Credit and Option Risk Premia

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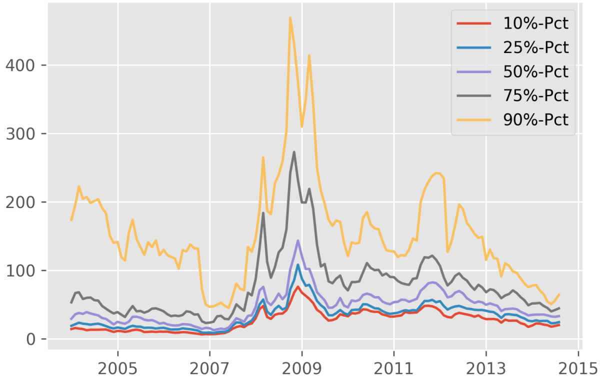
Motivation

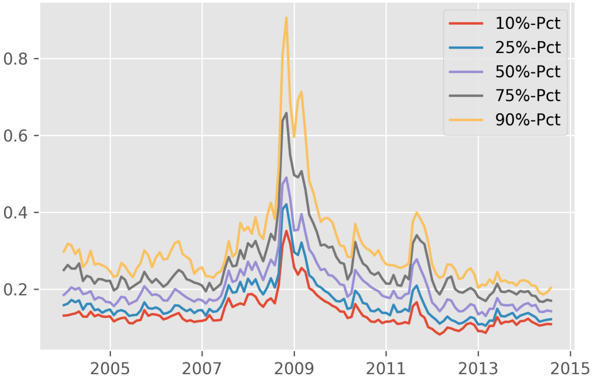
- Credit spread puzzle
  - Firms have low leverage and low actual default probabilities.
  - But credit spreads are large.
- Bankruptcy cost puzzle
  - Andrade and Kaplan (1998) estimate distress costs of 10-23% of firm value.
  - Glover (2016) estimates distress costs of 45% of firm value.
  - Chen (2010) estimates time varying distress costs.
- CDS rate $=$ Probability of default $\times$ Loss given default

CDS Rates

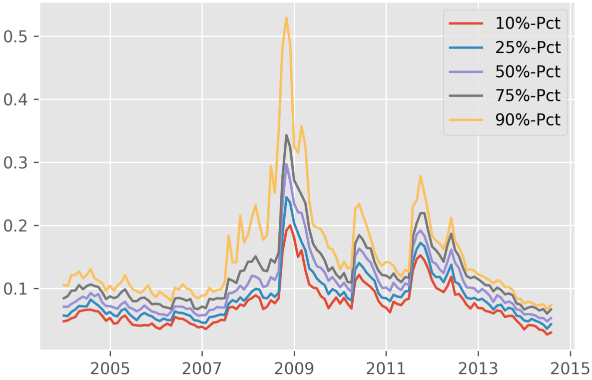


10%-Pct
25%-Pct
50%-Pct
75%-Pct
90%-Pct
400
300
200
100
0
2005
2007
2009
2011
2013
2015

Implied Volatility



Implied Volatility Skew



Contribution

- Solve a structural model of credit risk
  - Epstein-Zin pricing kernel with Markov switching fundamentals
  - Price debt and equity
  - Price CDS and option contracts
- New generalized solution approach
- Estimate time variation in bankruptcy costs at the firm-level
- Use joint information of CDS rates and implied volatilities
- IV moments are informative about the composition of risk

Literature

- Reduced-form credit risk models: Duffie, Singelton (1999); Berndt, et al. (2008)
- Structural credit risk models: Hackbarth, Miao, and Morellec (2006); Chen, Collin-Dufresne, Goldstein (2009); Bhamra, Kuehn, Strebulaev (2010), Chen (2010)
- Structural estimation: Hennessy, Whited (2007), Glover (2016)
- Credit and option pricing: Carr, Wu (2009, 2011); Collin-Dufresne, Goldstein, Yang (2012); Seo, Wachter (2016); Culp, Nozawa, Veronesi (2017); Kelly, Manzo, Palhares (2016); Reindl, Stoughton, Zechner (2016)
- Consumption-based option pricing: Drechsler, Yaron (2010); Backus, Chernov, Martin (2011); Schreindorfer (2014); Seo, Wachter (2015)
- Asset pricing with disaster risk: Barro (2006); Gabaix (2012); Gourio (2012); Wachter (2013)

Model

- Exogenous pricing kernel
- Firms issue perpetual debt and choose optimal leverage
- Firms can raise equity and issue more debt
- Firms can default

Pricing Kernel

- Log aggregate consumption growth $g_{c,t+1}$ follows

$$g_{c,t+1} = \mu_{c,t} + \sigma_{c,t}\varepsilon_{c,t+1}$$

- Drift and volatility of consumption growth depend on the aggregate Markov state ξ_t.

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- Epstein-Zin pricing kernel is

$$M_{t,t+1} = \beta^{\theta} \left(\frac{\lambda_{t+1}^{c} + 1}{\lambda_{t}^{c}} \right)^{-(1-\theta)} e^{-\gamma g_{c,t+1}}$$

- λ_t^c is the wealth-consumption ratio.

Unlevered Firm Value

- Log earnings growth $g_{i,t+1}$ follows

$$g_{i,t+1} = \mu_t + \sigma_t \varepsilon_{t+1} + \zeta \nu_{i,t+1}$$

- Drift and volatility of earnings growth depend on the aggregate Markov state ξ_t.
- Systematic ε_{t+1} and idiosyncratic $\nu_{i,t+1}$ Gaussian shocks.

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- Drift and volatility of earnings growth depend on the aggregate Markov state ξ_t.
- Systematic ε_{t+1} and idiosyncratic $\nu_{i,t+1}$ Gaussian shocks.
- Corporate income tax rate is η.
- After-tax asset value is

$$A_{i,t} = (1-\eta)E_{i,t} + \mathbb{E}_t[M_{t,t+1}A_{i,t+1}] \qquad E_{i,t+1} = e^{g_{i,t+1}}E_{i,t}$$

Debt Value

- Firms can issue perpetual debt to take advantage of the tax benefits of debt financing.
- The interest coverage ratio is defined as

$$\kappa_{i,t} = \frac{E_{i,t}}{c_{i,s}}$$

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- The debt value is given by

$$\begin{aligned} D_{i,t} &= 1_{\{\kappa_{i,t} \leq \kappa_t^D\}}(1-\omega_t)A_{i,t} \\ &+ 1_{\{\kappa_t^D < \kappa_{i,t} < \kappa_t^I\}}\big(c_{i,s} + \mathbb{E}_t[M_{t,t+1}D_{i,t+1}]\big) \\ &+ 1_{\{\kappa_t^I \leq \kappa_{i,t}\}}\left(c_{i,s} + \frac{c_{i,s}}{c_{i,t}}\mathbb{E}_t[M_{t,t+1}D_{i,t+1}]\right) \end{aligned}$$

- Bbankruptcy costs vary with the aggregate economy

$$\omega_t = \frac{\bar{\omega}}{1 + e^{a + b\mu_{c,t}/\sigma_{c,t}}}$$

Equity Value

- Equity holders decide about the optimal timing of default by maximizing the equity value

$$
\begin{aligned}
S_{i,t} &= \max\Big\{0, 1_{\{\kappa_{i,t}<\kappa_t^I\}}\Big((1-\eta)(E_{i,t}-c_{i,s}) \\
&+ \psi_e(E_{i,t}-c_{i,s})1_{\{E_{i,t}<c_{i,s}\}} + \mathbb{E}_t[M_{t,t+1}S_{i,t+1}]\Big) \\
&+ 1_{\{\kappa_t^I\leq\kappa_{i,t}\}}\Big((1-\eta)(E_{i,t}-c_{i,s}) + \Delta_{i,t} + \mathbb{E}_t[M_{t,t+1}S_{i,t+1}]\Big)\Big\}
\end{aligned}
$$

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\end{aligned}
$$

- Debt issuances proceeds $\Delta_{i,t}$ are net of debt issuance costs ψ_d.

- Firms face equity issuance costs ψ_e.

- The optimal state depend default threshold satisfies

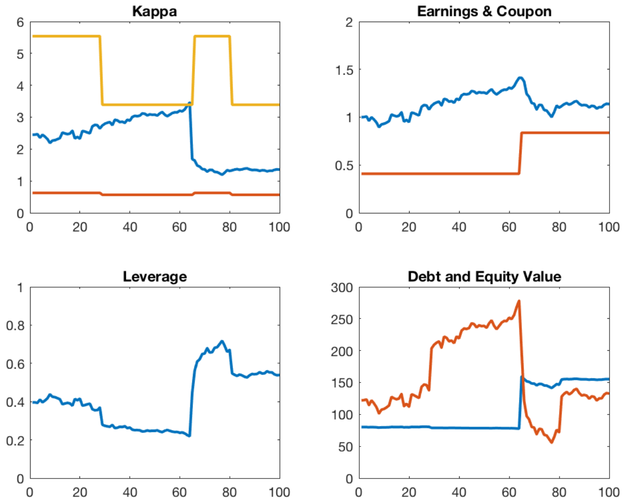
$$\kappa^D(\xi_t) = \max\{\kappa_{i,t} : S(\kappa_{i,t}, \xi_t) \leq 0\}$$

Levered Firm Value

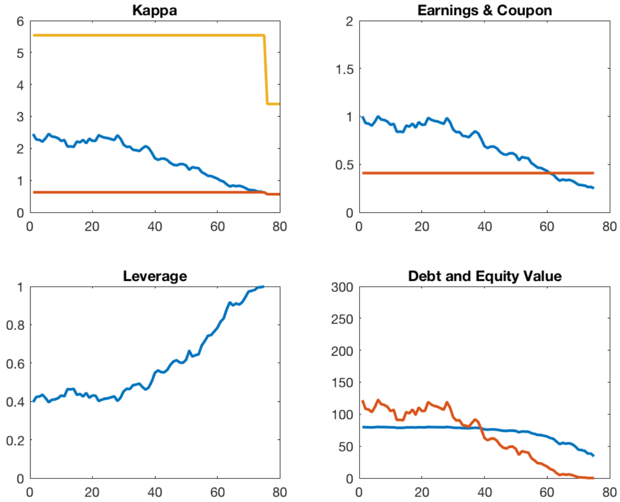
- Levered firm value is the sum of the value of debt and equity.
- Management chooses the optimal issuance threshold κ_t^I and the optimal coverage ratio $\bar{\kappa}_t$ to maximize levered firm value

$$
\begin{aligned}
F_{i,t} &= 1_{\{\kappa_{i,t} \leq \kappa_t^D\}}(1-\omega_t)A_{i,t} \\
&+ 1_{\{\kappa_t^D < \kappa_{i,t} < \kappa_t^I\}}\Big((1-\eta)E_{i,t} + \eta c_{i,s} \\
&+ \psi_e(E_{i,t} - c_{i,s})1_{\{E_{i,t} < c_{i,s}\}} + \mathbb{E}_t\big[M_{t,t+1}F_{i,t+1}\big]\Big) \\
&+ 1_{\{\kappa_t^I \leq \kappa_{i,t}\}}\Big((1-\eta)E_{i,t} + \eta c_{i,s} - \psi_d D_{i,t}^{ex} + \mathbb{E}_t\big[M_{t,t+1}F_{i,t+1}\big]\Big)
\end{aligned}
$$

Simulation: Debt Issuance



Simulation: Default



Credit Default Swaps

- Firm i defaults at time τ_i when its interest coverage ratio $\kappa_{i,t}$ drops below the default threshold κ_t^D such that

$$\tau_i = \inf\{t : \kappa_{i,t} \leq \kappa_t^D\}$$

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- PV of issuance seller cash-flow

$$\sum_{h=1}^{T} \mathbb{E}_t \left[M_{t,t+h} 1_{\{\tau_i = t+h\}} x_{i,t+h,s} \right] \qquad x_{i,t+h,s} = 1 - \frac{(1-\omega_{t+h}) A_{i,t+h}}{D_{i,s}}$$

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- PV of issuance buyer cash-flow

$$z_{i,s,t}^{T} \sum_{h=1}^{T} \mathbb{E}_t \left[M_{t,t+h} (1 - 1_{\{\tau_i \leq t+h\}}) \right]$$

Credit Default Swaps

- The log one-period CDS rates can be approximated by

$$\ln(z^1_{i,s,t}) \approx \ln(q^1_{i,t}) + \ln(L^{\mathbb{Q}}_{i,t,s})$$

where

$$q^1_{i,t} = \mathbb{E}^{\mathbb{Q}}_t \left[1_{\{\tau_i = t+1\}}\right] \qquad L^{\mathbb{Q}}_{i,t,s} = \mathbb{E}^{\mathbb{Q}}_t[x_{i,t+1,s} | \tau_i = t+1]$$

is the risk-neutral one-period default probability and the risk-neutral loss rate given default.

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is the risk-neutral one-period default probability and the risk-neutral loss rate given default.

- The variance of the log-linearized one-period CDS rate is

$$\mathrm{Var}(\ln z^1_{i,s,t}) = \mathrm{Var}(\ln q^1_{i,t}) + \mathrm{Var}(\ln L^{\mathbb{Q}}_{i,t,s}) + 2\mathrm{Cov}(\ln q^1_{i,t}, \ln L^{\mathbb{Q}}_{i,t,s})$$

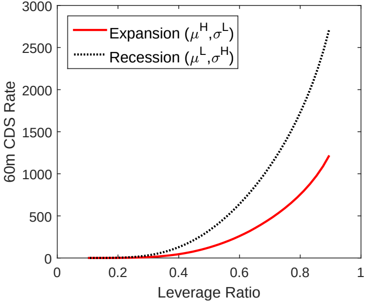
Equity Option Pricing

- The value of a European put option with maturity T and strike price X is given by

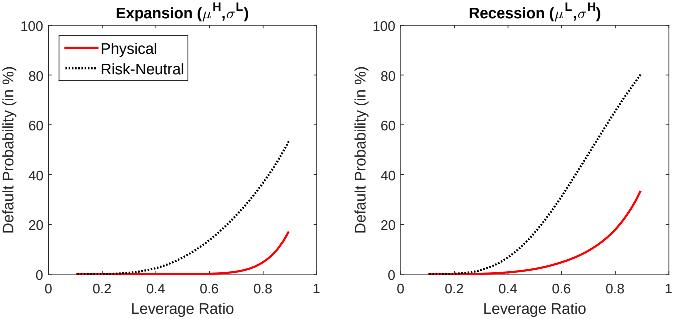
$$P_{i,t} = \mathbb{E}_t[M_{t,T} \max\{X - S_{i,T}, 0\}]$$

- Using the Black-Scholes model, we solve for implied volatilities.
- Option prices are not sensitive to loss rates because equity holders recover nothing in the case of default.
- Equity options are compound options.

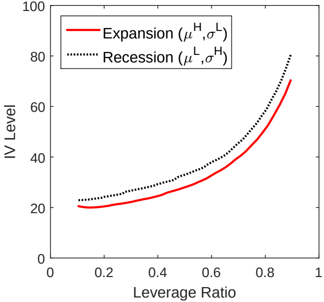
CDS Rates


3000
2500
2000
1500
1000
500
0
60m CDS Rate
0
0.2
0.4
0.6
0.8
1
Leverage Ratio
Expansion (μ^H,σ^L)
Recession (μ^L,σ^H)

Default Probabilities



Option Moments



Data

- Credit Market Analysis (CMA)
  - Monthly data from 2004 to 2014
  - S&P 100 constituents
  - 5-year tenor, senior debt, dollar denominated, XR or MR
- OptionMetrics
  - Monthly data from 2004 to 2014
  - S&P 100 constituents
  - IV surface adjusted for early exercise
- CRSP-Compustat
  - Debt: DLCQ + DLTTQ
  - Earnings: OIBDPQ
  - Monthly returns and market capitilization
- BEA NIPA
  - Monthly real non-durable and service consumption growth

Consumption Dynamics

Consumption States				
	$\mu_{c,h}$	$\mu_{c,l}$	$\mu_{c,d}$	
	0.2935	0.0932	-0.6180	
	$\sigma_{c,l}$	$\sigma_{c,h}$	$\sigma_{c,d}$	
	0.1855	0.4211	0.8422	

Transition Matrix				
(μ_h, σ_l)	(μ_l, σ_l)	(μ_h, σ_h)	(μ_l, σ_h)	(μ_d, σ_d)
0.9912	0.0029	0.0059	0.0000	0
0.0223	0.9718	0.0001	0.0058	0
0.0061	0.0000	0.9910	0.0029	0
0.0001	0.0060	0.0223	0.9567	0.0149
0	0	0	0.0225	0.9775

Calibrated Parameters

EIS	ψ	2
Time discount rate	β	0.996
Consumption-earnings correlation	ρ	0.1
Drift scaling	ϕ_μ	2
Bankruptcy costs maximum	$\bar{\omega}$	0.6
Debt issuance costs	ψ_d	0.005
Equity issuance costs	ψ_e	0.1

Estimated Parameters

		Model 1	Model 2
Risk aversion	γ	8.97	9.39
Aggregate volatility scaling	ϕ_σ	12.65	6.74
Idiosyncratic volatility	ζ	0.05	0.07
Tax rate	τ	0.22	0.22
Bankruptcy cost level	a	-4.84	-5.91
Bankruptcy cost cyclicality	b	0.83	6.33

SMM Moments

	Data	Model 1	Model 2
Average leverage	25.46		25.38
Average excess returns	0.47		0.78
Average 1-year CDS	0.44		0.26
Average 5-year CDS	0.80		0.84
Average ATM-IV	26.92		
Average IV Skew	4.32		
S.D. of leverage	2.16		2.58
S.D. of returns	4.68		3.40
S.D. of 1-year CDS	0.59		0.43
S.D. of 5-year CDS	0.50		0.53
S.D. of ATM-IV	10.51		
S.D. of IV Skew	2.15		

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Average leverage	25.46		25.38
Average excess returns	0.47		0.78
Average 1-year CDS	0.44		0.26
Average 5-year CDS	0.80		0.84
Average ATM-IV	26.92		37.41
Average IV Skew	4.32		5.48
S.D. of leverage	2.16		2.58
S.D. of returns	4.68		3.40
S.D. of 1-year CDS	0.59		0.43
S.D. of 5-year CDS	0.50		0.53
S.D. of ATM-IV	10.51		2.78
S.D. of IV Skew	2.15		1.63

SMM Moments

	Data	Model 1	Model 2
Average leverage	25.46	25.45	25.38
Average excess returns	0.47	0.65	0.78
Average 1-year CDS	0.44	0.15	0.26
Average 5-year CDS	0.80	0.72	0.84
Average ATM-IV	26.92	32.08	37.41
Average IV Skew	4.32	4.26	5.48
S.D. of leverage	2.16	2.31	2.58
S.D. of returns	4.68	2.79	3.40
S.D. of 1-year CDS	0.59	0.32	0.43
S.D. of 5-year CDS	0.50	0.52	0.53
S.D. of ATM-IV	10.51	5.23	2.78
S.D. of IV Skew	2.15	1.47	1.63

CDS Decomposition

	Model 1	Model 2
Average bankruptcy costs	58.83	29.58
S.D. of bankruptcy costs	0.49	25.23
Average LGD under $\mathbb{P}$	95.92	97.76
Average LGD under $\mathbb{Q}$	95.97	97.79
S.D. of LGD under $\mathbb{P}$	0.92	0.33
S.D. of LGD under $\mathbb{Q}$	0.96	0.31
Average 5-year def. probability under $\mathbb{P}$	0.54	0.75
Average 5-year def. probability under $\mathbb{Q}$	3.60	3.73
S.D. of 5-year def. probability under $\mathbb{P}$	0.63	0.62
S.D. of 5-year def. probability under $\mathbb{Q}$	2.16	1.82

Conclusion

- Solve a structural model of credit risk
  - Epstein-Zin pricing kernel with Markov switching fundamentals
  - Price debt and equity
  - Price CDS and option contracts
- Estimate time variation in bankruptcy costs
- Use joint information of CDS rates and implied volatilities
- IV moments are informative about the composition of risk