Can the Market Multiply and Divide?
Non-Proportional Thinking in Financial Markets

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Proportional thinking in financial markets

In financial markets, rational investors should react to news about firm value in terms of proportional price changes, i.e. ***returns***

Market value of the firm:

Size ≡ number of shares × share price

Holding firm size constant, nominal price of a financial security has no real meaning

- Price depends on the number of shares, and can easily be changed through splits or reverse splits

But changes in the value of stocks in response to news are frequently reported and discussed in dollar units…

Wall Street Journal (1970s)

Tuesday's Volume, 16,056, Shares

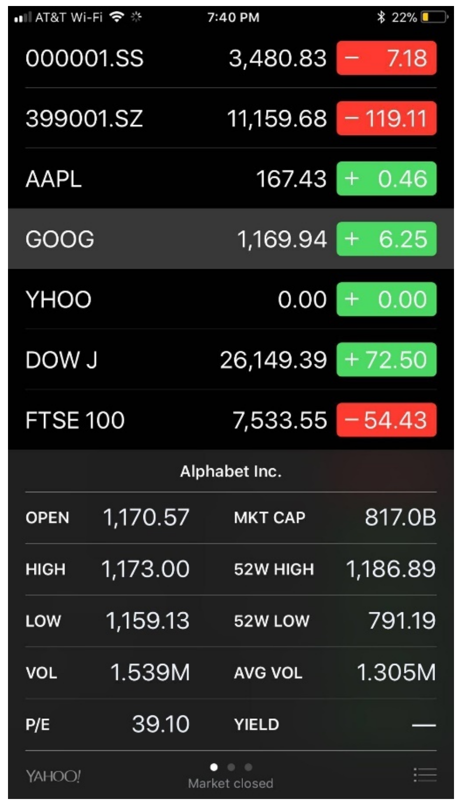
Volume since Jan. 1:	1970	1969	1968
Total sales	258,654,878	291,760,651	301,114,208

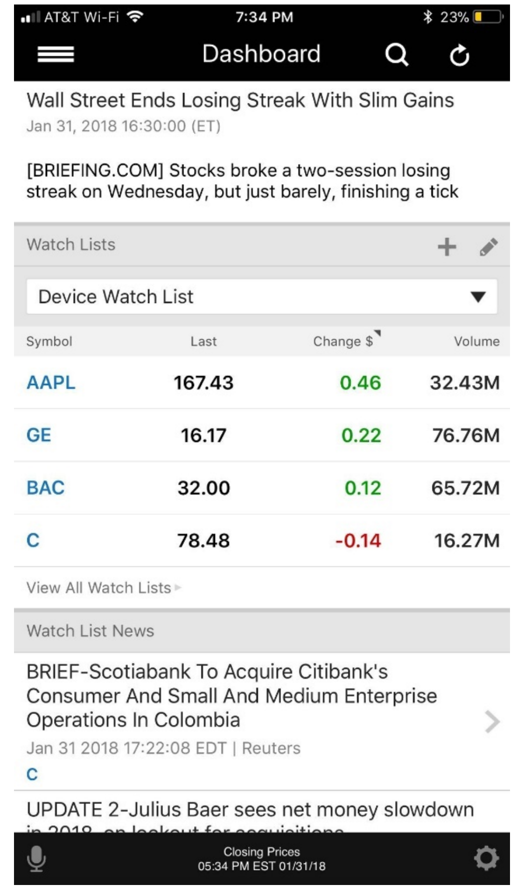
MOST ACTIVE STOCKS

	Open	High	Low	Close	Chg.	Volume
Chrysler	24¼	25⅝	24¼	25½	+1¼	335,100
Cont Data	71¼	71¼	63½	67⅛	−4½	277,500
CNA Finl	18⅜	18⅜	17	18⅛	− ¼	230,500
Texaco	25⅜	25⅞	24¾	25¾	+ ½	67,700
Telex Corp	134¼	135¾	126½	134⅜	−1¾	61,400
Am Airlin	23⅜	23⅞	21	22½	−1½	50,300
Itek Corp	79½	82	76½	79¾	−2⅛	42,500
Mid So Util	21	21	20	20	− ¾	33,100
Unvsty Cmp	56⅞	61⅞	53¼	59½	+1⅜	32,500
Teledyne	28¼	28½	20	27½	− ¾	30,700

Average closing price of most active stocks: 48.01.

Android, Apple, and Etrade apps (2010s)





CNBC (2010s)


SQUAWK BOX
THE PRIVATE EQUITY LANDSCAPE
S&P FUT (Jun) -3.25
S&P FV 2.83
S&P CLOSE 2,047.63
ap Bear 3X (TZA) 43.33 ▲ 0.39 iPath S&P VIX (VXX) 15.82 ▲ 0.24 Direxi
MGT Capital Invest (MGT) 2.82 ▲ 0.20 MGT Capital Invest (MGT) 2.81 ▲
CNBC
8:10A EASTERN

Our hypothesis

Non-proportional thinking: Investors think that news should correspond to a dollar change in price rather than a percentage change

Consider two otherwise identical stocks, one trading at $20/share, another at $30/share

- Investors may think the same piece of news should correspond to a $1 increase in price for both stocks

$1/$20 = 5% **$1/$30 = 3.3%**

→ Absolute magnitude of return reactions to the same news will be larger for lower priced stocks

Volatility predictions

Measures of a stock's volatility:

1. Total volatility: standard deviation of daily returns
2. Idiosyncratic volatility: standard deviation of returns in excess of market returns
3. Market beta: scaled covariance between the stock's return and the market return

Stronger return reactions to news for lower-priced stocks → These stocks will have greater total volatility, idiosyncratic volatility, and market beta

Preview of results

A doubling in share price corresponds to 20-30% decline in volatility

- ***Not driven by size***—rather, the size-volatility relation flattens by 80% after controlling for price
- To identify a causal effect of price, we show that volatility jumps after stock splits and drops after reverse splits

Lower-priced stocks have stronger return responses to news events

Price can explain overreaction, underreaction, reversals, and drift

Not driven by tick-size limitations, volume, liquidity, or catering / changes to a speculative investor base

Implications

A new explanation of under and overreaction to news

- Complements other behavioral explanations which focus on limited attention, biased beliefs about persistence

Offers insight into the determinants of volatility and drift

- Size doesn't matter: asset pricing facts such as "small stocks have higher volatility and market beta" are mostly driven by price
- Potential explanation for the "**leverage effect**" puzzle in which volatility is negatively related to past returns
- Reversals and predictability

N**ew explanation** of some **known asset pricing facts/puzzles**, e.g. Black (1976), Ohlson and Penman (1985)

Baseline volatility regression

$$log(vol_{it}) = \beta_0 + \beta_1 log\left(price_{i,t-1}\right) + controls + \tau_t + \varepsilon_{it}$$

- Stock i in year-month t
- vol_{it}: **total volatility**, **idiosyncratic volatility** or **absolute market beta**
- $controls$ can include size (linear control or 20 size categories), sales volatility, volume, bid-ask spread, institutional ownership, market-to-book, leverage, past returns, firm FE
- Standard errors double-clustered by stock and year-month

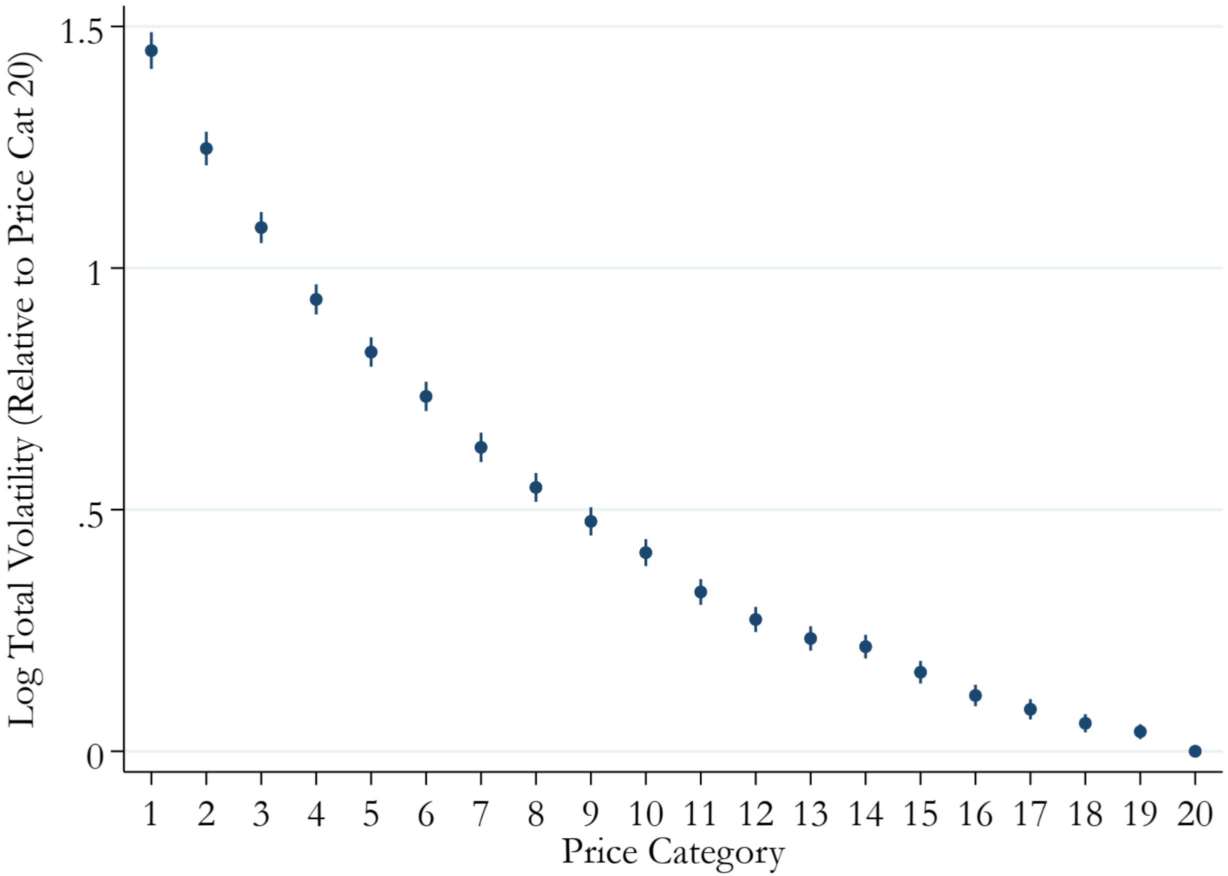
- **Non-proportional thinking predicts $\boldsymbol{\beta_1 < 0}$**

Baseline volatility results

	Log(Total Volatility)				Log(IVol)	Log(|Beta|)
	(1)	(2)	(3)	(4)	(5)	(6)
Log(Lagged Price)	-0.326***		-0.332***	-0.339***	-0.346***	-0.319***
	(0.00339)		(0.00446)	(0.00405)	(0.00399)	(0.00465)
Log(Lagged Size)		-0.146***	0.00431			
		(0.00235)	(0.00311)			
Year–Month FE	Yes	Yes	Yes	Yes	Yes	Yes
Size Category FE	No	No	No	Yes	Yes	Yes
R-squared	0.442	0.328	0.442	0.445	0.473	0.103
Observations	3,254,302	3,254,302	3,254,302	3,254,302	3,254,302	3,254,302

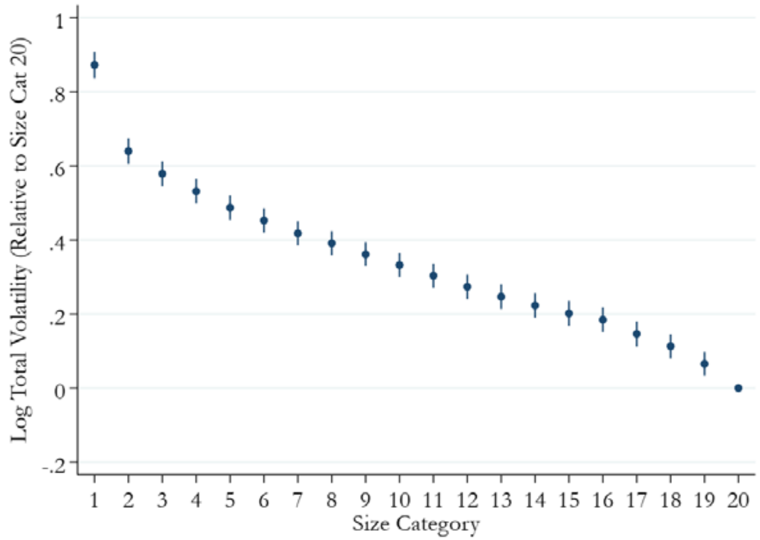
- Doubling price corresponds to a $>30\%$
- Holds after controlling flexibly for size, yet the size-volatility relation becomes insignificant once we control for price

Volatility-price relation

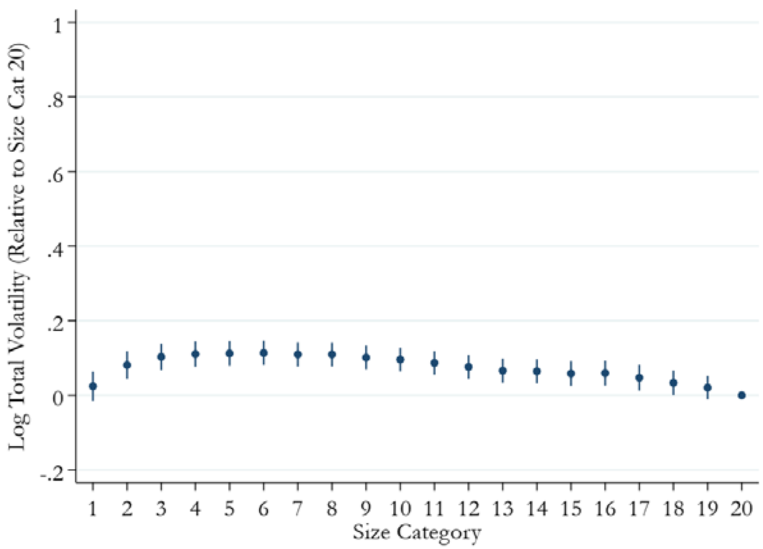


- Plots volatility against 20 price categories (omitted category 20), controlling for size categories and time
- Shows that negative relation is not driven only by low-priced stocks that are subject to tick-size distortions

Price can explain the size-volatility relation

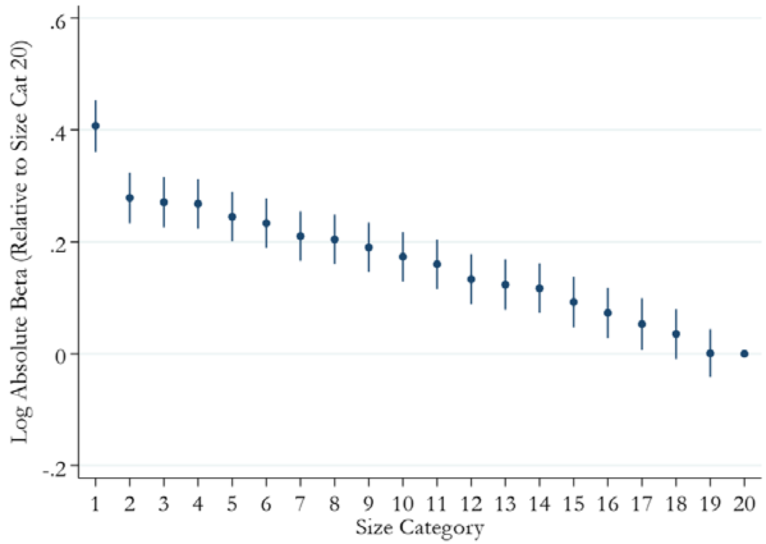


- Without controlling for $log(price_{i,t-1})$

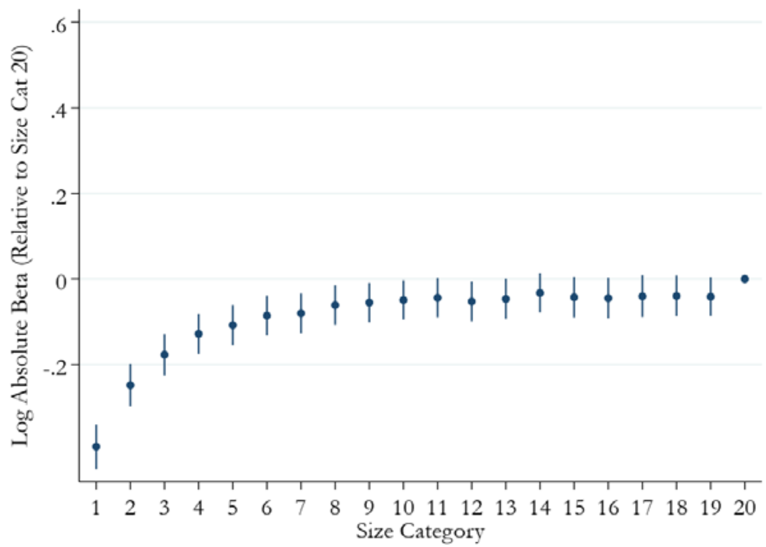


- Controlling for $log(price_{i,t-1})$
- Size-volatility relation flattens by $\sim 80\%$

Price can explain the size-beta relation



- Without controlling for $log(price_{i,t-1})$



- Controlling for $log(price_{i,t-1})$
- Size-beta relation flattens by $\sim 80\%$

The leverage effect puzzle

Puzzle: Past returns are strongly negatively related to volatility

Several potential explanations, including…

- *Leverage Effect*: Holding the debt level constant, if asset value falls, then the equity becomes more levered and consequently more risky

Non-proportional thinking

- Negative returns imply a decline in price
- Reacting to news in nominal units leads to higher volatility for lower-priced stocks

The leverage effect puzzle

	Log(Total Volatility)			
	(1)	(2)	(3)	(4)
Log(Lagged Price)	-0.339***		-0.332***	-0.331***
	(0.00412)		(0.00465)	(0.00466)
Log(Past 12-Month Return)		-0.240***	-0.0309***	
		(0.0101)	(0.00902)	
Year–Month FE	Yes	Yes	Yes	Yes
Size Category FE	Yes	Yes	Yes	Yes
Past 12 Monthly Returns	No	No	No	Yes
R-squared	0.458	0.346	0.458	0.459
Observations	2,966,196	2,966,196	2,966,196	2,966,196

Heterogeneity by size

	Log(Total Volatility)									
	(1) Size1	(2) Size2	(3) Size3	(4) Size4	(5) Size5	(6) Size6	(7) Size7	(8) Size8	(9) Size9	(10) Size10
Log(Lagged Price)	-0.363***	-0.386***	-0.370***	-0.364***	-0.346***	-0.325***	-0.310***	-0.298***	-0.281***	-0.270***
	(0.00562)	(0.00659)	(0.00746)	(0.00771)	(0.00800)	(0.00816)	(0.00870)	(0.00870)	(0.00949)	(0.00996)
Year–Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.393	0.368	0.363	0.362	0.354	0.349	0.347	0.351	0.343	0.350
Observations	1,111,769	333,979	226,006	173,581	146,796	128,577	117,145	107,699	99,812	92,354

	Log(Total Volatility)									
	(1) Size11	(2) Size12	(3) Size13	(4) Size14	(5) Size15	(6) Size16	(7) Size17	(8) Size18	(9) Size19	(10) Size20
Log(Lagged Price)	-0.248***	-0.227***	-0.207***	-0.201***	-0.184***	-0.168***	-0.166***	-0.124***	-0.123***	-0.139***
	(0.00969)	(0.0103)	(0.0118)	(0.0123)	(0.0135)	(0.0139)	(0.0179)	(0.0165)	(0.0206)	(0.0215)
Year–Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.355	0.359	0.351	0.357	0.359	0.388	0.403	0.412	0.450	0.526
Observations	85,267	81,440	78,053	75,213	72,894	70,567	67,662	65,314	62,743	57,431

Magnitude of volatility-price relation declines with size, but still strong for the largest 5% of stocks

Heterogeneity by decade

	Log(Total Volatility)									
	(1) 1920s	(2) 1930s	(3) 1940s	(4) 1950s	(5) 1960s	(6) 1970s	(7) 1980s	(8) 1990s	(9) 2000s	(10) 2010s
Log(Lagged Price)	-0.227*** (0.0140)	-0.275*** (0.0108)	-0.350*** (0.00989)	-0.191*** (0.0147)	-0.324*** (0.0126)	-0.462*** (0.00862)	-0.317*** (0.00646)	-0.369*** (0.00768)	-0.353*** (0.00995)	-0.251*** (0.00554)
Year–Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Size Category FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.560	0.652	0.609	0.289	0.396	0.353	0.315	0.440	0.449	0.356
Observations	22,843	82,661	97,620	118,906	209,217	452,864	624,639	751,051	586,932	307,569

Some variation over time, but the relation is not only driven by the early sample period

Splits as an event study

Potential concern: poor performance leads to low price and high vol

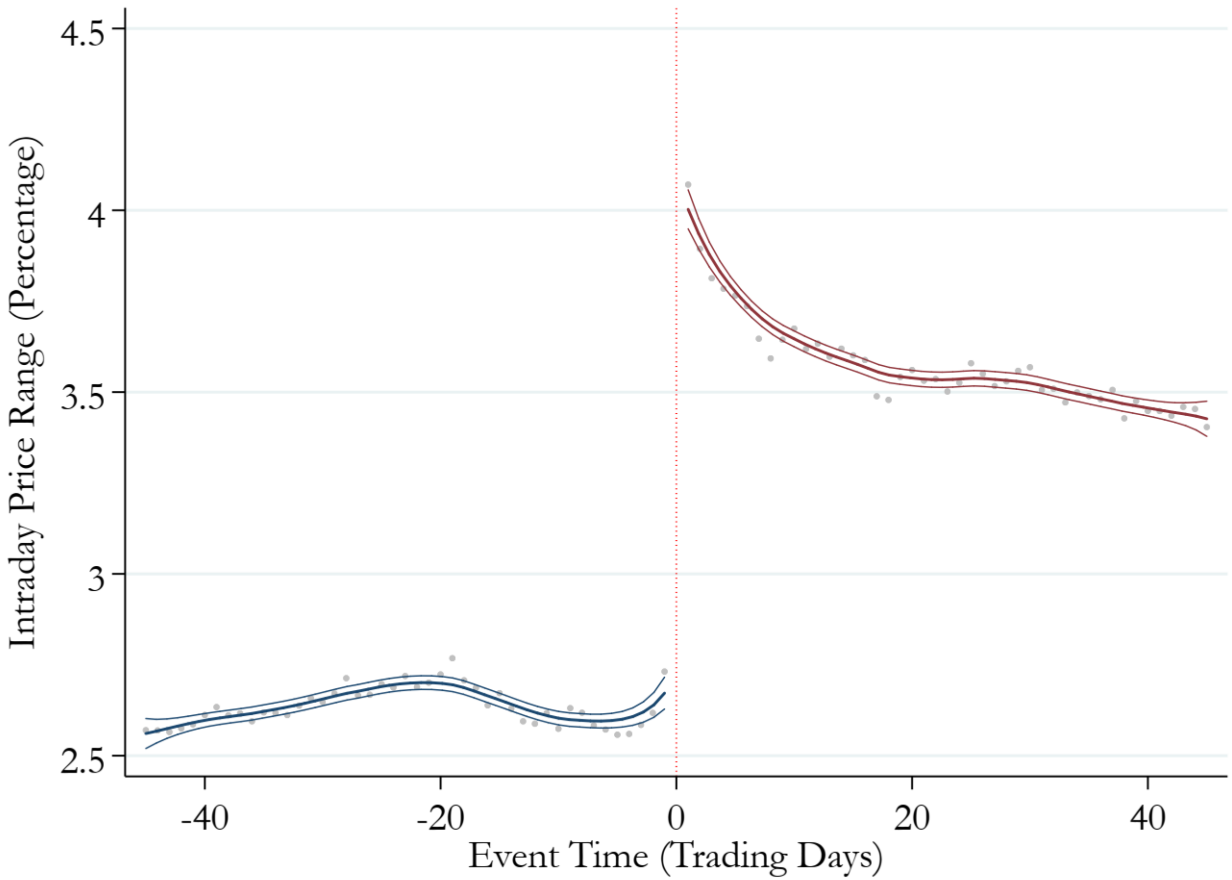
To identify a causal effect of price, we conduct a regression discontinuity around stock splits

- Following a standard 2-for-1 stock split, the price falls by half

Splits are not random (they tend to follow good performance), but splits are pre-scheduled and fundamentals are unlikely to change exactly on the split execution date

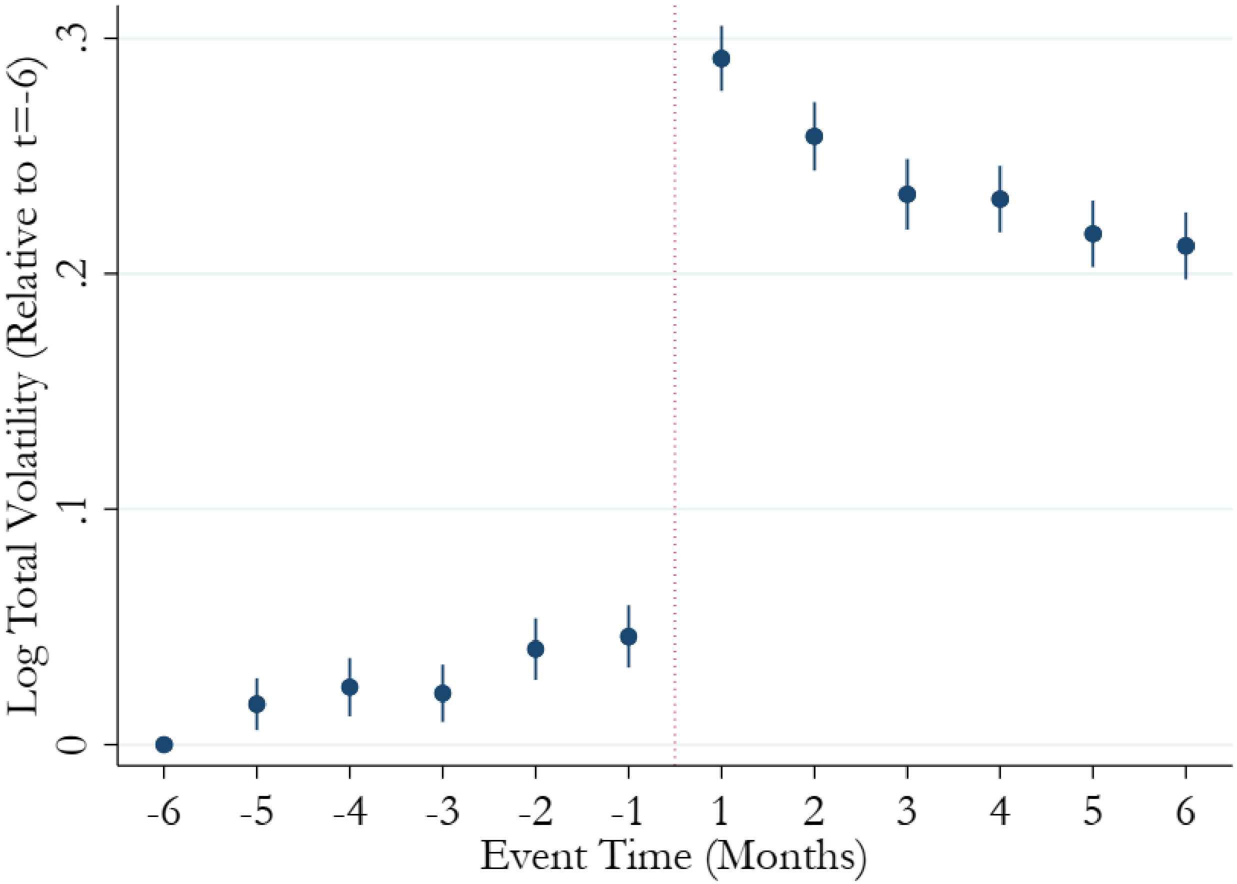
Expect the opposite patterns for reverse splits

Regression discontinuity around stock splits



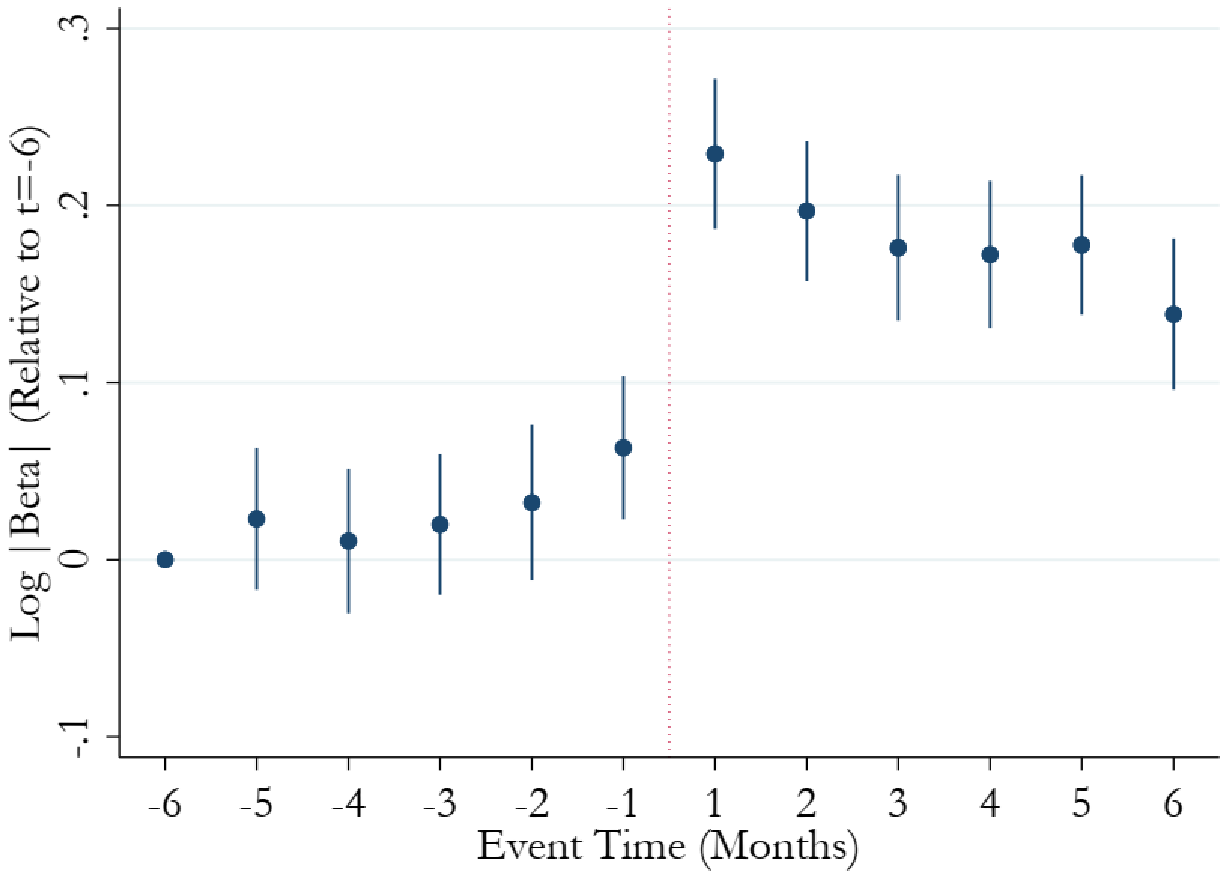
- Proxy for daily volatility using intraday price range percentage
- $> 30\%$ persistent increase in intraday price range after splits

Splits (total volatility)

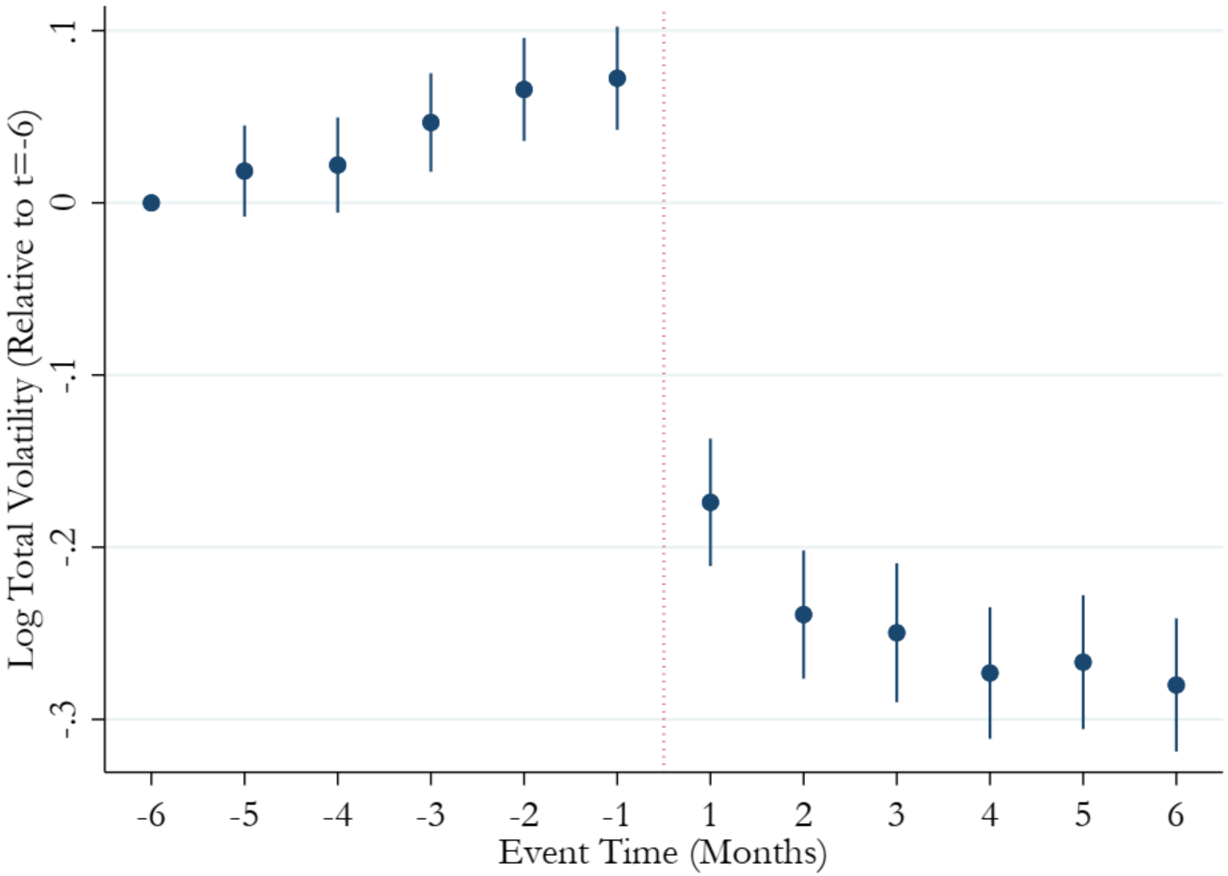


Regression coefficients on event months ($t = -6$ is omitted), controlling for stock and calendar year-month FE

Splits (market beta)



Reverse splits



Following reverse splits: prices↑, volatility↓

Remaining alternative explanations

Low prices may attract speculative investors who push up volatility

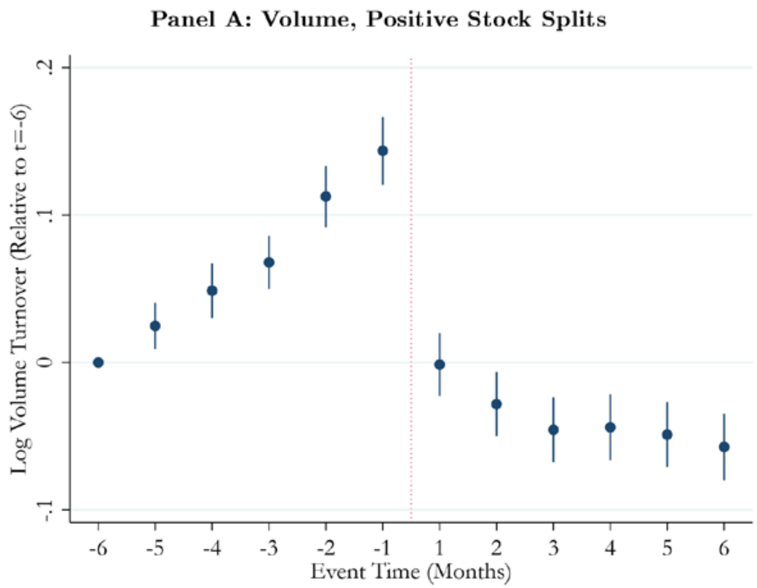
- Unlikely that investor base changes in a single day after split
- Not obvious that speculative investors would overreact to news, leading to higher betas and subsequent reversals

Firms may announce splits when they expect changes in firm strategy/performance, which could affect volatility

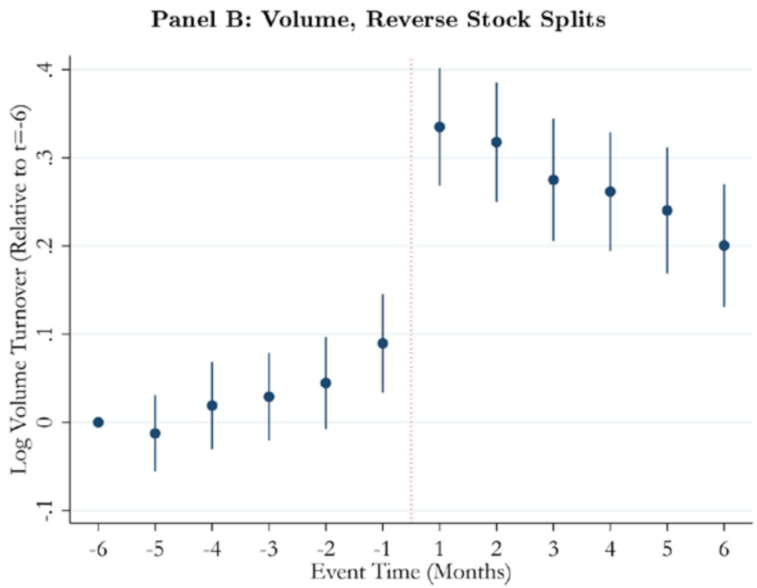
- However, splits are usually announced one month ahead

We can also examine these stories in more detail…

Volume after splits

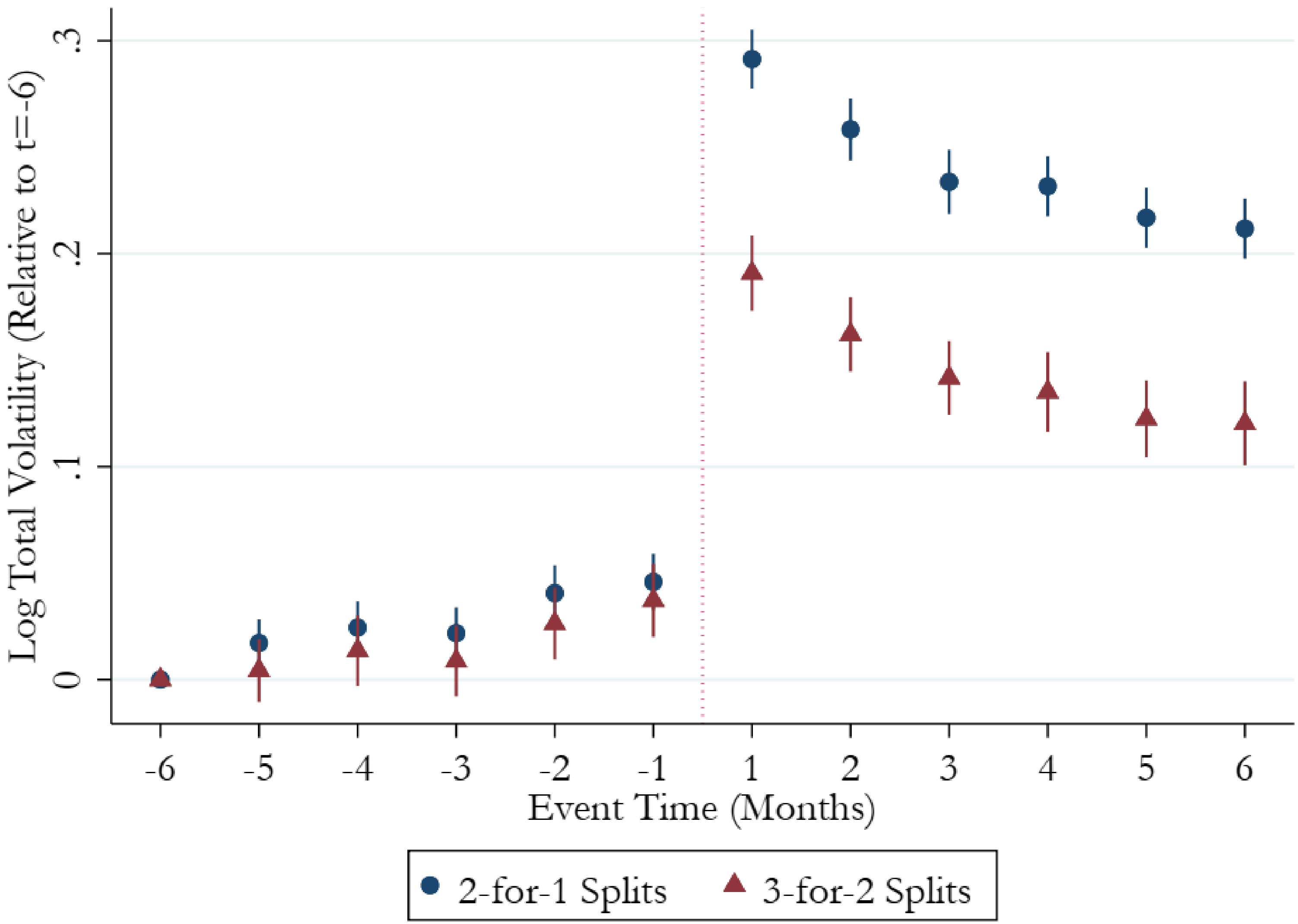


- Holding a stock’s market cap constant, increased speculation should lead to higher volume
- Instead, volume drops after splits, consistent with some investors trading fixed numbers of shares

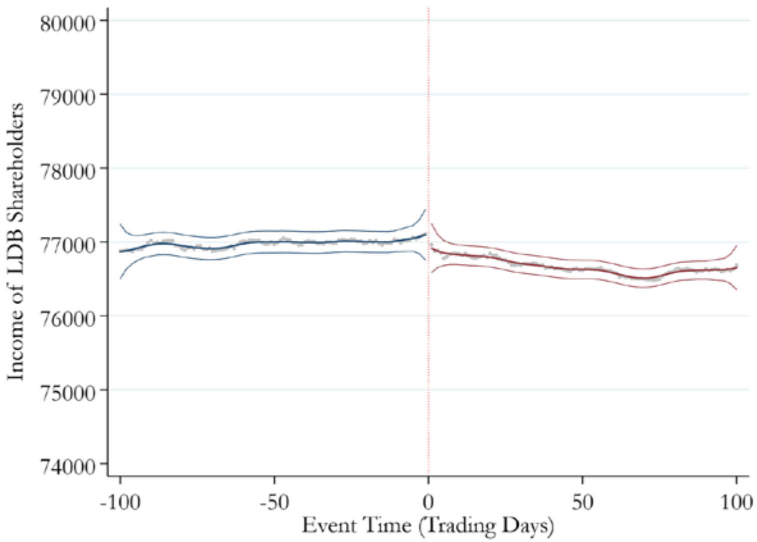


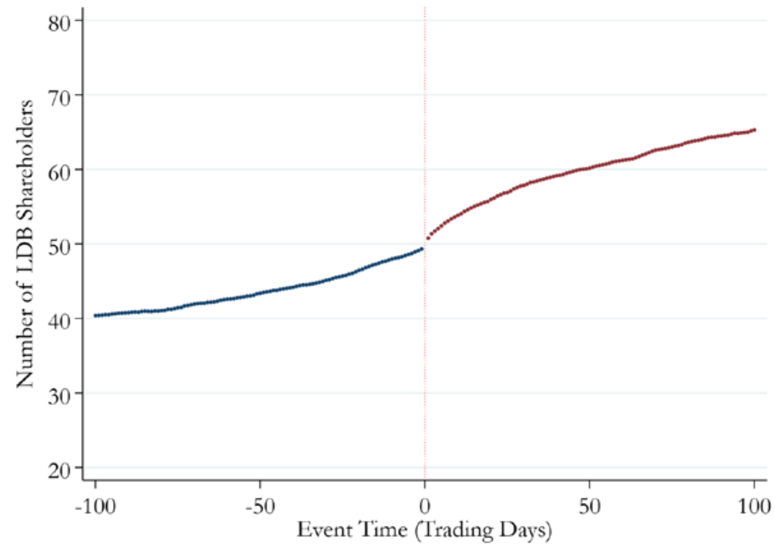
- Volume increases following reverse splits, consistent with some investors trading fixed numbers of shares

Effect size can be ranked by the type of split



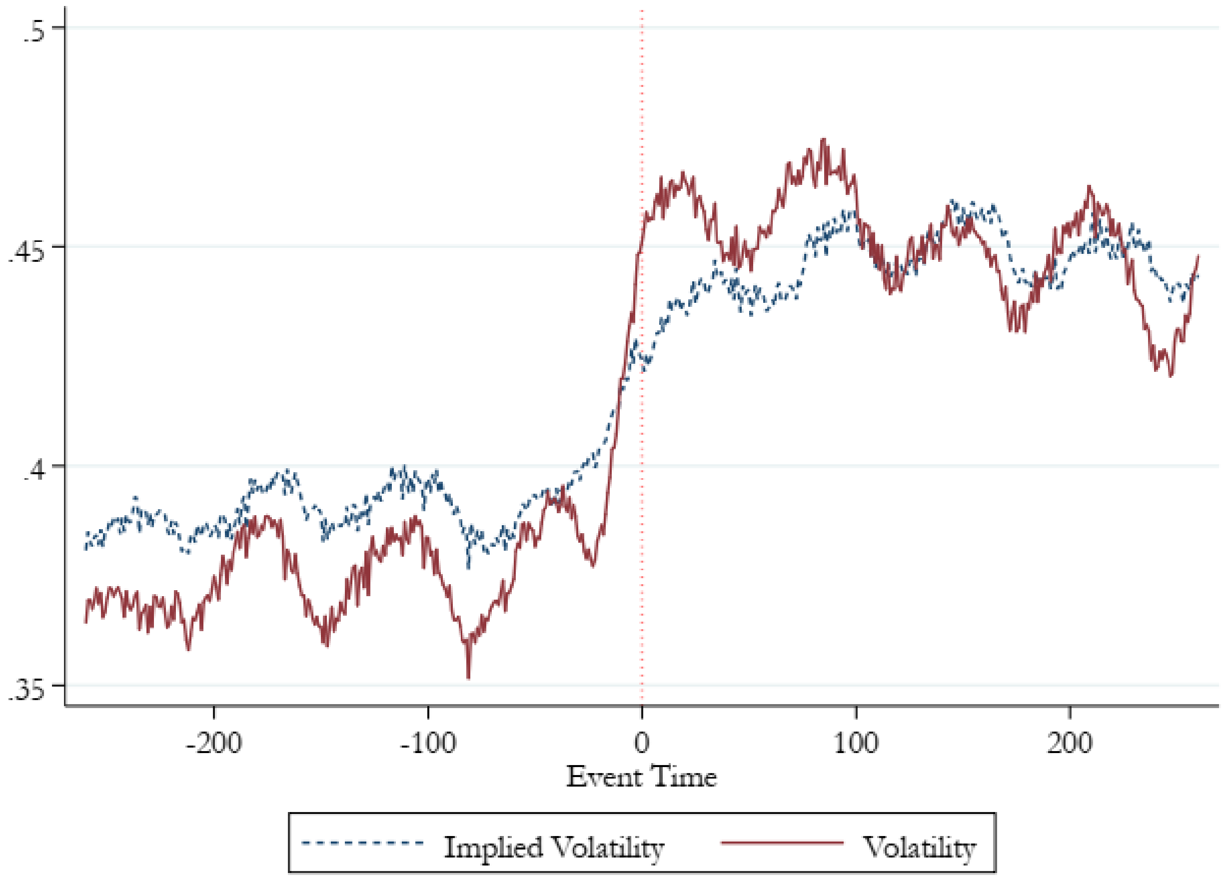
Small changes in retail investor base





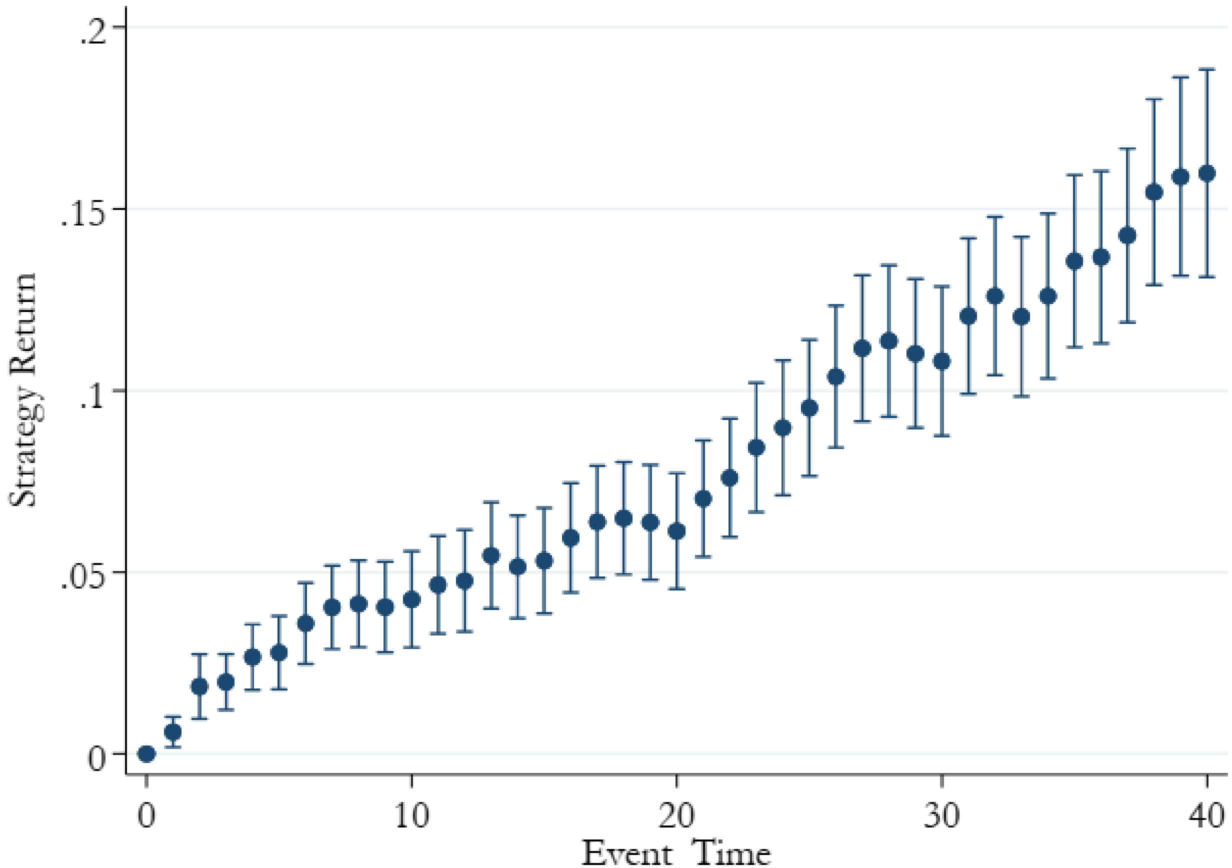
The 20-30% jump in volatility is ***too large*** to be explained by the small change in retail trading

Option implied vol and actual vol



Option traders under-estimate the increase in volatility after splits

Trading strategy: buy straddles on split date



Average 15 percent return within 40 days after split (does not account for transaction costs)

How do managers think about splits?

Are CEOs aware that splits lead to a 20-30% increase in volatility?

- Self-interested CEOs should love splits, because the jump in volatility would increase the value of executive stock options
- Other CEOs may avoid splits to keep volatility and beta low

Commonly-discussed reason for splits: Attract a broader investor base and increase liquidity

- In reality, a greater number of different investors trade the stock after a split, but volume turnover falls because some investors don’t realize they should double the number of shares traded

Responsiveness to news

So far, we've focused on volatility

Non-proportional thinking also predicts

- Stronger return reaction to news for lower-priced stocks

Caveat: We don't know whether stocks overreact or underreact in an absolute sense

- Investors could underreact for other reasons (e.g. limited attention), but underreact to a *lesser extent* for lower priced stocks due to non-proportional thinking

News identified from textual analysis

Textual analysis of firm-specific news, **S&P 500 firms in 2000s**

- Categorized news: value-relevant events, e.g. M&A, products

	Log(|CAR|)			
	Categorized News		Other News	
	(1)	(2)	(3)	(4)
Log(Lagged Price)	-0.299***	-0.217***	-0.279***	-0.208***
	(0.0161)	(0.0161)	(0.0156)	(0.0169)
Year–Month FE	Yes	Yes	Yes	Yes
Size Category FE	No	Yes	No	Yes
R-squared	0.105	0.115	0.106	0.111
Observations	377,454	377,454	375,123	375,123

Reactions to earnings news

Quarterly earnings are an important type of firm-specific news

	Log(|CAR|)			
	(1)	(2)	(3)	(4)
Log(Lagged Price)	-0.223***	-0.232***	-0.271***	-0.210***
	(0.00747)	(0.0102)	(0.00791)	(0.00944)
Year–Month FE	Yes	Yes	Yes	Yes
Size Category FE	No	Yes	No	Yes
Analyst Count FE	No	No	Yes	Yes
R-squared	0.067	0.071	0.072	0.082
Observations	339,736	339,736	339,736	339,736

A doubling in share price corresponds to a >20% reduction in the absolute return response in a 3-day window around earnings announcements

Reactions to earnings news

Advantage: can control for the actual earnings news

	Log(1+CAR)			
	All SUE Deciles		Extreme SUE Deciles	
	(1)	(2)	(3)	(4)
SUE Decile Rank	0.00994***	0.00995***		
	(0.000532)	(0.000533)		
SUE Decile Rank × Lagged Price Quintile 2	-0.000366	-0.000385		
	(0.000512)	(0.000514)		
SUE Decile Rank × Lagged Price Quintile 3	-0.00180***	-0.00180***		
	(0.000550)	(0.000551)		
SUE Decile Rank × Lagged Price Quintile 4	-0.00360***	-0.00359***		
	(0.000538)	(0.000539)		
SUE Decile Rank × Lagged Price Quintile 5	-0.00443***	-0.00442***		
	(0.000548)	(0.000549)		

Controlling for the actual earnings news, the return response is stronger for lower priced stocks

Reversals

Non-proportional thinking predicts

- Potential overreaction to news for low-priced stocks, followed by *eventual reversal*

Classic evidence of long run reversals: De Bondt and Thaler (1985)

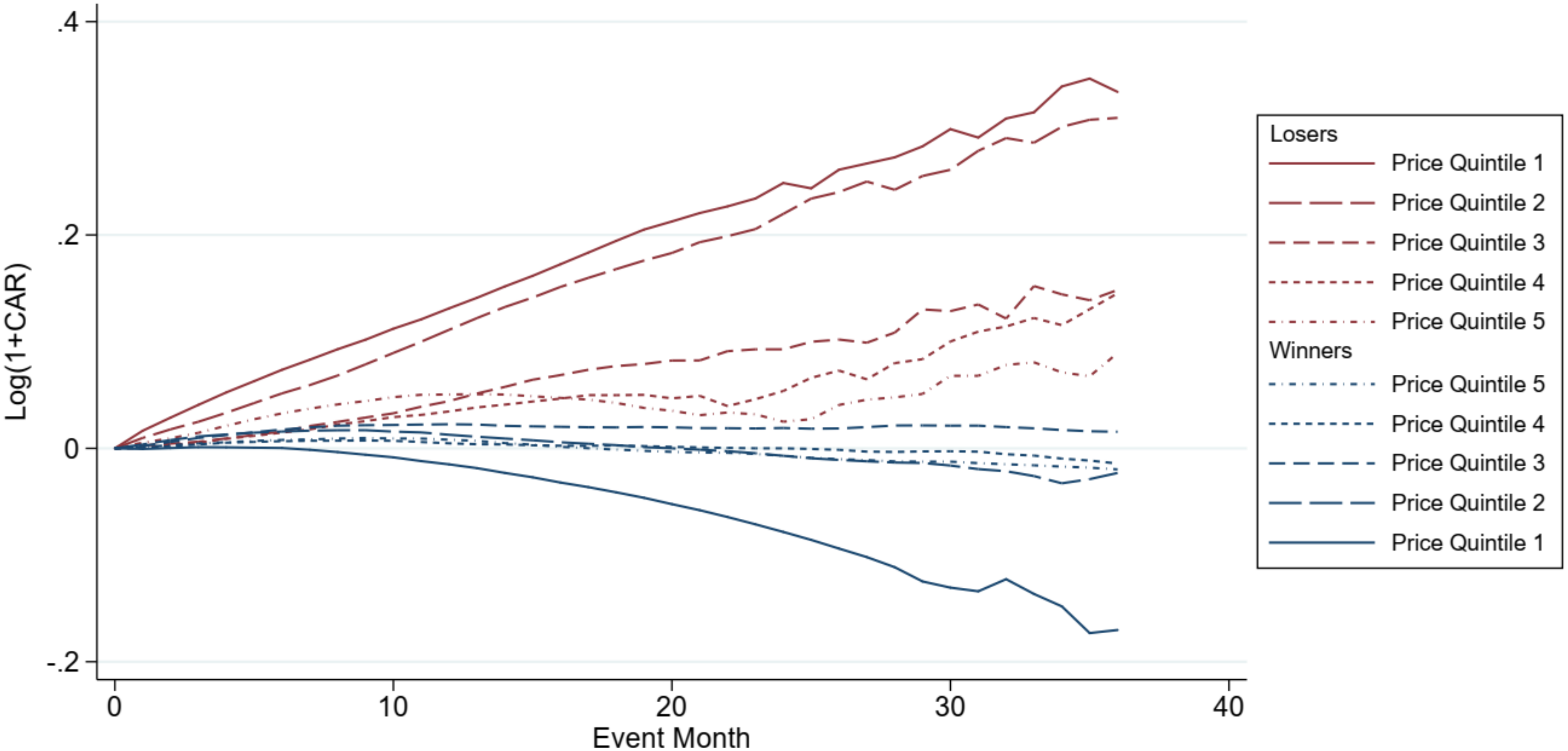
- Past winners underperform, past losers outperform

We can replicate De Bondt and Thaler (1985), and find

- The reversal is driven by low-priced stocks
- The magnitude of the reversal can be better sorted by price than size

Short run reversal (Jegadeesh 1990) is similarly better sorted by price than by size

Long run reversals: by price



- Overreaction to news for low-priced stocks and eventual reversal
- Underreaction to news for high-priced stocks and eventual drift

Reversals: sorting by **price** vs **size**

	Log(1 + 36 Month CAR)			Log(1 + 1 Month CAR)		
	(1)	(2)	(3)	(4)	(5)	(6)
Log(1 + Prev 36 Month CAR)	-0.154***	-0.186***	-0.158***			
	(0.00598)	(0.00982)	(0.00984)			
Log(1 + Prev 36 Month CAR) × Log(Price_{t-37})	0.0311***		0.0314***			
	(0.00183)		(0.00216)			
Log(1 + Prev 36 Month CAR) × Log(Size_{t-37})		0.0112***	0.000188			
		(0.00106)	(0.00124)			
Log(1 + Prev 1 Month CAR)				-0.124***	-0.205***	-0.152***
				(0.00579)	(0.0108)	(0.0110)
Log(1 + Prev 1 Month CAR) × Log(Price_{t-2})				0.0284***		0.0238***
				(0.00187)		(0.00226)
Log(1 + Prev 1 Month CAR) × Log(Size_{t-2})					0.0156***	0.00409***
					(0.00114)	(0.00136)
Fama-MacBeth	Yes	Yes	Yes	Yes	Yes	Yes
Avg R-squared	0.073	0.069	0.084	0.048	0.039	0.056
Observations	1,717,530	1,717,530	1,717,530	3,256,589	3,256,589	3,256,589
Time Periods	1,014	1,014	1,014	1,084	1,084	1,084

- Past returns negatively predict future returns
- The strength of this reversal varies more by price than by size

Relation to the proportional-thinking bias

Consumers are often willing to drive to another store to save $10 off a $20 calculator but not to save $10 off a $100 jacket

Our results suggest people think partly in dollars and partly in percents, leading to mistakes in settings in which:

- They should think entirely in dollars (consumer choice)
- They should think entirely in percents (financial markets)

Conclusion

Non-proportional thinking: Investors think that news should correspond to a dollar change in price rather than a percentage change in price

- Stronger return reactions to news for lower-priced stocks

Economic magnitudes are large

- NPT can explain a significant portion of the “leverage effect” puzzle as well as the volatility-size and beta-size relations in the data

Offers insight into the determinants of over- and underreaction, volatility, drift, and reversals

New view on the proportional-thinking bias

Practical takeaways

Many trading strategies bet on drift (momentum) or reversals

- Strategies are known to perform better for small-caps
- Downside of small caps: High transaction costs and price impact
- Our findings suggest that it would be more cost-effective to target low-priced stocks for reversals and high-priced stocks for momentum

Investors should be cautious of news reported in the wrong units

- Dollars vs. percents
- Real dollars vs. nominal dollars
- Index points vs. percents
- S&P 500 index does not include dividend payouts
- Dow is a price-weighted index
- Earnings per share surprise of $0.05 per share is typically bigger news for a $20 stock than $30 dollar stock