_1$ and $\hat{\alpha}_2$ allows estimating the market that least adjusts to transitory deviations. This market will stand as the closest to the fundamental value of credit risk. Following the literature on credit price discovery (e.g., Narayan, Sharma, and Thuraisamy, 2014), I use the component share (CS) metric to measure the relative shares of each market in the permanent component:

$$
\text{CS}_{\text{CDS}} := \frac{\alpha_2}{\alpha_2 - \alpha_1}, \qquad \text{and} \qquad \text{CS}_{\text{Stock}} := \frac{\alpha_1}{\alpha_1 - \alpha_2}, \tag{28}
$$

provided that $\hat{\alpha}_1 \neq \hat{\alpha}_2$.[12] I also rely on the information share (IS) metric to measure the relative adjustment speed of each market to innovations in the permanent component. The lower bound and upper bound of information share are given as follows (Hasbrouck, 1995):

$$
\text{IS}^{\text{Low}} := \frac{\alpha_2^2(\sigma_1^2 - \sigma_{12}^2/\sigma_2^2)}{\alpha_2^2\sigma_1^2 - 2\alpha_1\alpha_2\sigma_{12} + \alpha_1^2\sigma_2^2}, \qquad \text{IS}^{\text{Up}} := \frac{(\alpha_2\sigma_1 - \alpha_1\sigma_{12}/\sigma_1)^2}{\alpha_2^2\sigma_1^2 - 2\alpha_1\alpha_2\sigma_{12} + \alpha_1^2\sigma_2^2}, \tag{29}
$$

[12]Notice that $0 \leqslant \text{CS}_{\text{CDS}} \leqslant 1$ as soon as $\hat{\alpha}_1$ and $\hat{\alpha}_2$ have the expected negative and positive sign, respectively.If $\hat{\alpha}_1 = 0$, there is no evidence of price discovery in the stock market ($\text{CS}_{\text{CDS}} \equiv 1$). If $\hat{\alpha}_2 = 0$ there is no evidence of price discovery in the CDS market ($\text{CS}_{\text{CDS}} \equiv 0$).

where $\text{Var}(u_t^1) := \sigma_1^2$, $\text{Var}(u_t^2) := \sigma_2^2$, and $\text{cov}(u_t^1, u_t^2) := \sigma_{12}^2$. We follow the standard practice in the price discovery literature (e.g., Baillie et al., 2002) to average the lower and upper bounds:

$$\text{IS}_{\text{CDS}} := \frac{\text{IS}_{\text{CDS}}^{\text{Low}} + \text{IS}_{\text{CDS}}^{\text{Up}}}{2}, \qquad \text{and} \qquad \text{IS}_{\text{Stock}} := \frac{\text{IS}_{\text{Stock}}^{\text{Low}} + \text{IS}_{\text{Stock}}^{\text{Up}}}{2}. \tag{30}$$

Table 6 reports price discovery metrics for the stock and CDS markets. We restrict the analysis to the sub-sample of 106 firms that reject the null hypothesis of non-co-integration at the 5% threshold.

Panel A reports the stock market's price discovery metrics. For the stock market to significantly impact the efficient price of credit, the adjustment coefficient $\hat{\alpha}_1$ must be negative and statistically significant. Co-integrated firms are thus ranked into deciles based first on their increasing component share, and second on their decreasing leverage. For more than half of the firms, a component share close or equal to 100% indicates a high degree of proximity of the stock market with the permanent component of the credit-equity price system. As a result, the stock market monopolizes the price discovery process with a market share of 74.8% on average. To a lesser extent, the information share metric confirms the informational leadership of the stock market, with an average market share of 55.5%. This result is consistent with the CDS "sideshow" hypothesis (Hilsher, Pollet, and Wilson, 2015) for which informed traders globally favor the stock market to the CDS market because of transaction costs. We notice that the last three deciles (D8–D10) concentrate firms with average leverage well below the overall sample mean (0.29).

Panel B reports the smaller set of firms whose CDS market heavily weighs on the permanent component of the credit-equity price system. In this case, the adjustment coefficient $\hat{\alpha}_2$ must be positive and statistically significant. We thus focus on the 49 firms whose $\hat{\alpha}_2$ turns out to be significant at the 5% level. Confined to the last three deciles (D8–D10), Panel C reveals strong evidence for the role of the leverage effect. With a component share near 100%, CDS prices are closely aligned with the fundamental value of credit as they preempt the bulk of informed trading.

Table 6. CDS and equity market shares of the price discovery process

This table reports price discovery metrics for the stock and CDS markets. In the first stage, we select the co-integrated firms for which the Johansen trace statistic rejects the null of non-co-integration at the 5% threshold and retrieve error-correcting terms $\widehat{\eta}_{i,t}$. In the second stage, we estimate the following VECM by OLS:

$$\begin{aligned}
(\text{CDS return})_{i,t} &= \alpha_1 \widehat{\eta}_{i,t-1} + \sum_{k=1}^{p_i} b_{1,k} (\text{Leverage})_{i,t} (\text{Stock return})_{i,t-k} + \sum_{k=1}^{p_i} c_{1,k} (\text{CDS return})_{i,t-k} + u^1_{i,t}, \\
(\text{Stock return})_{i,t} &= \alpha_2 \widehat{\eta}_{i,t-1} + \sum_{k=1}^{p_i} b_{2,k} (\text{Stock return})_{i,t-k} + \sum_{k=1}^{p_i} c_{2,k} \frac{(\text{CDS return})_{i,t-k}}{(\text{Leverage})_{i,t}} + u^2_{i,t},
\end{aligned} \tag{27}$$

We assess the statistical significance of the adjustment coefficients $\hat{\alpha}_1$ and $\hat{\alpha}_2$ via robust standard errors corrected for heteroscedasticity and serial correlation (Newey-West, 1987). Panel A reports co-integrated firms for which $\hat{\alpha}_1$ is significant at the 5% level, ranked into deciles based on the second-stage estimate of the stock market's component share, CS_{Stock}. Panel B reports firms for which $\hat{\alpha}_2$ is significant at the 5% level, ranked into deciles based on the second-stage estimate of the CDS market's component share, CS_{CDS}. Panel C reports firms for which $\hat{\alpha}_2$ is significant at the 5% level, ranked into deciles based on the second-stage estimate of the CDS market's information share, IS_{Stock}. The summary statistics reported for each quintile are the averages (across firms) of the time-series means of the characteristics for each firm. Data source: Thomson Reuters.

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	Total
Panel A: stock market											
Average component share, CS_{Stock} (%)	0.0	26.8	53.5	80.3	96.8	100.0	100.0	100.0	100.0	100.0	74.8
Average information share, IS_{Stock} (%)	7.4	25.7	49.2	61.4	76.4	65.8	67.9	66.0	65.3	78.6	55.5
Average firm leverage	0.41	0.30	0.21	0.30	0.58	0.45	0.31	0.23	0.17	0.08	0.31
Average CDS level (bps)	142	108	87	121	192	131	141	88	62	51	115
Number of firms	11	11	11	11	11	11	11	11	11	7	106
Panel B: CDS market's component share											
Average component share, CS_{CDS} (%)	7.0	13.8	23.6	38.3	50.0	50.0	71.0	100.0	100.0	100.0	54.5
Average firm leverage	0.32	0.17	0.44	0.18	0.20	0.41	0.29	0.17	0.34	0.60	0.31
Average CDS level (bps)	82	78	174	58	94	155	115	51	134	184	111
Number of firms	5	5	5	5	5	5	5	5	5	4	49
Panel C: CDS market's information share											
Average information share, IS_{CDS} (%)	18.6	30.9	40.6	47.2	53.4	59.2	75.6	91.2	96.5	99.1	60.5
Average firm leverage	0.20	0.21	0.27	0.39	0.35	0.40	0.43	0.38	0.18	0.24	0.31
Average CDS level (bps)	63	98	133	105	102	195	84	154	66	108	111
Number of firms	5	5	5	5	5	5	5	5	5	4	49

The top decile (D10) contains highly-leveraged firms with leverage above the 95$^{\text{th}}$ percentile (0.58) of the average debt-to-asset distribution by firms. In this decile, the average CDS level rises above 200 basis points, a level corresponding to a credit rating lower than Baa2/BBB.

Panel C examines the same set of firms as Panel C from a slightly different standpoint. Whereas Panel C focuses on the CDS market's relative avoidance of noise, the focus now shifts to the relative speed of adjustment to innovations in the permanent component of credit. The average information share of the CDS market (60.5%) is in line with market shares reported in the recent study by Kryzanowski, Perrakis, and Zhong (2017).

6. Conclusions

A parsimonious structural framework is sufficient to build a theoretical model connecting the firm's financial leverage and the variance-equity elasticity. This elasticity amounts to twice the debt-to-assets ratio—a standard measure of the corporate leverage. This key feature enables putting the so-called "leverage effect" into a credit risk perspective, thus giving its full meaning to a four-decade-old term (Black, 1976). It provides a non-linear mechanism of information transmission between the equity and credit markets.

An empirical analysis over a large dataset of S&P 500 firms and an extended timeframe (2008-2018) highlights the non-linear role of the corporate leverage in the transmission of price information between stock and CDS markets. The study shows that such activity is intense and robust to market conditions. It affects all firms uniformly, irrespective of their level of indebtedness or their CDS spread quoted in the market. As the corporate leverage increases, it stimulates market activity in capital structure arbitrage and strengthens the co-integration of credit and equity markets. For co-integrated firms, the strength of the long-run equilibrium between CDS spread and stock price correlates with the debt-to-asset ratio.

In line with previous studies, two-thirds of the firms in the sample see their price discovery process widely dominated by the equity market, with stocks impounding more than 70% of the process. However, I find a significant portion of highly-leveraged firms for which half of the discovery process or more is occurring in the CDS market. The leverage effect could explain some of the pricing discrepancies observed between stock and CDS markets. The recent literature usually attributes these mispricings to various CDS market inefficiencies such as illiquidity or opaqueness. By contrast, the leverage effect provides an economic rationale for the limits to capital structure arbitrage and the lack of integration between equity and credit markets.

Appendix A. Proof of Lemma 1

A standard application of Ito's lemma shows that the firm's asset volatility is linked to the equity instantaneous volatility by the relationship:

$$\sigma S = \frac{\partial S}{\partial V} \sigma_V V. \tag{A1}$$

This can be differentiated with respect to the equity market value to get:

$$\frac{\partial \sigma}{\partial S} = \frac{\partial}{\partial S}\left(S_V \sigma_V \frac{V}{S}\right) = \frac{\partial S_V}{\partial S} \times \sigma_V \frac{V}{S} + S_V \sigma_V \times \frac{S-V}{S^2}, \tag{A2}$$

where we have used the fact that the firm's business risk, σ_V, is a constant independent from S and the firm's capital structure. We can apply the chain rule

$$\frac{\partial S_V}{\partial S} = \frac{\partial S_V}{\partial V} \frac{\partial V}{\partial S} = \frac{S_{VV}}{S_V}, \tag{A3}$$

which yields after substitution into Equation (A2):

$$S \frac{\partial \sigma}{\partial S} = \frac{S_{VV}}{S_V} \sigma_V V - S_V \sigma_V \frac{V-S}{S}. \tag{A4}$$

Equation (A1) also enables to express the unknown asset volatility $\sigma_V = \sigma S/(S_V V)$ which can now be eliminated from Equation (A4). Noticing that the debt-to-asset ratio $\ell = (V-S)/V$, we find:

$$S \frac{\partial \sigma}{\partial S} = \sigma \frac{S S_{VV}}{S_V^2} - \sigma \ell. \tag{A5}$$

which yields Equation (3) in a straightforward way.

It remains to be checked that the adjusting term $\varepsilon_\ell = (S S_{VV})/S_V^2$ to the leverage is bounded on

$[V_B;\infty)$. In the structural setting of Duffie and Lando (2001), the bankruptcy boundary is given by:

$$V_B = -\frac{C\gamma}{A(1+\gamma)}, \tag{A6}$$

while the optimal equity market value is available as an explicit function of the asset value:

$$S = AV - AV_B\left(\frac{V}{V_B}\right)^{-\gamma} + C\left[1 - \left(\frac{V}{V_B}\right)^{-\gamma}\right], \tag{A7}$$

where:

$$A := \frac{\delta}{r-\mu_V}, \quad C := (\theta-1)\frac{c}{r}, \quad \gamma := \frac{\mu_V - \sigma_V^2/2 + \sqrt{\left(\mu_V - \sigma_V^2/2\right)^2 + 2r\sigma_V^2}}{\sigma_V^2}.$$

To prove *(a)*, simple calculations show that when V goes to ∞, we have $S \sim AV$, $S_V \sim A$, and $S_{VV} \sim 1/V^{\gamma+2}$ so that:

$$\frac{SS_{VV}}{S_V^2} \sim \frac{1}{AV^{\gamma+1}} \longrightarrow 0. \tag{A8}$$

To prove *(b)*, we notice that when V tends to V_B, there is an indeterminate form $\frac{0}{0}$. Applying l'Hôpital's rule yields:

$$\lim_{V\to V_B} \varepsilon_\ell = \lim_{V\to V_B} \frac{S}{S_V^2} \cdot \lim_{V\to V_B} S_{VV} = \lim_{V\to V_B} \frac{1}{2S_{VV}} \cdot \lim_{V\to V_B} S_{VV} = \frac{1}{2}. \tag{A9}$$

Table 7 reports numerical simulations for ε_ℓ computed with the numerical assumptions from Duffie and Lando (2001). In accordance with the previous asymptotic results (A8) and (A9), ε_ℓ is always less than half the debt-to-asset ratio. It takes low asset values and deep states of financial distress to produce values of the same magnitude as the firm's leverage.

Table 7. Numerical magnitude of the financial leverage adjustment ε_ℓ

This table reports the numerical magnitude of ε_ℓ as a function of the firm's asset value V for five different bankruptcy boundaries corresponding to five different levels of asset volatility. The corresponding debt-to-asset ratio ℓ is also reported. Pricing assumptions are those of Duffie and Lando (2001): annual debt interest charge $c = 8.00$, corporate tax rate $\theta = 0.35$, bankruptcy costs 0.3, payout rate $\delta = 0.05$, risk-free rate $r = 0.06$ per annum, expected asset growth rate $\mu_V - \sigma_V^2/2 \equiv 0.01$ per annum.

	Bankruptcy boundary									
	$\sigma_V = 0.25$ $V_B = 45.34$		$\sigma_V = 0.20$ $V_B = 49.30$		$\sigma_V = 0.15$ $V_B = 54.88$		$\sigma_V = 0.10$ $V_B = 63.37$		$\sigma_V = 0.05$ $V_B = 78.01$	
V	ε_ℓ	ℓ	ε_ℓ	ℓ	ε_ℓ	ℓ	ε_ℓ	ℓ	ε_ℓ	ℓ
V_B	0.50	1.00	0.50	1.00	0.50	1.00	0.50	1.00	0.50	1.00
50	0.44	0.99	0.49	1.00	-	-	-	-	-	-
60	0.35	0.93	0.38	0.96	0.43	0.99	-	-	-	-
80	0.22	0.82	0.23	0.84	0.24	0.88	0.27	0.92	0.44	1.00
100	0.15	0.73	0.15	0.75	0.14	0.77	0.13	0.82	0.09	0.89
120	0.11	0.65	0.10	0.67	0.09	0.69	0.06	0.72	0.01	0.78
140	0.08	0.59	0.07	0.60	0.05	0.62	0.03	0.65	0.00	0.70
160	0.06	0.53	0.05	0.55	0.04	0.56	0.02	0.59	0.00	0.63
180	0.05	0.49	0.04	0.50	0.03	0.51	0.01	0.53	0.00	0.58
200	0.04	0.45	0.03	0.46	0.02	0.47	0.01	0.49	0.00	0.53

Appendix B. Proof of Lemma 2

In the sequel, I simplify the argument of Hagan and Woodward (1999) to prove Equation (5) by singular perturbation theory. For the sake of notational simplicity, I will assume zero interest rates. The stock pays no dividends, which implies a zero drift under the risk-neutral probability measure associated with the money market account. The stock price diffuses according to the dynamics:

$$\mathrm{d}S_t = \alpha(t)\sigma(S_t)S_t\mathrm{d}W_t. \tag{B1}$$

The undiscounted risk-neutral value $C(S,t) = \mathbb{E}\{(S_T - K)^+|S\}$ of a European-style call option with strike K and time to maturity T evolves according to the Black-Scholes-Merton partial differential equation (PDE):

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2(S)S^2\frac{\partial^2 C}{\partial S^2} = 0, \tag{B2}$$

subject to appropriate boundary and terminal conditions.

- *Re-scaling procedure.* Denoting $f : x \longmapsto x\sigma(x)$ and $\varepsilon := f(K) \ll 1$, I introduce the following change of variables:

$$\tau := \int_t^T \alpha^2(u)\mathrm{d}u, \qquad x := \frac{S-K}{\varepsilon}, \tag{B3}$$

in order to re-scale the call value as $\widetilde{C}(\tau, x) := C(t,S)/\varepsilon$. The new PDE in the variables (τ, x) verified by the re-scaled call value is as follows:

$$-\frac{\partial \widetilde{C}}{\partial \tau} + \frac{1}{2}\frac{f^2(K+\varepsilon x)}{f^2(K)}\frac{\partial^2 \widetilde{C}}{\partial x^2} = 0. \tag{B4}$$

Expanding in power series of ε, we note that:

$$f^2(K+x\varepsilon) = f^2(K)\left(1 + 2\frac{f'(K)}{f(K)}x\varepsilon\right) + O(\varepsilon^2). \tag{B5}$$

Substituting in Equation (B4), the PDE can now be written at first order in ε:

$$\frac{\partial \widetilde{C}}{\partial \tau} - \frac{1}{2}\frac{\partial^2 \widetilde{C}}{\partial x^2} = \nu x \varepsilon \frac{\partial^2 \widetilde{C}}{\partial x^2} + O(\varepsilon^2), \tag{B6}$$

where $\nu := f'(K)/f(K)$. Expanding the re-scaled price $\widetilde{C}$ in power series of ε as $\widetilde{C}^0 + \varepsilon \widetilde{C}^1 + O(\varepsilon^2)$, we are led to solve the following system of PDEs at first order in ε:

$$\begin{cases} \dfrac{\partial \widetilde{C}^0}{\partial \tau} - \dfrac{1}{2}\dfrac{\partial^2 \widetilde{C}^0}{\partial x^2} = 0, & \widetilde{C}^0(0,x) = x^+, \\ \dfrac{\partial \widetilde{C}^1}{\partial \tau} - \dfrac{1}{2}\dfrac{\partial^2 \widetilde{C}^1}{\partial x^2} = \nu x \dfrac{\partial^2 \widetilde{C}^0}{\partial x^2}, & \widetilde{C}^1(0,x) = 0. \end{cases} \tag{B7}$$

- *Solving the re-scaled problem.* Standard techniques apply to solve the first heat-like PDE. The solution for $\widetilde{C}^0$ is given by:

$$\widetilde{C}^0(\tau, x) = xN\left(\frac{x}{\sqrt{\tau}}\right) + \sqrt{\frac{\tau}{2\pi}}e^{-x^2/2\tau}, \tag{B8}$$

as it can be checked by means of the following elementary calculations:

$$\frac{\partial \widetilde{C}^0}{\partial x} = N\left(\frac{x}{\sqrt{\tau}}\right), \qquad \frac{\partial^2 \widetilde{C}^0}{\partial x^2} = \frac{e^{-x^2/2\tau}}{\sqrt{2\pi\tau}}, \qquad \frac{\partial \widetilde{C}^0}{\partial \tau} = \frac{e^{-x^2/2\tau}}{2\sqrt{2\pi\tau}}. \tag{B9}$$

In the same way, the solution for $\widetilde{C}^1$ is given by:

$$\widetilde{C}^1(\tau,x) = \nu x \tau \frac{e^{-x^2/2\tau}}{2\sqrt{2\pi\tau}}, \tag{B10}$$

as it can be checked by means of the following elementary calculations:

$$\frac{\partial \widetilde{C}^1}{\partial \tau} = \frac{(\nu x \tau + \nu x^3)e^{-x^2/2\tau}}{4\tau\sqrt{2\pi\tau}}, \qquad \frac{\partial^2 \widetilde{C}^1}{\partial x^2} = \frac{(-3\nu x \tau + \nu x^3)e^{-x^2/2\tau}}{2\tau\sqrt{2\pi\tau}}. \tag{B11}$$

Moreover, we notice that:

$$\widetilde{C}^1(\tau,x) = \tau \nu x \frac{\partial \widetilde{C}^0}{\partial \tau}. \tag{B12}$$

Substituting Equation (B12) in the re-scaled price expansion of $\widetilde{C}$, we obtain the solution for the re-scaled price at first order in ε:

$$\widetilde{C}(\tau,x) = \widetilde{C}^0(\tau,x) + \varepsilon \tau \nu x \frac{\partial \widetilde{C}^0}{\partial \tau} + O(\varepsilon^2) = \widetilde{C}^0\left(\tau + \varepsilon \tau \nu x + O(\varepsilon^2), x\right). \tag{B13}$$

- *Solving for the option price in the physical space.* The unscaled call price may then be deduced as follows:

$$C(t,S) = \varepsilon \widetilde{C}(\tau,x) = \varepsilon \widetilde{C}^0(\tau(1+\varepsilon\nu x) + O(\varepsilon^2), x) = \widetilde{C}^0\left(\varepsilon^2\tau(1+\varepsilon\nu x) + O(\varepsilon^4), \varepsilon x\right). \tag{B14}$$

Noting that $\varepsilon x = S - K$, we obtain the option price with respect to physical variables:

$$C(t,S) \approx \widetilde{C}^0(\tau^*, S-K), \tag{B15}$$

where $\tau^* \simeq \varepsilon^2 \tau\left(1 + \nu(S-K)\right)$. We also note that $\varepsilon = f(K)$ may be developed around the midpoint $(K+S)/2$ for spot prices close to the call strike K:

$$f^2(K) = f^2\left(\frac{K+S}{2}\right)\left(1+\frac{f'\left(\frac{K+S}{2}\right)}{f\left(\frac{K+S}{2}\right)}(K-S)\right)+o(K-S), \tag{B16}$$

which gives at leading order:

$$\tau^* = \tau f^2\left(\frac{K+S}{2}\right)+O(K-S). \tag{B17}$$

- *The Black-Scholes-Merton case.* The preceding whole line of reasoning may be applied to the pure Black-Scholes model, which means performing the same calculations for the following stock price dynamics:

$$\mathrm{d}S_t = \widehat{\sigma}_{K} S_t \mathrm{d}W_t, \tag{B18}$$

where $\widehat{\sigma}_K$ is the constant Black-Scholes implied volatility at strike K and expiry T. In this specific case we note that f is the identity function while $\tau = \widehat{\sigma}_K^2(T-t)$, $\nu = 1/K$ and $\varepsilon = K$. Applying Equation (B15) with the previous parameters, the Black-Scholes price is then given by $\widetilde{C}^0(\tau^*_{BS}, S-K)$ where we have at leading order:

$$\tau^*_{BS} \simeq \widehat{\sigma}_K^2(T-t)\left(\frac{K+S}{2}\right)^2+O(K-S). \tag{B19}$$

- *Linking local volatility with implied volatility.* As the option price observed in the market is both given by the local volatility model (B15) and the Black-Scholes model, we can write:

$$\widetilde{C}^0(\tau^*, S-K) = \widetilde{C}^0(\tau^*_{BS}, S-K). \tag{B20}$$

As $\widetilde{C}^0$ is strictly increasing in the re-scaled time to maturity variable τ, we thus obtain $\tau^* = \tau^*_{BS}$. Substituting Equations (B17) and (B19) in this last relationship, we get at leading order the following relationship which is valid for stock prices in the vicinity of the strike price:

$$\widehat{\sigma}_{K}^{2}(T-t) \simeq \sigma^{2}\left(\frac{K+S}{2}\right)\int_{t}^{T}\alpha^{2}(u)\mathrm{d}u. \tag{B21}$$

This is Equation (5).

Appendix C. Proof of Proposition 1

By definition of the variance-equity elasticity:

$$e_{v} = \frac{\mathrm{d}\sigma^{2}/\sigma^{2}}{\mathrm{d}S/S} = \frac{2}{\sigma}\cdot\frac{\mathrm{d}\sigma}{\mathrm{d}\ln S}. \tag{C1}$$

But Lemma 2 ensures that the slope of the local volatility $\sigma(\cdot)$ at S is twice the slope of the implied volatility $\widehat{\sigma}_{T}(\cdot)$ at K, ignoring the factor $\overline{\alpha}_{T}$. As a consequence, we have:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\ln S} = \frac{2}{\overline{\alpha}_{T}}\cdot\frac{\mathrm{d}\widehat{\sigma}}{\mathrm{d}\ln K}, \tag{C2}$$

which yields after substitution of the local volatility slope in (C1):

$$e_{v} = \frac{4}{\overline{\alpha}_{T}\sigma}\cdot\frac{\mathrm{d}\widehat{\sigma}}{\mathrm{d}\ln K}. \tag{C3}$$

Substituting the structural formulation (7) of the implied volatility skew into (C3) yields:

$$e_{v} = \frac{4}{\overline{\alpha}_{T}\sigma}\left(-\frac{\sigma}{2}\left(\ell-\varepsilon_{\ell}\right)\overline{\alpha}_{T}\right) = -2\left(\ell-\varepsilon_{\ell}\right), \tag{C4}$$

which is Equation (8).

Appendix D. Proof of Lemma 3

A natural option structure matching the moments of a default swap instrument is the *credit risk reversal* (Ilinski, 2003). This optional structure combines long out-of-the-money put options with short at-the-money call options. The out-of-the-money put option is intended to replicate the default swap payoff on the occurrence of a credit event, that is, upon a jump to zero of the stock price. Simultaneously, the at-the-money call option is intended to provide exposure to the third moment of the implied volatility surface. It turns out that a specific choice for the geometry of the credit risk reversal structure offers no entry cost and as little convexity as possible between the two option exercise prices.[13] This last feature ensures an approximate static replication of the default swap instrument.

To match the first two moments of a *binary* default swap,[14] let us show that once the put strike, K_p, has been chosen arbitrarily, the call strike, K_c, and the put (resp. call) quantity n_p (resp. n_c) should be chosen as follows:

$$\begin{cases} K_c = F_T^2/K_p, \\ n_p = F_T/K_p, \\ n_c = -1. \end{cases} \tag{D1}$$

Hedging the structure with forward contracts, we can assume no dividends, no carrying costs as well as no implied volatility skew for the sake of simplicity. Let define $P := n_p p - c$ as the upfront premium for the credit risk reversal, where p (resp. c) is the put (resp. call) theoretical price. The usual Black-Scholes formulae can be used to calculate this upfront cost:

$$\begin{aligned} P &= [n_p K_p N(d_1) - n_p F_T N(d_2) - F_T N(d_1) - K_c N(d_2)]e^{-rT} \\ &= [(n_p K_p - F_T)N(d_1) - (n_p F_T - K_c)N(d_2)]e^{-rT} \\ &= 0, \end{aligned} \tag{D2}$$

[13]It is still possible to use more complex structures, such as combinations of risk reversals or put spreads, to match the higher-order sensitivities of the default swap instrument more closely.

[14]A binary default swap instrument is an instrument making a single payment of 1$ in case of a default event.

where $d_1 = \ln(F_T/K_c)/(\sigma\sqrt{T}) + \sigma\sqrt{T}/2$ and $d_2 = d_1 - \sigma\sqrt{T}$. Similarly, the convexity γ of the credit risk reversal is zero since:[15]

$$\frac{\gamma_c}{\gamma_p} = \frac{N'(d_1)}{N'(d_2)} = \exp\left(-\frac{\ln(F_T/K_c) + \ln(K_p/F_T)}{2}\right) = \exp\left(\frac{2\ln n_p}{2}\right) = n_p. \tag{D3}$$

Let us now consider the expected payoff of the delta-hedged credit risk reversal upon a default event, denoted L. This expected loss appears to be tightly constrained by the credit risk reversal geometry. Indeed, in case of a jump to zero of the stock price, the delta-hedged credit risk reversal pays off the put notional $n_p K_p$ minus its initial delta δF_T in cash:

$$L = F_T - \delta F_T = (1 - n_p \delta_p + \delta_c) F_T, \tag{D4}$$

where δ_p (resp. δ_c) is the initial hedge ratio of the put (resp. call). Denoting δ_p^0 (resp. δ_c^0) the delta of the put (resp. call) option struck at F_T, the call-put parity yields $\delta_c^0 - \delta_p^0 = 1$. With a strike K_p sufficiently close to F_T, we have $n_p \delta_c^0 - \delta_c^0 \simeq 1$. Substituting into Equation (D5) yields:

$$L \simeq [n_p(\delta_p^0 - \delta_p) - (\delta_c^0 - \delta_c)] F_T. \tag{D5}$$

Recall now that $\partial\delta/\partial K = -\gamma F_T/K$ then gives the sensitivity of the delta in the Black-Scholes model. Applying this general result for K_p and K_c sufficiently close to F_T, a first-order Taylor expansion yields:

$$\begin{aligned} \delta_{0,p} - \delta_p &\approx -F_T \overline{\gamma}_p \ln(F_T/K_p), \\ \delta_{0,c} - \delta_c &\approx -F_T \overline{\gamma}_c \ln(F_T/K_c), \end{aligned} \tag{D6}$$

where $\overline{\gamma}_p$ (resp. $\overline{\gamma}_c$) is the average convexity between F_T and K_p (resp. K_c). Substituting into Equation (D5), the expected payoff upon default turns out to depend explicitly on the log-distance

[15] The Black-Scholes convexity is $\gamma_p = \gamma_c = N'(d_1)/(S_0\sigma\sqrt{T})$, where $N'(x) = \exp(-x^2/2)/\sqrt{2\pi}$.

between the strikes:

$$L \simeq \overline{\gamma} F_T^2 \ln\left(K_c/K_p\right), \tag{D7}$$

where $\overline{\gamma} = n_p \overline{\gamma}_p = \overline{\gamma}_c$ is the average convexity between the strikes.

In the presence of an implied volatility skew $\widehat{\sigma}_p > \widehat{\sigma}_{ATM} > \widehat{\sigma}_c$, the upfront premium of the credit risk reversal has to be locally adjusted for the put (resp. call) implied volatility $\widehat{\sigma}_p$ (resp. $\widehat{\sigma}_c$). At first order, the adjustment cost to the premium is:

$$(\widehat{\sigma}_p - \widehat{\sigma}_{ATM}) \times n_p \nu_p - (\widehat{\sigma}_{ATM} - \widehat{\sigma}_c) \times \nu_c, \tag{D8}$$

where ν_p (resp. ν_c) is the put (resp. call) sensitivity to volatility[16] calculated at $\widehat{\sigma}_{ATM}$. Using the fact that $n_p \nu_p = n_p \widehat{\sigma}_{ATM} \gamma_p F_T^2 T = \widehat{\sigma}_{ATM} \gamma_c F_T^2 T = \nu_c$, the upfront premium becomes:

$$P \simeq (\widehat{\sigma}_p - \widehat{\sigma}_c) \widehat{\sigma}_{ATM} \overline{\gamma} F_T^2 T. \tag{D9}$$

Finally, the fair spread s_T of a binary default swap instrument of maturity T may be assimilated to the annualized premium to be paid for protection, P/T, against the expected payoff upon default, L. Dividing Equations (D7) and (D9), the fair spread is given by:

$$s_T \simeq \widehat{\sigma}_{ATM} \cdot \frac{\widehat{\sigma}_p - \widehat{\sigma}_c}{\ln(K_c/K_p)}. \tag{D10}$$

Substituting Equation (D10) into the standard credit identity $s_T = \lambda_T \times (1 - \mathrm{R})$, where R is the expected recovery rate on debt, yields Equation (9).

Appendix E. Proof of Proposition 2

Letting K converge to S in Equation (5) shows that the implied volatility and the local volatility coincide at the money. Inserting $\widehat{\sigma}_{ATM} = \overline{\alpha}_T \sigma$ and substituting Equation (7) for $|\Sigma_T|$ into Equa-

[16]The Black-Scholes sensitivity to the implied volatility $\widehat{\sigma}$ is given by $\widehat{\sigma} \gamma F_T^2 T$.

tion (9) yields:

$$\lambda_T = \frac{k\overline{\alpha}_T^2\,(\ell - \varepsilon_\ell)}{2}\sigma^2, \tag{E1}$$

which leads to:

$$\frac{\mathrm{d}\lambda_T}{\lambda_T} = \frac{\mathrm{d}\sigma^2}{\sigma^2}. \tag{E2}$$

An immediate consequence in terms of elasticity is $e_\lambda = e_v$.

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