Credit and Option Risk Premia

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Motivation

- **Credit spread puzzle**
  - Firms have low leverage and low actual default probabilities.
  - But credit spreads are large.

- **Bankruptcy cost puzzle**
  - Andrade and Kaplan (1998) estimate distress costs of 10-23% of firm value.
  - Glover (2016) estimates distress costs of 45% of firm value.
  - Chen (2010) estimates time varying distress costs.

- CDS rate = Probability of default × Loss given default
CDS Rates
Implied Volatility
Implied Volatility Skew
Contribution

- Solve a structural model of credit risk
  - Epstein-Zin pricing kernel with Markov switching fundamentals
  - Price debt and equity
  - Price CDS and option contracts

- New generalized solution approach

- Estimate time variation in bankruptcy costs at the firm-level

- Use joint information of CDS rates and implied volatilities

- IV moments are informative about the composition of risk
Literature

- Reduced-form credit risk models: Duffie, Singelton (1999); Berndt, et al. (2008)

- Structural credit risk models: Hackbarth, Miao, and Morellec (2006); Chen, Collin-Dufresne, Goldstein (2009); Bhamra, Kuehn, Strebulaev (2010), Chen (2010)


- Credit and option pricing: Carr, Wu (2009, 2011); Collin-Dufresne, Goldstein, Yang (2012); Seo, Wachter (2016); Culp, Nozawa, Veronesi (2017); Kelly, Manzo, Palhares (2016); Reindl, Stoughton, Zechner (2016)

- Consumption-based option pricing: Drechsler, Yaron (2010); Backus, Chernov, Martin (2011); Schreindorfer (2014); Seo, Wachter (2015)

- Asset pricing with disaster risk: Barro (2006); Gabaix (2012); Gourio (2012); Wachter (2013)
Model

- Exogenous pricing kernel
- Firms issue perpetual debt and choose optimal leverage
- Firms can raise equity and issue more debt
- Firms can default
Log aggregate consumption growth $g_{c,t+1}$ follows

$$g_{c,t+1} = \mu_{c,t} + \sigma_{c,t} \varepsilon_{c,t+1}$$

Drift and volatility of consumption growth depend on the aggregate Markov state $\xi_t$. 

Epstein-Zin pricing kernel is

$$M_{t,t+1} = \beta \theta (\lambda_{c,t} + 1 - \lambda_{c,t}) - (1 - \theta) e^{-\gamma g_{c,t+1}}$$

$\lambda_{c,t}$ is the wealth-consumption ratio.
Pricing Kernel

- Log aggregate consumption growth $g_{c,t+1}$ follows
  
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- Drift and volatility of consumption growth depend on the aggregate Markov state $\xi_t$.

- Epstein-Zin pricing kernel is
  
  $$M_{t,t+1} = \beta^\theta \left( \frac{\lambda_{t+1}^c + 1}{\lambda_t^c} \right)^{-(1-\theta)} e^{-\gamma g_{c,t+1}}$$

- $\lambda_t^c$ is the wealth-consumption ratio.
Log earnings growth $g_{i,t+1}$ follows

$$g_{i,t+1} = \mu_t + \sigma_t \varepsilon_{t+1} + \zeta \nu_{i,t+1}$$

Drift and volatility of earnings growth depend on the aggregate Markov state $\xi_t$.

Systematic $\varepsilon_{t+1}$ and idiosyncratic $\nu_{i,t+1}$ Gaussian shocks.
Log earnings growth $g_{i,t+1}$ follows

$$g_{i,t+1} = \mu_t + \sigma_t \varepsilon_{t+1} + \zeta \nu_{i,t+1}$$

Drift and volatility of earnings growth depend on the aggregate Markov state $\xi_t$.

Systematic $\varepsilon_{t+1}$ and idiosyncratic $\nu_{i,t+1}$ Gaussian shocks.

Corporate income tax rate is $\eta$.

After-tax asset value is

$$A_{i,t} = (1 - \eta) E_i, t + E_t[M_{t,t+1} A_{i,t+1}] = e^{g_{i,t+1}} E_i, t$$
Debt Value

- Firms can issue perpetual debt to take advantage of the tax benefits of debt financing.

- The interest coverage ratio is defined as
  \[ \kappa_{i,t} = \frac{E_{i,t}}{c_{i,s}} \]
Debt Value

- Firms can issue perpetual debt to take advantage of the tax benefits of debt financing.

- The interest coverage ratio is defined as
  \[ \kappa_{i,t} = \frac{E_{i,t}}{c_{i,s}} \]

- The debt value is given by
  \[
  D_{i,t} = 1_{\{\kappa_{i,t} \leq \kappa^D_t\}} (1 - \omega_t) A_{i,t} \\
  + 1_{\{\kappa^D_t < \kappa_{i,t} < \kappa^I_t\}} \left( c_{i,s} + \mathbb{E}_t[M_{t,t+1}D_{i,t+1}] \right) \\
  + 1_{\{\kappa^I_t \leq \kappa_{i,t}\}} \left( c_{i,s} + \frac{c_{i,s}}{c_{i,t}} \mathbb{E}_t[M_{t,t+1}D_{i,t+1}] \right)
  \]

- Bankruptcy costs vary with the aggregate economy
  \[
  \omega_t = \frac{\bar{\omega}}{1 + e^{a+b\mu_{c,t}/\sigma_{c,t}}}
  \]
Equity Value

- Equity holders decide about the optimal timing of default by maximizing the equity value

\[
S_{i,t} = \max \left\{ 0, 1_{\kappa_{i,t} < \kappa^I_t} \right\} \left( (1 - \eta)(E_{i,t} - c_{i,s}) + \psi_e (E_{i,t} - c_{i,s}) 1_{E_{i,t} < c_{i,s}} + \mathbb{E}_t [M_{t+1} S_{i,t+1}] \right) \\
+ 1_{\kappa^I_t \leq \kappa_{i,t}} \left( (1 - \eta)(E_{i,t} - c_{i,s}) + \Delta_{i,t} + \mathbb{E}_t [M_{t+1} S_{i,t+1}] \right)
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Equity Value

- Equity holders decide about the optimal timing of default by maximizing the equity value

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S_{i,t} = \max \left\{ 0, 1_{\kappa_{i,t} < \kappa^I_t} \right\} \left( (1 - \eta)(E_{i,t} - c_{i,s}) + \psi_e (E_{i,t} - c_{i,s}) 1_{E_{i,t} < c_{i,s}} + \mathbb{E}_t [M_{t,t+1} S_{i,t+1}] \right) \\
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\]

- Debt issuances proceeds \( \Delta_{i,t} \) are net of debt issuance costs \( \psi_d \).

- Firms face equity issuance costs \( \psi_e \).

- The optimal state depend default threshold satisfies

\[
\kappa^D(\xi_t) = \max \{ \kappa_{i,t} : S(\kappa_{i,t}, \xi_t) \leq 0 \}
\]
Levered Firm Value

- Levered firm value is the sum of the value of debt and equity.

- Management chooses the optimal issuance threshold $\kappa^I_t$ and the optimal coverage ratio $\bar{\kappa}_t$ to maximize levered firm value

$$F_{i,t} = 1_{\{\kappa_{i,t} \leq \kappa^D_t\}}(1 - \omega_t)A_{i,t} + 1_{\{\kappa^D_t < \kappa_{i,t} < \kappa^I_t\}}(1 - \eta)E_{i,t} + \eta c_{i,s} + \psi_e(E_{i,t} - c_{i,s})1_{\{E_{i,t} < c_{i,s}\}} + \mathbb{E}_t[M_{t,t+1}F_{i,t+1}]$$

$$+ 1_{\{\kappa^I_t \leq \kappa_{i,t}\}}((1 - \eta)E_{i,t} + \eta c_{i,s} - \psi_d D^e_{i,t} + \mathbb{E}_t[M_{t,t+1}F_{i,t+1}])$$
Simulation: Debt Issuance

**Kappa**

**Earnings & Coupon**

**Leverage**

**Debt and Equity Value**
Simulation: Default
Firm $i$ defaults at time $\tau_i$ when its interest coverage ratio $\kappa_{i,t}$ drops below the default threshold $\kappa^D_t$ such that

$$\tau_i = \inf\{t : \kappa_{i,t} \leq \kappa^D_t\}$$
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$$\tau_i = \inf \{ t : \kappa_{i,t} \leq \kappa^D_t \}$$

PV of issuance seller cash-flow

$$\sum_{h=1}^{T} \mathbb{E}_t \left[ M_{t+h} 1_{\{\tau_i=t+h\}} x_{i,t+h,s} \right]$$

$$x_{i,t+h,s} = 1 - \frac{(1 - \omega_{t+h})A_{i,t+h}}{D_{i,s}}$$
Credit Default Swaps

- Firm $i$ defaults at time $\tau_i$ when its interest coverage ratio $\kappa_{i,t}$ drops below the default threshold $\kappa_t^D$ such that

  $$\tau_i = \inf\{t : \kappa_{i,t} \leq \kappa_t^D\}$$

- PV of issuance seller cash-flow

  $$\sum_{h=1}^{T} \mathbb{E}_t \left[ M_{t,t+h} 1\{\tau_i = t+h\} x_{i,t+h,s} \right] \quad x_{i,t+h,s} = 1 - \frac{(1 - \omega_{t+h}) A_{i,t+h}}{D_{i,s}}$$

- PV of issuance buyer cash-flow

  $$\tilde{z}_{i,s,t}^{T} \sum_{h=1}^{T} \mathbb{E}_t \left[ M_{t,t+h} (1 - 1\{\tau_i \leq t+h\}) \right]$$
The log one-period CDS rates can be approximated by

$$\ln(z_{i,s,t}^1) \approx \ln(q_{i,t}^1) + \ln(L_{i,t,s}^Q)$$

where

$$q_{i,t}^1 = \mathbb{E}_t^Q \left[ 1\{\tau_i = t+1 \} \right] \quad L_{i,t,s}^Q = \mathbb{E}_t^Q \left[ x_{i,t+1,s} \mid \tau_i = t + 1 \right]$$

is the risk-neutral one-period default probability and the risk-neutral loss rate given default.
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\ln(z_{i,s,t}^1) \approx \ln(q_{i,t}^1) + \ln(L_{i,t,s}^Q)
\]

where

\[
q_{i,t}^1 = \mathbb{E}_t^Q \left[ 1\{\tau_i = t+1\} \right] \quad L_{i,t,s}^Q = \mathbb{E}_t^Q \left[ x_{i,t+1,s} | \tau_i = t + 1 \right]
\]

is the risk-neutral one-period default probability and the risk-neutral loss rate given default.

The variance of the log-linearized one-period CDS rate is

\[
\text{Var}(\ln z_{i,s,t}^1) = \text{Var}(\ln q_{i,t}^1) + \text{Var}(\ln L_{i,t,s}^Q) + 2\text{Cov}(\ln q_{i,t}^1, \ln L_{i,t,s}^Q)
\]
The value of a European put option with maturity $T$ and strike price $X$ is given by

$$P_{i,t} = \mathbb{E}_t[M_{t,T} \max\{X - S_{i,T}, 0\}]$$

Using the Black-Scholes model, we solve for implied volatilities.

Option prices are not sensitive to loss rates because equity holders recover nothing in the case of default.

Equity options are compound options.
Default Probabilities

**Expansion** ($\mu^H, \sigma^L$)

**Recession** ($\mu^L, \sigma^H$)

![Graphs showing default probabilities and leverage ratios in expansion and recession scenarios.](image-url)
Option Moments

- Expansion ($\mu^H, \sigma^L$)
- Recession ($\mu^L, \sigma^H$)
Data

- Credit Market Analysis (CMA)
  - Monthly data from 2004 to 2014
  - S&P 100 constituents
  - 5-year tenor, senior debt, dollar denominated, XR or MR

- OptionMetrics
  - Monthly data from 2004 to 2014
  - S&P 100 constituents
  - IV surface adjusted for early exercise

- CRSP-Compustat
  - Debt: DLCQ + DLTTQ
  - Earnings: OIBDPQ
  - Monthly returns and market capitalization

- BEA NIPA
  - Monthly real non-durable and service consumption growth
### Consumption Dynamics

#### Consumption States

<table>
<thead>
<tr>
<th>$\mu_{c,h}$</th>
<th>$\mu_{c,l}$</th>
<th>$\mu_{c,d}$</th>
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<tr>
<td>0.2935</td>
<td>0.0932</td>
<td>-0.6180</td>
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<table>
<thead>
<tr>
<th>$\sigma_{c,l}$</th>
<th>$\sigma_{c,h}$</th>
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<tbody>
<tr>
<td>0.1855</td>
<td>0.4211</td>
<td>0.8422</td>
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#### Transition Matrix

$$
\begin{pmatrix}
(\mu_h, \sigma_l) & (\mu_l, \sigma_l) & (\mu_h, \sigma_h) & (\mu_l, \sigma_h) & (\mu_d, \sigma_d) \\
0.9912 & 0.0029 & 0.0059 & 0.0000 & 0 \\
0.0223 & 0.9718 & 0.0001 & 0.0058 & 0 \\
0.0061 & 0.0000 & 0.9910 & 0.0029 & 0 \\
0.0001 & 0.0060 & 0.0223 & 0.9567 & 0.0149 \\
0 & 0 & 0 & 0.0225 & 0.9775
\end{pmatrix}
$$
## Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>EIS</td>
<td>$\psi$ 2</td>
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<tr>
<td>Time discount rate</td>
<td>$\beta$ 0.996</td>
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<tr>
<td>Consumption-earnings correlation</td>
<td>$\rho$ 0.1</td>
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<tr>
<td>Drift scaling</td>
<td>$\phi_{\mu}$ 2</td>
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<td>Bankruptcy costs maximum</td>
<td>$\bar{\omega}$ 0.6</td>
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<tr>
<td>Debt issuance costs</td>
<td>$\psi_d$ 0.005</td>
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<tr>
<td>Equity issuance costs</td>
<td>$\psi_e$ 0.1</td>
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## Estimated Parameters

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<tr>
<th>Parameter</th>
<th>( \gamma )</th>
<th>( \phi_{\sigma} )</th>
<th>( \zeta )</th>
<th>( \tau )</th>
<th>( a )</th>
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<td>12.65</td>
<td>0.05</td>
<td>0.22</td>
<td>-4.84</td>
<td>0.83</td>
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<td>Aggregate volatility scaling</td>
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<td>6.74</td>
<td>0.07</td>
<td>0.22</td>
<td>-5.91</td>
<td>6.33</td>
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<td>Idiosyncratic volatility</td>
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<td>12.65</td>
<td>0.05</td>
<td>0.22</td>
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## SMM Moments

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<th></th>
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<tbody>
<tr>
<td>Average leverage</td>
<td>25.46</td>
<td>25.38</td>
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</tr>
<tr>
<td>Average excess returns</td>
<td>0.47</td>
<td></td>
<td>0.78</td>
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<tr>
<td>Average 1-year CDS</td>
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<td>Average 5-year CDS</td>
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<tr>
<td>S.D. of leverage</td>
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<td>2.58</td>
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<tr>
<td>S.D. of returns</td>
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<td></td>
<td>3.40</td>
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<td>Average 5-year CDS</td>
<td>0.80</td>
<td>0.72</td>
<td>0.84</td>
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## CDS Decomposition

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<tbody>
<tr>
<td>Average bankruptcy costs</td>
<td>58.83</td>
<td>29.58</td>
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<tr>
<td>S.D. of bankruptcy costs</td>
<td>0.49</td>
<td>25.23</td>
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<tr>
<td>Average LGD under $P$</td>
<td>95.92</td>
<td>97.76</td>
</tr>
<tr>
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<td>95.97</td>
<td>97.79</td>
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<tr>
<td>Average 5-year def. probability under $P$</td>
<td>0.54</td>
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<tr>
<td>Average 5-year def. probability under $Q$</td>
<td>3.60</td>
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<tr>
<td>S.D. of 5-year def. probability under $P$</td>
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<td>2.16</td>
<td>1.82</td>
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Conclusion

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- Use joint information of CDS rates and implied volatilities

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